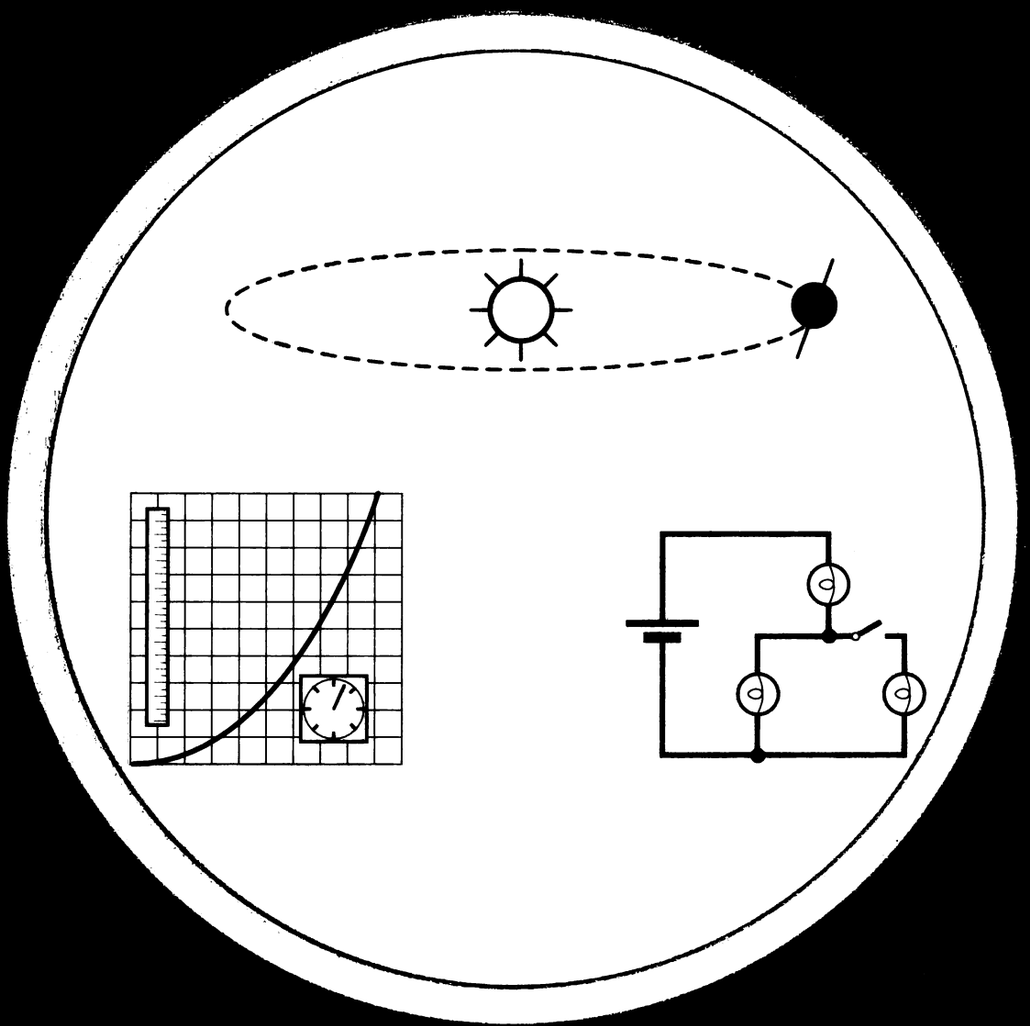
PHYSICS BY INQUIRY

Volume I



Lillian C. McDermott

and the

Physics Education Group at the University of Washington

PHYSICS BY INQUIRY

An introduction to physics and the physical sciences

Volume I

Lillian C. McDermott
Professor of Physics

with

Peter S. Shaffer and Mark L. Rosenquist

and the

Physics Education Group
University of Washington

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Preface

Physics by Inquiry is a set of laboratory-based modules that provide a step-by-step introduction to physics and the physical sciences. Through in-depth study of simple physical systems and their interactions, students gain direct experience with the process of science. Starting from their own observations, students develop basic physical concepts, use and interpret different forms of scientific representations, and construct explanatory models with predictive capability. All the modules have been explicitly designed to develop scientific reasoning skills and to provide practice in relating scientific concepts, representations, and models to real world phenomena.

Physics by Inquiry is not meant to be passively read. The modules do not provide all the information and reasoning included in a conventional text. There are gaps that must be bridged by the student. The process of science cannot be learned by reading, listening, memorizing, or problem-solving. Effective learning requires active mental engagement.

Physics by Inquiry contains narrative, experiments and exercises, and supplementary problems. As the course progresses, student notebooks become an important resource.

- Narrative

  The narrative is double-spaced. It includes statements of fact, definitions, and examples of the kind of reasoning that is expected of students.

- Experiments and exercises

  Experiments and exercises are inset from the narrative. They should be done as they are encountered.

- Supplementary problems

  A collection of problems at the end of each module provides additional practice in applying physical concepts and scientific reasoning skills.

- Student notebooks

  Students maintain notebooks in which they record observations, do exercises and problems, and reflect on how their understanding is evolving. In this way, they create an indispensable reference that complements the text and serves as an individualized study guide.

Note to the instructor

Physics by Inquiry consists of three volumes. The first two are subtitled: *An introduction to physics and the physical sciences.* Volume I develops fundamental concepts and basic reasoning skills essential for the physical sciences. The material included in Volume II provides a foundation for the study of introductory physics. With the exception of *Electromagnets* and *Astronomy by Sight,* Volumes I and II can be used independently. Volume III, which is subtitled: *An introduction to physics,* introduces additional topics from the standard introductory course.

Physics by Inquiry has been designed for courses in which the primary emphasis is on discovering rather than on memorizing and in which teaching is by questioning rather than by telling. Such a course allows time for open-ended investigations, dialogues between the instructor and individual students, and small group discussions. A major goal is to help students think of physics not as an established body of knowledge, but rather as an active process of inquiry in which they can participate.

Physics by Inquiry is particularly appropriate for preparing preservice and inservice K–12 teachers to teach science as a process of inquiry. The modules can also be used to help underprepared students succeed in the mainstream science courses that are the gateway to majors in science, mathematics, engineering, and technology. For these student populations, as well as for those in the liberal arts, the curriculum helps establish a sound foundation for the building of scientific literacy.

Physics by Inquiry has an accompanying Instructor's Guide for college and university faculty. It has several purposes: to suggest how the materials can be most effectively used with different student populations, to help the instructor anticipate student difficulties, to provide information about the equipment, to describe the demonstrations referred to in the text, and to convey the purpose of unusual exercises and experiments.

Development of *Physics by Inquiry*

Physics by Inquiry is the product of an intensive, collaborative effort by the Physics Education Group in the Physics Department at the University of Washington. Directed by Lillian C. McDermott, the group includes faculty, research associates, and graduate students. Members of the group conduct in-depth investigations of student understanding through which they identify and analyze specific difficulties that students encounter in studying physics. This research has provided the foundation for the design of the instructional strategies that are incorporated in *Physics by Inquiry.*

Physics by Inquiry has been developed through an iterative, interactive process of research, curriculum development, and instruction. The participation of the Physics Education Group in the instructional program of the Physics Department has made it possible to design, test, and modify the curriculum in a continuous cycle on the basis of regular feedback from the classroom. In addition to monitoring the effectiveness with students at the University of Washington, the Physics Education Group has been able to draw on the experience of instructors at other institutions who have pilot-tested earlier editions of the modules. The extensive testing and subsequent revision that have been an integral part of the development process have helped ensure that *Physics by Inquiry* is well-matched to the students for whom it is intended.

Physics by Inquiry has evolved into its present form over a long period of time, during which the co-authors have shared intellectual leadership and editorial responsibility. In the early versions of the curriculum, Mark L. Rosenquist helped establish the instructional approach that underlies all of the modules. In all the later versions and in the additional modules that appear in this first published edition of *Physics by Inquiry,* Peter S. Shaffer has played a pivotal role.

Acknowledgments

Substantive contributions to the design, testing, and modification of the modules have been made by many members of the Physics Education Group, past and present:

Bradley Ambrose, Patricia Chastain, James Evans, Gregory Francis, Diane Grayson, Randal Harrington, Stephen Kanim, Christian Kautz, Pamela Kraus, Ronald Lawson, Michael Loverude, Martha Means, Tara O'Brien Pride, Graham Oberem, Brian Popp, Mark Somers, Richard Steinberg, David Trowbridge, Emily van Zee, Stamatis Vokos, Betty Windham, and Karen Wosilait.

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Colleagues at other institutions who have contributed to the development of *Physics by Inquiry* include:

Davene T. Eyres (North Seattle Community College), T. Dean Gaily (University of Western Ontario), Fred M. Goldberg (San Diego State University), Eunsook Kim and Beth Thacker (The Ohio State University), Suzanne M. Lea (University of North Carolina at Greensboro), William Moore (University Preparatory Academy in Seattle), and Robert A. Morse (St. Albans School in Washington, D.C.).

The development of *Physics by Inquiry* has drawn on several published instructional materials. One important source has been *The Various Language* by Arnold Arons. Some of the precollege curricula developed with NSF support, such as *ESS, SCIS, IPS, PSSC Physics,* and *Project Physics,* have also been excellent resources.

Support by the National Science Foundation and the Physics Department has enabled the Physics Education Group to conduct the coordinated program of research, curriculum development, and instruction that has produced *Physics by Inquiry.* The encouragement of Clifford W. Mills at John Wiley & Sons, Inc., has also been deeply appreciated.

Lillian C. McDermott
August 1995

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PHYSICS BY INQUIRY

Properties of Matter

In *Properties of Matter,* we investigate some basic properties of matter. We develop the concepts of mass, volume, and density and use these in interpreting simple physical phenomena. We extend our inquiry to include some of the properties of solutions. Our investigation provides a context for developing important scientific skills, such as proportional reasoning, reasoning by analogy, and control of variables.

Part A: Measurements of matter

The emphasis in Part A is on making measurements in the laboratory that can be used as a basis to construct the scientific concepts of mass and volume. Both the process used to make the measurements and the numbers obtained from this process are important in establishing an unambiguous meaning for these terms.

Section 1. Mass

Experiment 1.1

The equipment for this experiment should be available in the classroom or will be provided by a staff member.

Working with your partner, assemble a balance. Sketch the balance in your notebook and label the parts of the balance.

Experiment with the balance and objects you have at your table and record your results. Some possible ideas for your explorations are:

Does it make a difference where the objects are placed in the pans?

Does it make a difference if you shorten or lengthen the chain(s) from which the pans are hung?

Does it make a difference if you switch the objects and put them in opposite pans?

Does it make a difference if you use a different balance?

Set up your equipment so that it appears to balance, then discuss with your partner what the term *balance* means. How should your balance look before loads are placed in the pans? Try to write a definition for *balance.*

Be sure to record your current ideas about balancing at this point. Later you may choose to modify your definition.

What similarities do you observe among the objects that balance one another? What differences do you observe among the objects that do not balance one another?

✔ Check your ideas with a staff member.

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Experiment 1.2

A. Take two objects and place them on opposite sides of the balance. Do they balance? If not, how can you use the balance to order or rank them?

   Select a third object and use the balance to come up with a ranking for all three objects.

   Add several more objects to the ranking you came up with above.

B. Obtain some square nuts. Take the objects that you ranked in Part A and balance them one at a time, with square nuts. Record your results.

   Do any two objects balance with the same number of square nuts? If so, what happens when those objects are put on opposite sides of a balance?

   Take any two objects that balance with a different number of nuts. What happens when those objects are placed on opposite sides of a balance?

   How can you use square nuts to order or rank objects? How does the ranking you obtain compare to the ranking you obtained in part A of this experiment?

In the preceding experiment, we found that the number of square nuts that balance with an object tells us something useful about that object. If two objects balance the same number of nuts, then the two objects will balance with each other. If one object balances with more nuts, then the two objects will not balance with each other. We will call the number of square nuts that balance an object the *mass* of the object. In the following experiments and sections we will explore the idea of mass in more detail.

Experiment 1.3

A. Take two objects that do not balance.

   How can you use your balance to determine how much more mass one object has than the other?

B. Take two objects that balance a different number of square nuts. What can you say about the masses of those objects?

   How could you determine how many times larger or smaller the mass of one object is compared to the other?

✔ Check your answers with a staff member.

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Experiment 1.4

For this experiment you will need several clothespins.

Imagine that students in another class have decided to use clothespins for their experiments instead of square nuts.

A. The students in the other class have found that one of their books will balance 6 identical clothespins.

> What answer would students in that class give to the question: What is the mass of the book?

Decide on a way to use the clothespins you have been given and your square nuts to answer the following question:

> What mass would *you* measure for that book if you were to find the mass of the book in nuts? Explain your reasoning.

Name a set of objects besides square nuts or clothespins that you could use to measure the mass of the book. What property must those objects have in order to be able to use them to measure mass?

B. Using square nuts, what mass would you measure for each of the following objects?

(1) An egg that balances 4 clothespins.

(2) A plant that balances 15 clothespins.

(3) A plant that has a mass of 15 clothespins.

(4) A glass of water with a mass of 30 clothespins.

C. What mass would the students in the other class measure for each of the following objects?

(1) A mouse that balances 10 square nuts.

(2) A cup that balances 4 square nuts.

(3) A shoe that has a mass of 100 square nuts.

✔ Explain your reasoning to a staff member.

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Section 2. Principles of balancing

Experiment 2.1

How can you make 6 square nuts balance 3 square nuts? If you need additional materials, ask the staff.

Is there only one way to make them balance?

Experiment 2.2

For this experiment, you will need some paper clips, and a balance like the one you used in Section 1. You will also need about 10 identical square nuts. Check to be sure the nuts all have about the same mass.

Remove the pans from the balance. For this section, hang objects directly from the balance with paper clips rather than placing the objects in the pans.

A. Hang square nuts from the beam with paper clips. Set up a number of different arrangements of nuts, some of which balance and some of which do not. Record both types of arrangements in your notebook.

B. After you have experimented on your own, answer the questions below. See if you can come up with other questions for your partner.

Can you balance 1 square nut with 2 square nuts?

Can you balance 2 square nuts at the same location on the right side with 2 nuts at different locations on the left side?

How many paper clips balance 1 square nut? Can you make different numbers of paper clips balance 1 nut? If so, what is the least number of paper clips you can make balance 1 square nut?

What is the greatest number of square nuts you can put on one side of the balance to balance 1 nut on the other side?

Does it make any difference if several nuts are hung directly from the hole or one below the other?

C. Consider an arrangement of square nuts and paper clips that balance.

What things can you change that *will* affect whether the two sides are balanced?

What things can you change that *will not* affect whether the two sides are balanced?

In each case, list as many different things as you can.

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Experiment 2.3

A. In part C of Experiment 2.2, you listed some things that you thought do affect whether two objects will balance. Select one of the items from your list. Plan and execute an experiment to show whether that item does, in fact, affect balancing.

   Write a description of your experiment and the experimental results in your notebook for later reference. Record a description or drawing of the balance as it looks throughout the experiment. Include an explanation of how your experiment helps you to decide whether the item you are testing does affect balancing.

B. Select one of the items from your list that you said *does not* affect balancing. Plan and execute an experiment to show whether that item does not affect balancing.

   Write out your plan in your notebook. Include a drawing or description of the balance as it looks throughout the experiment. Explain how your experiment helps you to show whether the item you are testing does not affect balancing.

Exercise 2.4

A. Consider the following discussion between two students.

   Student 1: *"I had my tomato plants inside the house on my windowsill. I just replanted them outside in the garden and they aren't doing as well. The temperature change must have affected their growth."*

   Student 2: *"You don't know that. Something else could have affected their growth."*

   With which student do you agree? Explain your reasoning.

   If you agree with student 2, describe an experiment the first student could perform to find out whether temperature change influences the growth of tomatoes.

B. In light of the discussion between the students in part A of this exercise, return to parts A and B of Experiment 2.3. If student 2's argument could be applied to either of your experiments, then come up with a different method for making your decision.

✔ Discuss your experiments with a staff member.

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The reasoning used in Experiment 2.3 is called *control of variables.* Roughly speaking, a *variable* is something that might influence the result of an experiment. In an experiment, there might be many variables: shape, mass, the hole from which the objects are hung, etc. If one arrangement of objects doesn't balance, it may not be possible to say which of these variables is responsible for the difference; several variables may combine to affect the balancing.

Suppose we have two objects that are hung the same distance out on either side of a balance beam, but one side of the beam is lower than the other side. The hole from which the objects are hung cannot possibly be responsible for the difference between the two sides, because the location of the hole on each side of the balance is exactly the same. Note that we cannot eliminate or remove variables, but it is not necessary to eliminate a variable. It is good enough to make the variable the same for each side of the balance. That way, even if that variable does have an influence on balancing, it influences each side of the balance identically. That variable cannot be responsible for any differences in behavior of the two sides of the balance.

Experiment 2.5

Perform experiments to decide whether each of the variables you listed in part C of Experiment 2.2 does or does not affect balancing. Be sure to discuss how you used the method of control of variables in each experiment. (If you have already performed the appropriate experiments, then discuss those experiments in terms of control of variables.)

Experiment 2.6

A. Hang a single square nut from a hole on the left side of the balance. Where do you think you will have to place another nut to reach a balance? Is there only one place or are there several locations you can place a nut to achieve balance? Check your prediction.

Leave the single nut in the same place on the left arm of the balance. Can you balance that nut with 2 square nuts on the right side? How many different ways can you do this?

Can you balance the single nut on the left side with 3 square nuts? Can you balance the single nut with four square nuts?

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B. Place 2 square nuts on the left arm of the balance. Predict a number of different ways to balance these 2 nuts with one or more nuts on the right side of the balance. Check your predictions. Record each arrangement and note whether it balances.

C. Try out more predictions of your own and check to see if they are correct. Record your results.

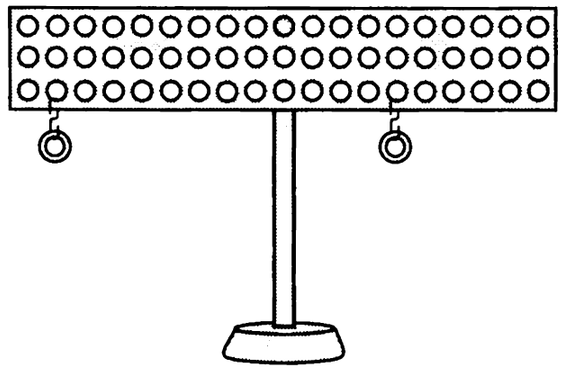
D. Do you see any pattern that would explain which arrangements of square nuts balance? Write down your ideas so that you can refer to them as you continue to work through this section. Try to use your ideas to make predictions in the following experiments.

✔ Check your ideas with a staff member.

Experiment 2.7

A. Predict whether the following arrangements of square nuts will balance. Then, set up each arrangement and check your predictions.

(1)



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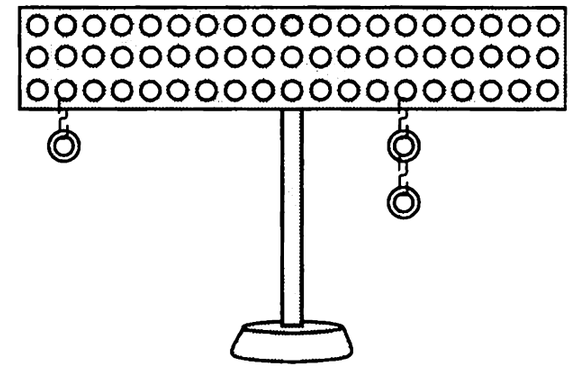

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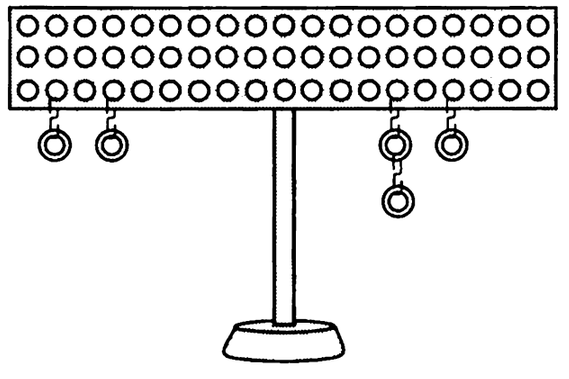

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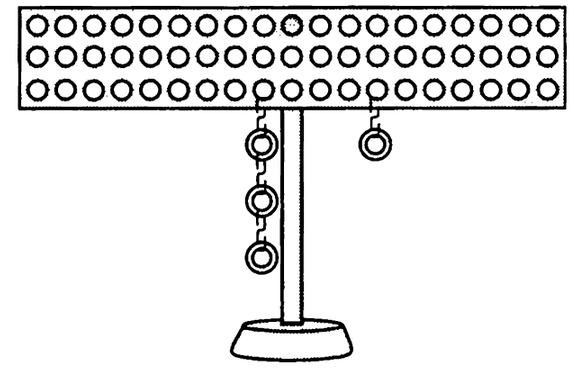

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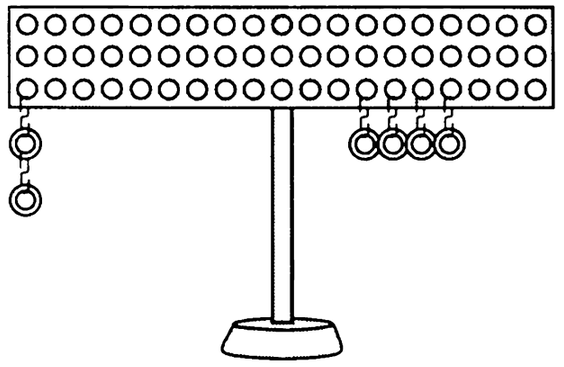

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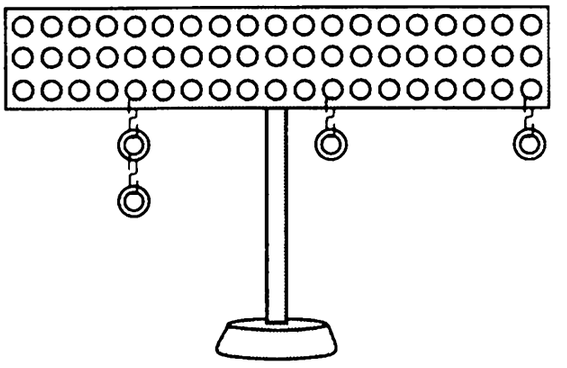
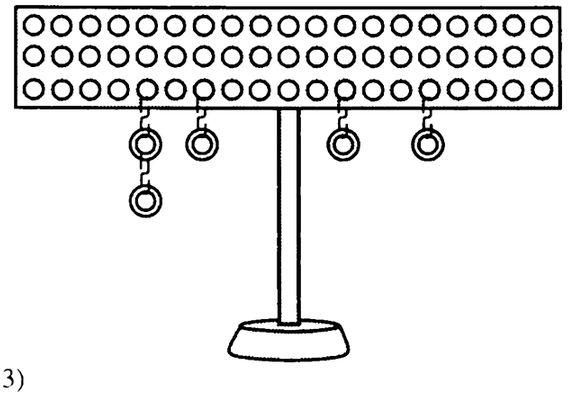


B. In the examples below, where could you add a single nut to make the beam balance?

(1)



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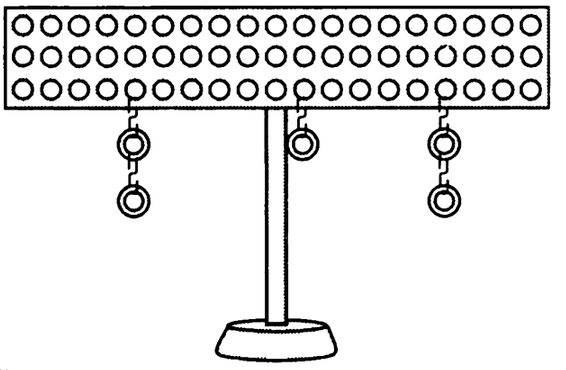

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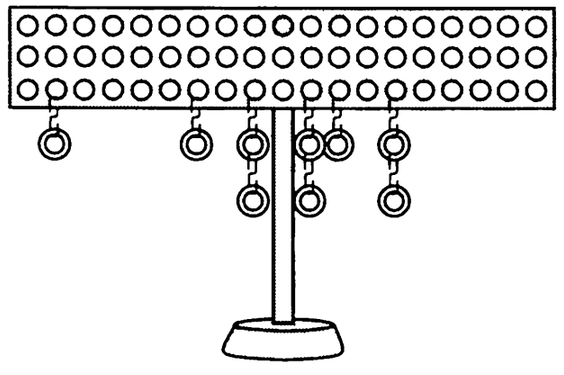

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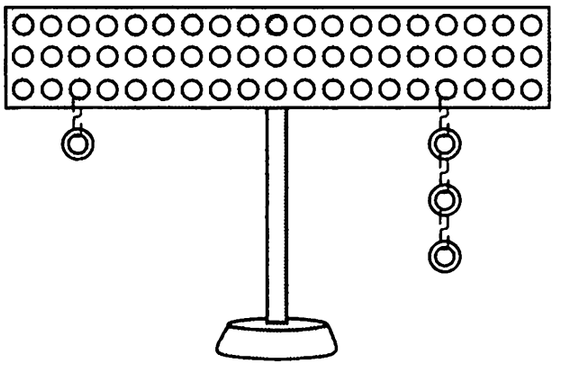

Exercise 2.8

A. A student has 4 square nuts on each side of her balance. What can you say about whether the two sides are balanced?

Are you able to predict balancing if you know only the mass that has been placed on each side of the balance?

B. A student has square nuts hung from the fourth hole on each side of his balance. What can you say about whether the two sides are balanced?

Are you able to predict balancing if you know only the hole(s) from which objects have been hung?

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In previous experiments you have found that both mass and the location of the mass along the balance beam affect balancing. However, knowing either one alone is not enough to predict balancing behavior.

Exercise 2.9

Make two lists, one showing the arrangements that balanced from your previous experiments and the other showing those that did not balance. Try to develop some form of shorthand for recording each of the arrangements, so that you do not have to draw a diagram for each one.

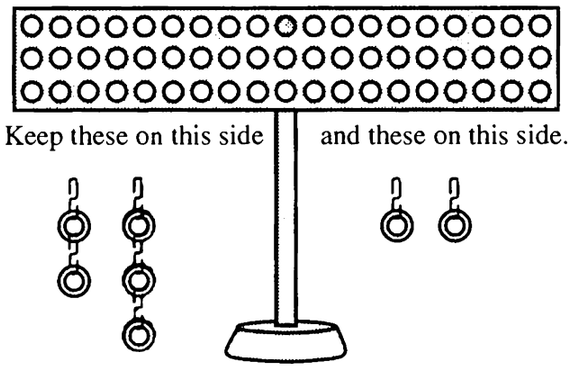
Examine your lists and try to come up with a mathematical equation that you can use to decide whether a given arrangement of square nuts will balance.

Is your equation consistent with your ideas in Experiment 2.6?

✔ Check your results with a staff member.

Experiment 2.10

Find two different ways to hang the mass arrangements shown below to make the system balance. Use all the nuts shown.



Check each of your arrangements, both with your balance and with your mathematical equation.

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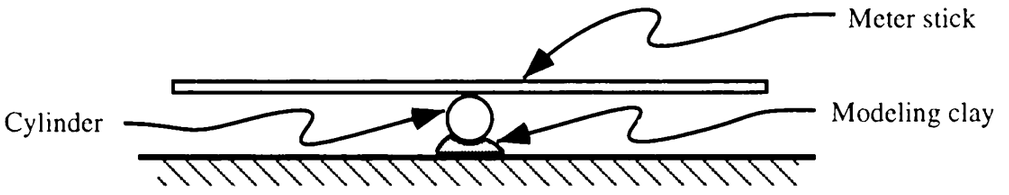
Experiment 2.11

Obtain a meterstick or a small board, a small cylinder, and some clay from a staff member and make a small seesaw out of these materials. Use the clay to hold the cylinder on the table so it will not roll. Place the meterstick on the cylinder so that the meterstick balances as shown below.

The point about which the meterstick turns (where the meterstick touches the cylinder) is called the *fulcrum* or *pivot*. Where is the pivot of the balance you used in the preceding experiments?



Set up a number of different arrangements of square nuts on the seesaw, some of which balance and some of which do not. Record both types of arrangements in your notebook.

Try to find a mathematical equation that explains the behavior of the seesaw.

How does that equation compare to the equation you used for the balance? How is it different? How is it the same?

Exercise 2.12

In this exercise, you will rewrite your equation for the seesaw in a more standard form. Assume there is only one object on each side of the seesaw.

Use the following variables in writing your equation:

Let M_1 be the mass of the object on the left.

Let L_1 be the distance from the fulcrum of the object on the left.

Let M_2 be the mass of the object on the right.

Let L_2 be the distance from the fulcrum of the object on the right.

✔ Check your answer with a staff member.

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The form of the mathematical equation you came up with in the preceding exercise says that something about the left side is equal to something about the right side when the stick is balanced. This "something" is not the mass and it is not the distance. Instead it is a new quantity, the mass times the distance from the fulcrum.

Exercise 2.13

In this exercise, we will assume that there is only one object on each side of the seesaw.

A. Discuss what will happen in each of the following cases:

(1) $M_1 \times L_1 < M_2 \times L_2$

(2) $M_1 \times L_1 > M_2 \times L_2$

(3) $M_1 \times L_1 = M_2 \times L_2$

B. If 8 square nuts were placed 30 cm from the fulcrum on the right side, and 12 nuts were placed 25 cm from the fulcrum on the left side, which side would turn down? Answer by thinking; do not do the experiment. Explain your reasoning.

✔ Check your answer with a staff member.

The quantity mass times length (distance from the fulcrum) plays an important role in balancing. The behavior of a balance could be summarized: "The side with the greater product of mass and length will turn down. The stick will only be balanced when the product of the mass and the length on the two sides are equal." We have developed a new concept that can be used to predict the direction of turning or rotation. Because the quantity mass times length is useful, we will give it a name. We will call the product of the mass and the length on each side of the balance the *turning effect* for that side.

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Exercise 2.14

A. Use the idea of a turning effect to discuss whether or not a balance will balance.

   How can you predict whether a balance is balanced if there are several objects on each side of the balance? Use the idea of turning effect in your answer.

B. Suppose a student has a balance with the same turning effect on both sides. What can you say about whether the sides are balanced?

   Are you able to predict balancing if the only thing you know about the balance is the turning effect on each side?

✔ Explain your reasoning to a staff member.

Exercise 2.15

A. Suppose you had an object that you suspected had a mass of one-half of one of your square nuts. Describe how you could use the balance and a single square nut to check this.

   How could you use the balance to check that an object had a mass of one-fifth of one of your square nuts?

B. Suppose you had a single square nut and an object that you suspected had a mass of 5 nuts. How could you use your balance to check this? Explain your reasoning.

Experiment 2.16

Obtain a pair of pans like the ones you used in Section 1. Hang the pans 8 holes from the center on each side of the balance. Check to be sure they balance with nothing in the pans.

Place an object in one of the pans and square nuts in the other pan until the sides are nearly balanced. How could you measure the mass of the object to eighths of a nut mass? Do so.

✔ Explain your reasoning to a staff member.

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Section 3. Measurements of mass

In this section we continue our study of mass. The experiments in this section and the next are concerned with the measurement of mass using a commercial equal arm balance.

Exercise 3.1

Two students are discussing their homework from the night before. They had been told to go home and find the mass of a pet with the wood blocks they had been using in class to measure mass.

The first student found that her pet had a mass of 50 blocks and the other student found that his pet had a mass of 65 blocks.

What information do you have about each student's pet? Is it possible that one of the pets is a gerbil? a German shepherd? Explain your reasoning.

Would you know more if you knew they were using square nuts to measure the masses of their pets?

In the preceding exercise, you saw that if other people used the same set of objects as you did to measure mass, then you knew more about their measurements than if they used a different set of objects. In this module we will use a set of standard objects that are available to many people. When mass is measured using this mass standard, the mass is said to be measured in *grams*. In the following experiments, we will explore grams as a measure of mass.

Experiment 3.2

Obtain a set of standard gram masses from a staff member. Examine the set you are given and record the number on the box. For the rest of this module, you should always use *your* set of masses.

What do you think the numbers on the larger pieces in your mass set mean? Use your balance from Section 1 to determine if you are correct.

We will explore the numbers on the smaller aluminum pieces in your mass set later in this section.

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Experiment 3.3

Balance a single square nut on your balance with the masses from your standard mass set. (Do not use the smaller aluminum pieces yet.) How many grams does it take to balance a single nut?

Measure the mass of an object using square nuts. Predict the number of gram pieces you would have to use to balance the object. Explain how you arrived at your answer. Then check your prediction.

Now measure the mass of a different object using your standard mass set. How many square nuts would it take to balance that object? Check your prediction.

✔ Explain your reasoning to a staff member.

In the preceding experiment, we saw that any measurement of mass using square nuts can be converted to a measurement using gram masses. Likewise, any measurement of mass in grams can be converted to a measurement of mass in square nuts. Rather than continue to use square nuts, which are unique to our class, we will use the standard mass set for the rest of this module.

Experiment 3.4

The purpose of this experiment is to learn how to use a commercial balance.

Obtain a commercial balance from the staff. Record the identification number of *your* balance. Be sure to use the same balance and standard masses throughout the course.

A. Adjust the riders on the balance arms so that the beam is balanced with nothing in the pans. It is not necessary for the beam to come to rest for it to be balanced. If the pointer swings an equal amount to each side, the system is balanced.

B. The numbers stamped on the flat pieces of aluminum in your mass set tell the masses of the aluminum pieces in *milligrams* (mg). There are 1000 milligrams in one gram. Another way to say this is that one milligram is one-thousandth of a gram.

   Write one-thousandth as a decimal. Then complete this statement:

   1 mg = _____ g. If you are unsure of the meaning of numbers like 1.0, 0.1, 0.01, 0.010, 0.001, ask your instructor for help.

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C. To find the mass of an object, follow this procedure: Put the object on one side of the balance. Then put enough standard masses on the other side so that the system balances. Usually, the beam will not balance exactly. When this happens, find the mass to the nearest 100 mg.

Find an object of about 20 to 40 grams and an object of less than one gram. In each case, determine the mass to the nearest 100 mg. Write the masses as decimal numbers of grams in the spaces below.

Mass of the first object _____

Mass of the second object _____

Does it matter on which side of the balance the object is placed?

Experiment 3.5

The riders on the arms of the balance can be used to make adjustments that are finer than the smallest standard masses you have available. In this experiment, you will calibrate a scale for use with the rider. The scale should run from 0 to either 100 mg or 200 mg, depending on the construction of the balance.

Even if your balance has already been calibrated for you, work through this experiment, as it will explain how to use the rider to measure mass.

A. Put tape along the right side of the beam. The tape should run from the center to just before the end of the beam. It should not touch anything having to do with the pan or pan support.

B. Set the right rider near the center of the balance beam, just to the right of the fulcrum needle. Mark its position with a single neat vertical line drawn on the tape at the tip of the rider. With the rider pointing to the line, move the left rider until the beam balances. Mark the location of the left rider on the beam. *Before using your balance, always check to see that the left rider points to this mark.*

C. Place the 200 mg standard mass (or 100 mg, if your balance is designed for a 100 mg scale) on the left pan of the balance. Move the right rider out until the system is balanced. Mark the position of the rider with a straight vertical line on the tape at the tip of the rider.

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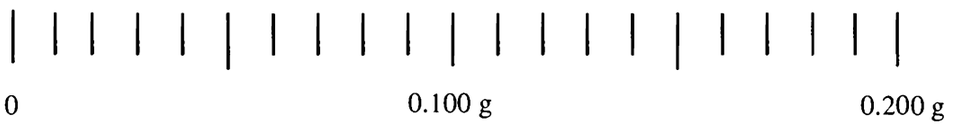
D. Carefully divide the interval between the two lines on the tape into 20 equal parts (10 for a 100 mg scale) with a set of neat vertical lines. (Consult a staff member for an easy way to do this.) This set of lines will be the rider scale. You may wish to make every fifth mark slightly longer than the others.

Take care in making your scale. You will be using it many times. When you are finished, have a staff member check your scale. The scale should look like this for a 200 mg scale:



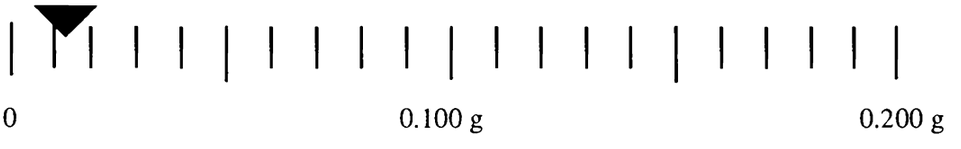
(This is not the actual size of the scale.)

Exercise 3.6

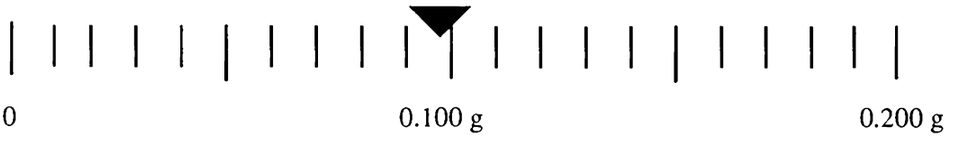
What does the rider scale read in each of the following cases?

A.

B.



C.



✔ Check your answers with a staff member.

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Exercise 3.7

In each of the following questions, assume that the system is balanced.

A. If the rider scale reads 0.100 g and the material in the right pan has a mass of 22.600 g, what is the mass of the material in the left pan?

B. If the material in the left pan has a mass of 22.600 g and the rider scale reads 0.100 g, what is the mass of the material in the right pan?

C. If the material in the right pan has a mass of 16.300 g and the rider scale reads 0.021 g, what is the mass of the material in the left pan?

D. If the rider scale reads 0.012 g and the material in the left pan has a mass of 4.700 g, what is the mass of the material in the right pan?

Your balance should now be ready to use. It is a good idea to always put the unknown object in the left pan. Then you can just add the rider scale reading to the known mass in the right pan to obtain the mass of the unknown object.

Experiment 3.8

✔ Ask a staff member to check your procedure for measuring the mass of an object.

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Section 4. Uncertainty

Experiment 4.1

Obtain an object with a mass between 2 and 10 grams and an object with a mass of about 30 to 40 grams. Find the mass of each object to the nearest 0.001 g. Record your results in the first line of the table below.

In this experiment, you are to determine the mass of both of your objects *alternately* over and over again. The measurements will probably be slightly different each time. We need to find out just how different they will be so that we will know how much we can trust the measurements.

If you are not able to finish this experiment in one class session, be sure to save both of your objects for the next session. Also, check that your balance is still balanced. Otherwise, you will have to start over.

Record your measurements to the nearest 0.001 g in the following table:

Less massive object	More massive object

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As you can see from the results of the preceding experiment, the value obtained for the mass of an object may vary each time the mass is measured. This means that we cannot completely trust a balance to give the mass exactly—a measurement might be off a little bit. All measuring instruments behave this way. For this reason, we say that there is some uncertainty in all measurements.

Because of this uncertainty in the measurement of mass, we must always distinguish between a measurement and a concept called the actual mass. We think of the object as having an actual mass, but our instruments rarely give the actual mass exactly.

Consider the 10 mass measurements of each object in Experiment 4.1. Although the measured numbers differ, it seems reasonable that all the measurements together could be used to estimate the actual mass of the object.

One idea is that the measurement that occurs most often is probably the correct one—but that could just be chance. A more reasonable way is to suppose that the balance probably gives too high a reading half of the time and too low a reading half of the time. Then the actual mass is probably in the middle, between the high and low readings. One way of finding a value in the middle is to calculate the average of all the measurements. The average is calculated by adding up all of the measurements and dividing by the number of measurements that were made.

Exercise 4.2

Calculate the averages of the values you obtained from your measurements of the masses of the two objects in the previous experiment.

Average of mass measurements for less massive object_____

Average of mass measurements for more massive object_____

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Of course, the average probably is not exactly the actual mass, but it is as good an estimate as we can get. It might be possible to improve the estimate by taking hundreds of measurements to make sure the too-high and too-low measurements balance each other out, but it would rarely be worth the effort.

Exercise 4.3

In the table below, list in order from largest to smallest all of your mass measurements from Experiment 4.1. Also write the average value in the space provided.

Less massive object		More massive object	
_____		_____	
_____		_____	
_____		_____	
_____		_____	
_____		_____	
_____	av _____	_____	av _____
_____		_____	
_____		_____	
_____		_____	
_____		_____	

For each object, find the measurement that is farthest from the best estimate of the actual mass (the average value). Subtract to find out how much this measurement differs from the average. The number obtained by subtracting gives an estimate of the largest error that is made in measuring with your balance. We will call this number the *uncertainty* of the balance. You should have a different uncertainty for your balance depending on whether it is heavily or lightly loaded.

Record the uncertainty of your balance for heavy and light loads below.

Uncertainty of the balance when heavily loaded: _____

Uncertainty of the balance when lightly loaded: _____

✔ Check your results with a staff member.

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The balance does not necessarily tell us the actual mass, but in the experiments performed so far, the error has never been more than the value just calculated for the uncertainty. This property of the balance must be taken into account when interpreting all measurements of mass.

It is important to tell how much uncertainty there is in a measurement when reporting the measurement. A common way of reporting uncertainties is to write the uncertainty right after the measured value using the "±" symbol. For example, if a mass is measured to be 12.342 g and the uncertainty in the measurement is 0.006 g, the measurement would be reported like this:

$$12.342 \pm 0.006 \text{ g}.$$

Exercise 4.4

Suppose you measure the mass of an object just once on your balance and obtain 30.642 g. This may not be the actual mass, but you know that the actual mass must be close. The actual mass cannot differ from the measured value by more than the uncertainty, because that is the largest error ever observed for the balance. Use the appropriate uncertainty of your balance to answer the following questions.

What is the highest the actual mass could possibly be?

What is the lowest the actual mass could possibly be?

To be fair, we probably should say "reasonably" instead of "possibly" in these questions, because it is conceivable we did not see the very biggest error the balance will ever make. After all, we measured the mass of each object only 10 times.

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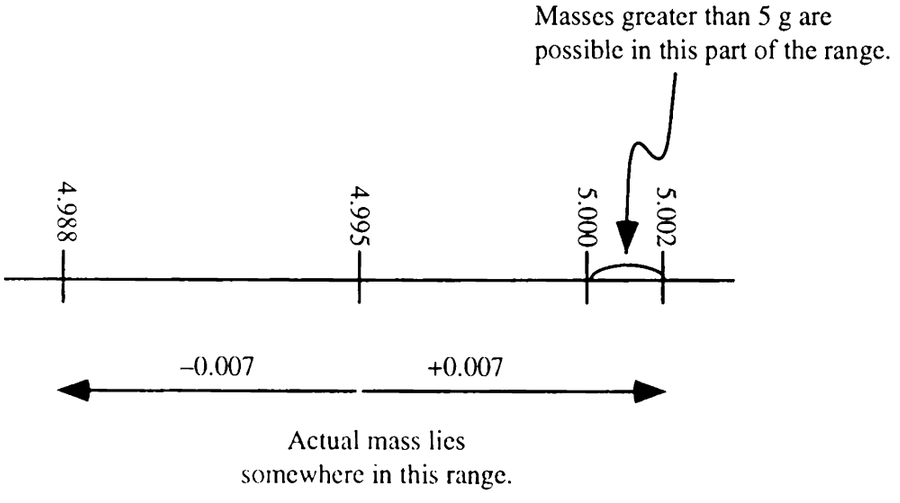
Following is an example of a problem involving reasoning with uncertainties of measurement:

Suppose there is a country where everyone must give the government 5 g of gold as a tax. It is up to the tax collector to decide whether each person has given enough, and there is a severe penalty for giving too little, namely one year in the labor camps. Since the tax collector's balance has an uncertainty of 7 mg or 0.007 g, deciding whether a piece of gold has a mass which is more than 5 g is not always simple.

Consider the case of Mrs. Freeman. The tax collector places Freeman's gold on the balance and gets 4.995 g. Is this definitely too little or is there a chance that the actual mass could be 5 g or more?

To answer this question we must consider all the possibilities. If the measured mass is 4.995 g and the uncertainty of measurement is 0.007 g, the actual mass might really be anywhere between 4.995 – 0.007 = 4.988 g and 4.995 + 0.007 = 5.002 g. This range is illustrated in the diagram below. Of course, we do not know what the actual mass is—the only conclusions we can draw are whether it is possible that the mass is greater than 5 g or less than 5 g. In this case, both are possible. Fortunately, the tax collector is reasonable and says: "Well, it is possible that your lump has a mass more than 5 g. I'll give you a break."



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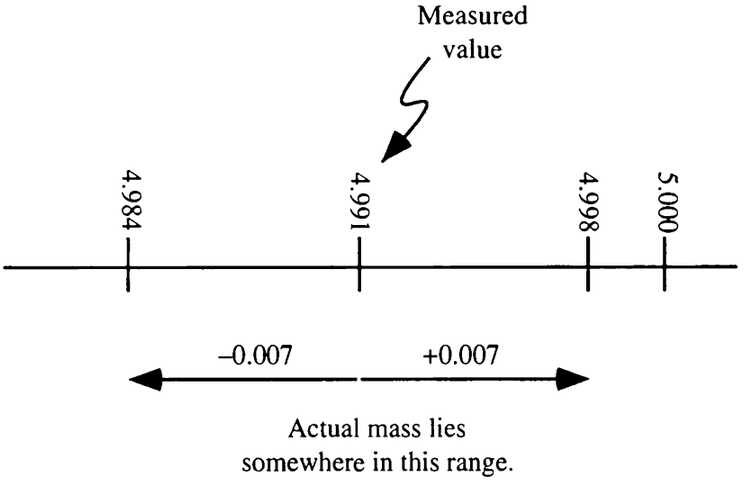
Now consider the unfortunate case of Mr. Slammer whose piece of gold is measured at 4.991 g. Is this definitely too little or is there a chance that the actual mass could be 5 g or more?

The largest mass that Slammer's piece of gold could have, by any reasonable chance, is 4.991 + 0.007 = 4.998 g. "The mass is definitely less than 5 g," says the tax collector. "You know what that means."



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Experiment 4.5

A. Find the mass of two nickels one time each. Fill in the following table using the appropriate uncertainty for your balance.

	First nickel	Second nickel
Measured value	_____	_____
Highest reasonable value for the actual mass	_____	_____
Lowest reasonable value for the actual mass	_____	_____

B. Make a diagram like those in the preceding sample problem. On this diagram show both measured values as well as both ranges.

C. Do you think the two nickels have the same mass? Explain your reasoning.

(1) Under what conditions is it valid to conclude from measurements that two masses *definitely differ?*

(2) Under what conditions is it valid to conclude from measurements that two masses *might be the same?*

(3) Under what conditions is it valid to conclude from measurements that two masses are *definitely the same?*

✔ Explain your reasoning to a staff member.

Exercise 4.6

A magician says: "I can recover lost coins from the sea. Give me a coin. I will toss it into the sea and then recover it."

Before giving the coin to the magician, you place it on your balance. You obtain 43.164 g. The coin seems to be tossed into the sea and then to reappear in a flourish of the magician's hand. You wipe off the coin, then place it on your balance and obtain 43.179 g.

You have correctly measured the mass both times. Are you justified in concluding that the returned coin is a different one?

Explain your reasoning clearly. Tell how you obtain each number you use in your explanation and tell what each means.

✔ Check your reasoning with a staff member.

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Section 5. Operational definitions

In everyday conversation, we often describe objects in general terms, such as amount, size, and heaviness. In science, however, clear communication is important. Precise and unambiguous definitions make it possible for scientists to check one another's findings and to build upon the work of others. *Operational definitions,* which leave no doubt about the meaning of a technical term, are necessary.

Exercise 5.1

A. For the simple balance you used in Section 1, the beginning of an operational definition for the term *balance* is given below. You and your partner should each complete the operational definition on your own and then discuss your answers with each other.

Operational definition for balance:

(1) Check the equal arm balance to see that the pin or fulcrum is positioned above the center of the balance arm.

(2) Attach one pan to each end of the balance arm. Next, observe which end of the arm is nearer the floor (not which pan).

(3) And so on...

B. Imagine that you have never used an equal arm balance before. Would you be able to read your operational definition and know how to make the arm balance?

C. Place your balance on a notebook or other slanted surface so that the base of the balance is no longer level. Does your operational definition from part A still work?

Suppose that the floor were not level. Would your operational definition from part A still work?

Modify your operational definition so that it does not require that the balance be placed on a level surface.

D. Compare your operational definition to the definition you gave in Experiment 1.1 for the term *balance.* What are the differences between the two definitions?

✔ Discuss this exercise with a staff member.

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The purpose of a scientific operational definition is to give the clearest possible idea of what is meant by a term. The operational definition of a quantity, such as mass, tells what steps, or operations, to take in order to obtain a numerical value for the quantity. It does not describe the applications that make a concept useful. The question of how a concept like mass can be put to use is a separate issue. The operational definition of a quantity should only tell how to measure that quantity, not its applications.

Exercise 5.2

Construct an operational definition for mass. You may wish to refer to Section 1 of the module.

✔ Check your answer with a staff member.

Definitions in the sciences are operational whenever possible. Of course, it is important to know both how to find mass and when to use it. An operational definition, however, does not cover both.

The essence of the operational definition for mass you gave in Exercise 5.2 is the comparison of the unknown object with standard objects by means of an equal arm balance. In actual practice, the use of a rider on the arm allows us to be more precise. We could achieve the same precision by using very light standard objects.

Exercise 5.3

A. Suppose you were on a desert island. You carefully made an equal arm balance but you needed to select a set of standard masses from the stones on the island. Tell how you would test your set of stones to make sure you had a good set of standard objects.

B. Could you say your measurements were made in grams?

C. Is your testing procedure in part A an operational definition of a *standard mass set?* Explain your answer.

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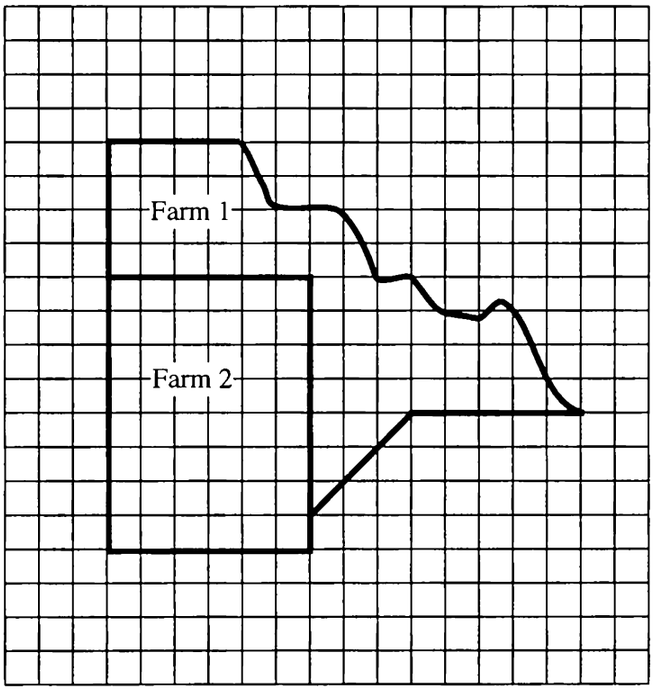
Exercise 5.4

Construct an operational definition for *length.* Your operational definition should include a description of the equipment you need in order to measure length.

Exercise 5.5

Suppose you want to buy a farm. The farm real-estate agent shows you two farms. Both are the same price, and the quality of the land is the same in every respect. Therefore, you decide to buy the bigger one. Suppose you obtained an aerial photograph of the region and were able to draw in the boundaries of the two farms as shown in the figure below. Determine which is the bigger farm.



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The best way to compare the size of the two farms in the preceding exercise is to cover each of the farms with identical squares and count the number of squares that fit inside the boundary of each farm. The farm that requires more squares is the larger farm.

In fact, this is exactly the way farms are measured. In the United States, the squares are called acres. For measuring smaller areas, smaller squares are used. For example, a square whose sides are one centimeter long is called a *square centimeter* (cm^2).

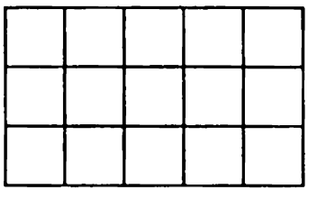
There are many situations in which it is necessary to know how large a surface is: the size of farms, the size of a wall that has to be painted, the size of a floor that has to be tiled. In all of these cases, the surface can be measured by the following procedure:

(1) Fit the standard squares inside the boundaries of the figure.

(2) Count the number of squares that fit inside the boundaries, estimating the contribution of partial squares.

The number obtained by this procedure is called the *area* of the figure. The meaning of the word *area* should not be confused with a formula for computing area under special circumstances. For example, many people have been told "area is length times width." This is usually false. The correct definition of area is given by the procedure outlined above. Length times width is just a quick way of counting the squares for the special case of rectangles; it is not correct for other shapes. The area of a figure is always obtained by counting the squares that fit inside its boundaries, but for rectangles this counting can be done more quickly by multiplying the length and width.

Exercise 5.6



Using the rectangle at right as an example, explain why multiplying the width times the length of a rectangle gives the same number as counting the squares that fit inside.

Note: Both counting and multiplying give the same number. That is not the issue. The issue is *why* multiplying is the same as counting for a rectangle.

✔ Explain your reasoning to a staff member.

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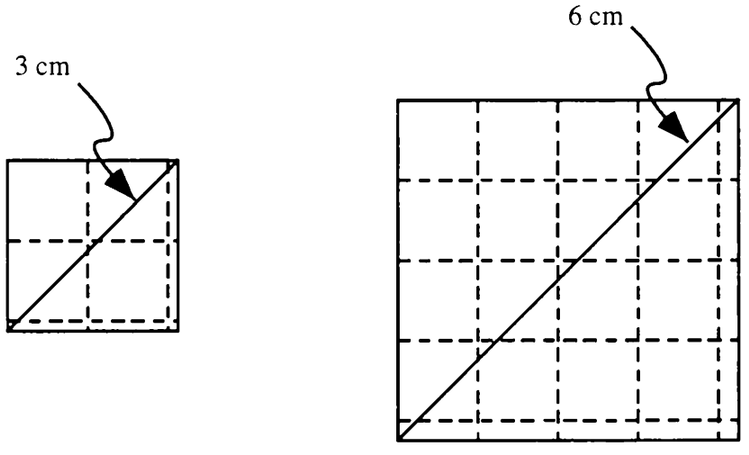
Most dictionary definitions are not operational. For example, the following nonoperational definition of area is taken from a dictionary: "the measure of a bounded region on a plane." This definition indicates that the concept of area is useful in describing the size of a figure. It does not, however, state how the size is to be determined. Thus, this definition leaves open the possibility of misunderstanding the exact meaning of area.

Why do we use operational definitions? Here is an example of how nonoperational definitions can lead to trouble. Suppose we use the nonoperational definition of area, "the measure of a bounded region on a plane," to compare the sizes of two television screens. Television screens are usually almost square, and the size of the screen is reported by giving the diagonal measure of the screen. A 19-inch television measures 19 inches from one corner to the opposite corner. Pretend there are two very small television screens. They are shown in the following diagram: a 3 cm TV and a 6 cm TV.



The nonoperational definition allows conflicting ways of finding "the measure of a bounded region on a plane." Measuring diagonally, we get 3 cm and 6 cm. Counting the squares, we get 4.5 cm^2 and 18 cm^2. According to the diagonal method, the big screen might sound as though it is twice as big as the small one. According to the counting-the-squares method, however, the big screen is four times as big as the small one. This will

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never do! In science we need a definition that will only lead to one result. That is why the operational definition of area tells exactly what to do: count the squares.

Operational definitions are often written in such a way that it is not apparent that a set of operations is being specified. For example, the operational definition of the area of a figure given in Exercise 5.5 might be condensed to something like this: "The area of a figure is the number of standard squares that fit inside it." Although this definition is not a list of steps to be taken to obtain the desired number, it is still clear what steps have to be taken.

To decide whether a definition is operational, it is necessary to see whether the definition makes clear just what operations need to be done to find a particular quantity or to make a specific test.

Sample problem

Which of the following definitions are operational?

A. value: the measure of the raw materials and labor used to produce a product and deliver it to market

B. value: the price for which a product can be bought or sold

Sample solution

The first definition is not operational because it does not tell how the measures of raw materials, labor, and delivery to market are to be taken. In fact, the best way to measure labor has been in dispute for a long time. In contrast, the second definition is operational. It clearly specifies one operation that must be done: the product must be sold. The definition also implies that the price should be noted and should be called the value of the product.

Both of these definitions have merit. Certainly both could be improved. The question of merit, however, is separate from question of whether a definition is operational.

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Exercise 5.7

Which of the following are operational definitions? Explain how you can tell.

A. mass – the quantity of matter in a body

B. momentum – the product of an object's mass and its linear velocity

C. temperature – the measure of hotness or coldness of a body or environment

D. area – the area of a ballfield is the amount of land it occupies

E. expert – a person with comprehensive knowledge

F. college sophomore – an undergraduate student with more than 45 credits and fewer than 90 credits

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Section 6. Volume

Experiment 6.1

A. Obtain a large block and a number of small cubes from the staff. Use the small cubes to build a model of the block with approximately the same dimensions as the block. Count the number of cubes required to build your model.

Construct models for several other objects by using the same procedure as above. Sketch each object in your notebook and record the number of cubes it took to build each model.

Retain your objects for use in Experiment 6.8.

B. Recall how you balanced objects and square nuts on opposite sides of an equal arm balance. How is the above activity similar to the balancing activity? How is it different from the balancing activity?

Exercise 6.2

Consider the following statement by a student:

"The concept of volume is useful for giving us some sense of how big things are. The volume of an object tells how much room the object takes up or how much space it occupies. The volume of a tank tells how much it will hold."

A. Discuss with your partners whether the statement above is an operational definition for volume. Explain your reasoning. If it is not an operational definition, write up an operational definition for volume in your notebook.

B. Compare your operational definition for volume to your definition for area. Discuss any similarities and differences.

✔ Check your definition with a staff member.

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When reporting the volume of an object, it is necessary to tell which standard cubes are being used as well as how many fit inside the object. The standard cube used is the *unit of volume.* The cubes you used in the first experiment are one centimeter long on each edge. When volume is measured with one-centimeter cubes as in the first experiment, the unit of volume is called the *cubic centimeter* (cm^3). If we counted 260 cubes of this size, we would say the volume is 260 cubic centimeters or 260 cm^3.

There are many ways of finding how many standard cubes fit inside an object. One way is to imagine cutting the object up into identical cubes and then count them. Another is to build a copy of the object with standard cubes and count how many are required.

Experiment 6.3

A. A staff member will show you a demonstration for this experiment.

B. Write out a description of the experiment in your notebook. Record your ideas before the staff member performs the experiment.

C. Record the results of the experiment.

Account for your observations.

Exercise 6.4

From the results of the previous experiment, suggest a different method for measuring volumes from that given on the previous page.

Exercise 6.5

Give an operational definition of volume in terms of the method for measuring volumes that you suggested in Exercise 6.4.

✔ Check your answer with a staff member.

Experiment 6.6

A. Fill a graduated cylinder to the 35 milliliter (mL) mark. Drop in seven of the plastic 1 cm cubes. How much does the water level rise? Explain your result.

What do your results indicate about the relationship between 1 cm^3 and 1 mL? Explain.

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B. How could you put the marks on the side of the cylinder if they were not already there?

As you observed in the previous experiments, another method of measuring volume is to submerge the object in a container of water and see how much the water level rises. A container with lines drawn on the side (such as a graduated cylinder) is used. The lines are numbered to show how many standard cubes fit in the container below each level. When an object is placed in the water, the water that used to be in that space is pushed out of the way. For each cubic centimeter of material that is submerged, one cubic centimeter of water is displaced. The water level rises accordingly. This method of measuring volumes is called the method of water displacement.

If 1000 cm^3 are piled together, they form a larger unit of volume called the *liter* (L). Since 1 cm^3 is one-thousandth of a liter, it is also called a *milliliter* (mL). A milliliter and a cubic centimeter are two names for the same unit of volume.

Experiment 6.7

Ask a staff member to show you what to do for this experiment.

After the class discussion, record values for the uncertainty in measurement for each of the graduated cylinders used.

Cylinder number	Total volume	Smallest division	Uncertainty

Exercise 6.8

A. Measure by water displacement the volume of one of the objects you used in Experiment 6.1.

B. How does your result compare with the result you obtained in Experiment 6.1? Which method do you think is more accurate? Why?

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Exercise 6.9

There are at least two different volumes associated with a tin can. What are they, and how would you measure each?

Experiment 6.10

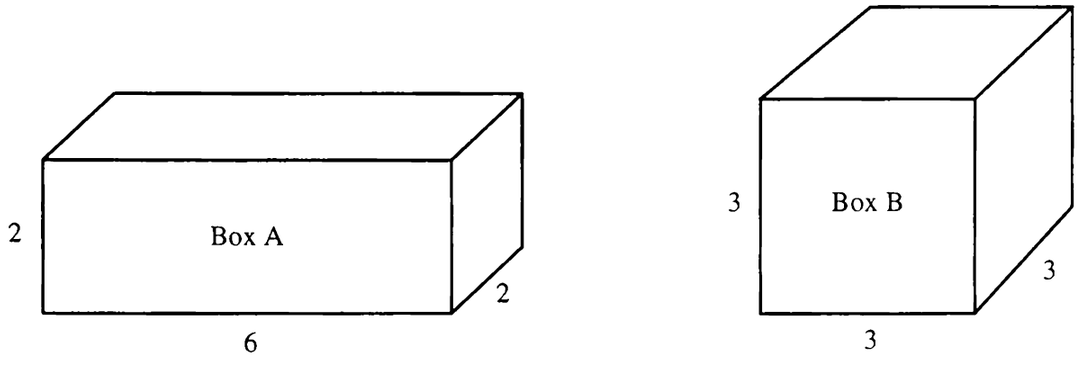
A. Obtain 60 plastic cubes and assemble them into a box shape with dimensions 3 by 4 by 5. Use this box of cubes as an example of a rectangular solid. Explain why multiplying length times width times height gives the same number as counting the cubes.

B. Explain why "volume is length times width times height" is unacceptable as a definition of volume.

✔ Explain your reasoning to a staff member.

Exercise 6.11

Consider the two boxes shown. Their dimensions are given in meters. Answer the following questions about the boxes. Explain your reasoning.



A. Suppose these boxes, including their lids, are to be made out of very thin plywood. Which box requires more wood?

B. Which box will hold more peanuts?

C. Which box is heavier (empty)?

D. Which box would you say is bigger and why?

Exercise 6.12

A. Does each gram of a submerged object displace a gram of water?

B. Could we measure mass of a submerged object by the water displacement method?

✔ Explain your reasoning to a staff member.

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Section 7. Changes in mass and volume

Before you begin Experiment 7.1, turn to the next page and start Experiment 7.3. While waiting for the ice in that experiment to melt, you should begin Experiment 7.1.

Experiment 7.1

Caution: The methanol (wood alcohol) used in this experiment is a lethal poison if ingested. Do not inhale large quantities of the vapor. Also, wear eye protection whenever using methanol.

A. Obtain two 10 mL graduated cylinders. Put 4 to 5 mL of methanol in one and 4 to 5 mL of water in the other. Read the volumes as precisely as possible and record them.

   What is the uncertainty in each measurement?

   What is the total volume of the two separate liquids? What is the uncertainty in the total volume?

B. Pour the water into the alcohol and record the combined volume. What is the uncertainty in the combined volume? Compare the initial and final measurements of the total volume of liquid. Take uncertainty into account in making this comparison. What can you conclude about whether the total volume changed when the liquids were mixed?

Experiment 7.2

Caution: See caution under Experiment 7.1.

Obtain two plastic pill bottles and caps for this experiment. Put about 5 mL of alcohol in one and about 5 mL of water in the other. Cap the bottles to prevent evaporation.

Place the two bottles and their contents all on the balance at the same time to obtain the total mass of the system. Why is less uncertainty incurred by finding the mass of everything together rather than finding the mass of each bottle separately?

Pour one liquid into the other and recap the bottles. Find the mass of the two bottles and the resulting mixture all at once. Compare the initial and final measurements. Take the uncertainty of your balance into account in making this comparison. What can you conclude about whether the total mass changed when the liquids were mixed?

Record your measurements and your conclusions.

✔ Discuss this and the preceding experiment with a staff member.

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Experiment 7.3

Obtain a plastic bottle and cap and some ice from the staff. In this experiment you will find the mass of some ice, let it melt, and find the mass of the resulting water to see what happens to the mass.

Put the ice in the bottle and put on the cap. Find the mass of the cap–bottle–ice system. Record the mass to the nearest milligram. Wait for the ice to melt. Do not shake the bottle; the cap might leak.

When the ice has melted, find the mass of the system again. Be sure that no water has collected on the outside of the bottle. If it has, wipe it off before placing it on your balance. Record the final mass of the system.

What can you conclude about whether the mass changed when the ice melted? Explain your reasoning.

Experiment 7.4

Caution: Wear eye protection for this experiment.

Obtain a thick-walled bottle and cap and one-quarter of an Alka-Seltzer. Be sure the cap has a gasket in it. We will investigate what happens to the mass of the system when the tablet dissolves in water.

A. Fill the bottle one-third full of water, but do not put the tablet in the bottle yet. Place the bottle, water, cap, and tablet on the balance at the same time and record the mass.

   The next step must be done quickly and correctly. Put the tablet in the bottle and screw on the cap *immediately.* You must screw the cap on very tightly—as tightly as possible without using a wrench.

   Wait until the tablet has almost entirely disappeared. Then place the system on the balance again and record the mass.

   What can you conclude about whether the mass changed when the tablet dissolved? Explain your reasoning.

B. Now slowly open the cap. If you open it too fast, water may spurt out.

   What does the sound you hear when you open the container suggest is happening? When you no longer hear hissing, find the mass of the system again.

   What can you conclude about whether the mass of the system changed when you opened the container?

   On the basis of this experiment, would you conclude that the gas has mass or not? If so, determine the mass of the gas.

✔ Explain your reasoning to a staff member.

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If your experiments were done carefully and there were no accidents or oversights, you should have found no detectable change in the mass of the changing systems, except in part B of Experiment 7.4 when the system was opened. In the language of science, we say that mass is *conserved* in a *closed system.* The word *conserved* has a special meaning in physics and chemistry. When some quantity remains the same before and after some process takes place, we say that the quantity is conserved in the process. Even when ice turns into water and the appearance and hardness change, the mass is still the same. Even though a solid Alka-Seltzer tablet disappears leaving only liquid and gas, the mass of the system stays the same so long as nothing is allowed in or out of the system. No one has ever been able to find a consistently repeatable process in which the mass of a closed system definitely changes. Although it is easy to imagine, for example, that ice might have less mass when it is melted, this never happens. It is almost as if there were a law against changes in the mass of a closed system. For this reason, it is said that there is a *law of conservation of mass.*

Exercise 7.5

Write a statement of the law of conservation of mass. Do not use the word *conserved* in your statement.

Exercise 7.6

Obtain a syringe from the staff. Compress the gas inside and observe what happens. Then resolve the following dispute between two students.

Student 1: *"I say the volume decreased. Volume is the number of standard cubes that fit inside, and fewer cubes would fit inside after the piston is compressed."*

Student 2: *"No, the volume is still the same. This is a closed system. Nothing could get in or out, so there are still the same number of cubes of air inside the syringe. Since there are the same number of cubes of air inside, the volume is the same."*

Tell which student is correct and explain what is wrong with the other student's reasoning.

✔ Check your reasoning with a staff member.

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Experiment 7.7

Obtain a small object that will fit into a graduated cylinder and a piece of clay. Both the object and the clay should sink when placed in water.

A. Drop the object into the graduated cylinder of water and measure the change in the level of the water.

Now mold the clay so that it has the same shape and size as the object.

Predict how the level of the water will change when the clay is dropped into the graduated cylinder. Explain your reasoning.

Perform the experiment and check your prediction.

B. Suppose you were to form the piece of clay that you used in part A into a different shape.

Predict how the level of the water will change when the clay is placed into the graduated cylinder. Explain your reasoning.

Perform the experiment and check your prediction. Choose a shape that is very different from the shape in part A. If the result is not what you predicted, how can you resolve the difference?

Exercise 7.8

Discuss whether it is possible to write a law of conservation of volume. Include in your response any experimental evidence you have for your answer.

✔ Check your reasoning with a staff member.

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Section 8. Distinguishing between mass and volume

Mass and volume both tell how much of something there is. We could either say "we have 10 cm^3 of aluminum," or we could say "we have 27 g of aluminum." Both of these statements tell how much aluminum we have.

But mass and volume are not the same. Their operational definitions are completely different. The definition of mass involves the use of a balance to compare the measured object with standard objects. The definition of volume involves fitting cubes inside the object being measured. The mass of an object tells us how heavy it is; the volume of an object tells us how much space it takes up. As we have seen, mass is conserved in a closed system, but volume sometimes is not.

What has happened here is that one concept, amount, has been replaced with two different, precisely defined concepts: mass and volume. The concept of an *amount of material* is too vague to be used in science. We must always specify whether we are talking about the mass of the material or the volume of the material. The statement "we have the same amount of water and mercury" is meaningless. We might have the same volume of water and mercury, but then the mercury would be heavier. We might have the same mass of water and mercury, but then the water would fill a larger container than the mercury.

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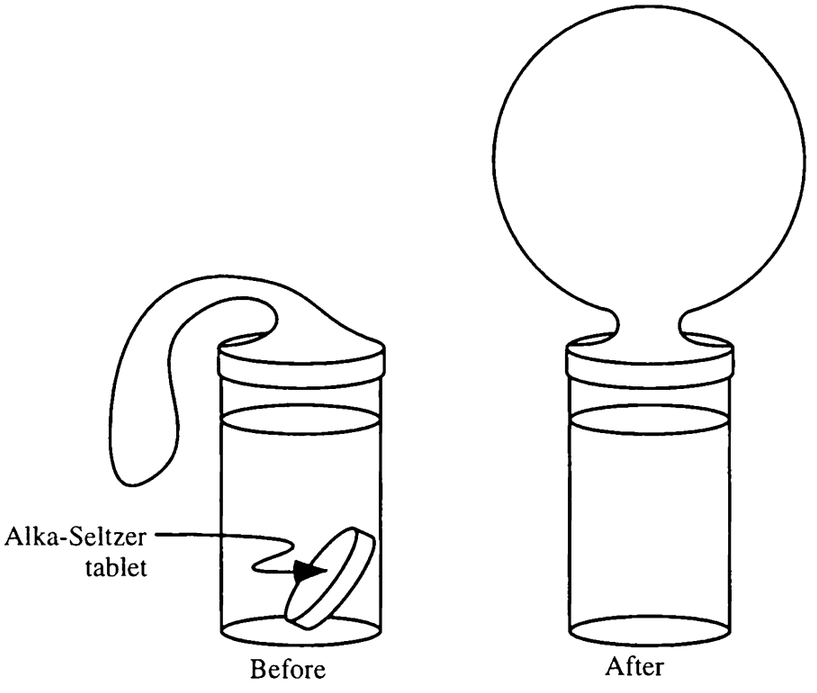
Exercise 8.1

Consider an experiment in which we dissolve an Alka-Seltzer in water. Suppose that immediately after the tablet is placed in a bottle, a tight-fitting balloon is fitted over the mouth of the bottle. The tablet then dissolves and the balloon becomes inflated as shown in the following diagram.



Consider the system consisting of everything inside the bottle and balloon.

A. When the tablet dissolves, does the volume of the system increase, decrease, or remain the same? Explain your reasoning.

B. When the tablet dissolves, does the mass of the system increase, decrease, or remain the same? Explain your reasoning.

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Part B: Pure substances

In Part B of this module, we examine the relationship between the mass and volume of an object made of a pure substance. We explore how this relationship can be used to help identify the substance of which the object is made and to make predictions about the behavior of the object under certain circumstances.

Section 9. Proportional reasoning with mass and volume

Experiment 9.1

A. Obtain a set of objects for this experiment from the staff. Find the mass and volume of each object. For each object, record the number you obtain when you divide its mass by its volume. Compare the results.

> Is the number you obtain from this division the same for all your objects or does it yield a different number for different objects?
>
> Are there any objects for which the numbers you obtain are the same? What similarities do you see among those objects? What differences do you see among objects for which the numbers you obtain are different?

In your notebook, write down a clear explanation of what you think the number you obtain by dividing the mass of an object by its volume tells you about the object to which it applies.

B. You should be able to break some of the objects you used in part A into smaller pieces. Do so now, and measure the mass and volume of some of the pieces. For each piece, divide the mass by the volume. Compare your results with your results from part A.

How do the numbers you obtained for the *pieces* of an object compare to the number you obtained for the entire object?

> Are there any objects for which the number you obtained is nearly the same as for the pieces of that object?
>
> Are there any objects for which the number you obtained is very different from the number you obtained for the pieces of that object?

C. Write down a rule that you can use to predict whether the number you obtain by dividing the mass by the volume for a *piece* of an object will be the same as that for the entire object.

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If an object or sample of liquid is the same throughout, it is said to be *homogeneous.* An aluminum block is homogeneous because all parts of it are the same material. The way to test the homogeneity of something is to compare tiny pieces of it taken from different parts. If this is done to a chocolate chip cookie, some of the tiny pieces will be chocolate and some will be cookie. A chocolate chip cookie is therefore said to be *inhomogeneous* because not all parts of it are made of the same material. Glass, pure water, and copper are homogeneous; tiny pieces taken from them are always the same material. Bread has air pockets, and dirt contains many things, so both of these are inhomogeneous.

For homogeneous materials, mass divided by volume always gives the same number for the same substance. This relation between mass and volume is called a *proportion.* It is said that the mass and volume of the substance are *proportional* to each other. Proportionality is discussed in detail in a later section of the module.

Exercise 9.2

For this exercise, use examples that are different from the examples given in the text and in Experiment 9.1.

A. Give an example of an object for which all of the 1 cm^3 pieces share the mass equally.

B. Give an example of an object for which the 1 cm^3 pieces do not share the mass equally.

C. What does an object have to be like in order for the 1 cm^3 pieces to share the mass equally?

This section is an introduction to quantitative reasoning. Every arithmetic step taken with numbers in solving a problem must have a justification. The reasoning that goes into a calculation is as important a part of science as laws like the law of conservation of mass. In the problems in this section, the reasoning that leads to the answer is more important than the answer itself. Because mass and volume are proportional, the reasoning used in this section is called *proportional reasoning.*

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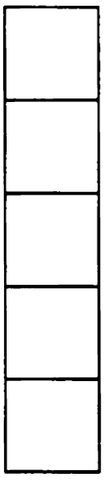
Sample problem

Suppose we have a stone with a mass of 15 g and a volume of 5 cm^3. What is the mass of 1 cm^3 of this stone?

Sample solution



At right is a diagram of the stone. It is *not* a picture of the stone. It is not supposed to look like the stone; it is just something to help us think about the problem. The actual stone might be any shape.

In the diagram, the stone is cut into five equal pieces, each one having a volume of 1 cm^3. If all these 1 cm^3 pieces share the mass equally, then we can find the mass of 1 cm^3 of stone by splitting up the total mass into 5 equal shares. The way to split up the total mass (15 g) into 5 equal shares using arithmetic is to divide (15 g)/5 = 3 g. Since each 1 cm^3 has one of these shares, the mass of each cubic centimeter of stone is 3 g.

Comment

This problem illustrates one of the reasons division is used in calculations. We know that the mass of 5 cm^3 is 15 g. We want to find the mass of just 1 cm^3. To do this, we divide 15/5. When we want to find out how much of one quantity goes with just one unit of a second quantity, we divide. The result of division in this kind of problem tells us the amount of the quantity in the numerator for each unit of the quantity in the denominator.

The mass of one unit of volume of a substance is an important quantity. Calculations we do with mass and volume often can be reasoned through by thinking about breaking objects into 1 cm^3 packages. For the stone above, the package has a volume of 1 cm^3 and a mass of 3 g. The following problems show how to reason with these packages.

Sample problem

Suppose we have a larger piece of the same kind of stone with a volume of 120 cm^3. What would its mass be?

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Sample solution

The problem says "with a volume of 120 cm^3." Using the operational definition of volume, we can translate this phrase to mean "there are 120 one cubic centimeter packages of stone." Now, we know that 1 cm^3 of this stone has a mass of 3 g, so all we have to do to find the mass of this large stone is add up 3 g 120 times—once for each 1 cm^3 package. A fast way to do this addition is to multiply 120 × 3 g, which gives 360 g.

Comment

This solution illustrates the reasoning that leads to the use of multiplication in a problem.

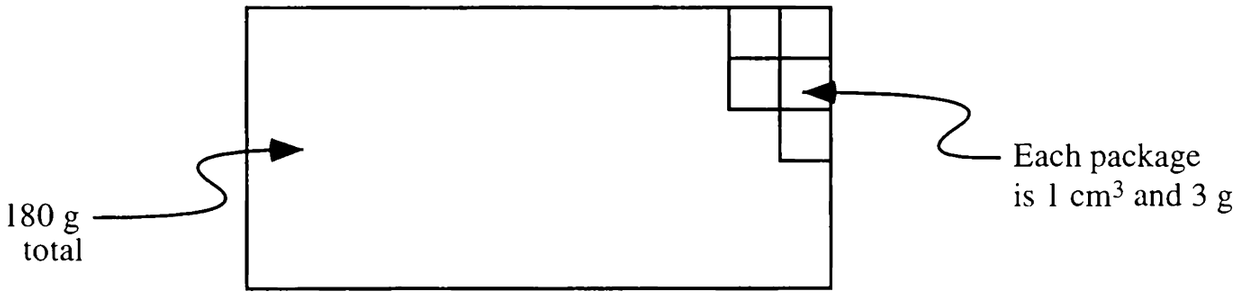
We use multiplication here because we want to add the same number repeatedly.

Sample problem

What is the volume of 180 g of the same kind of stone?

Sample solution

In this problem we are asked to find the volume. First, we translate this request into, "Find the number of 1 cm^3 packages of stone it takes to have a mass of 180 g." We again use our knowledge that the mass of each 1 cm^3 package of stone is 3 g. Here is a diagram of the 180 g piece of stone, shown cut up into 1 cm^3, 3 g packages.



The problem is to find out how many 1 cm^3, 3 g packages there are in 180 g. We can calculate how many 3 g packages there are in 180 g by dividing: 180/3 = 60. Thus, there are 60 of these 3 g packages in 180 g, and each one of them has a volume of 1 cm^3. Therefore, the volume (number of 1 cm^3 packages) of the 180 g stone is 60 cm^3.

Comment

Here we see another, different justification for dividing. We are not trying to split up 180 into three equal shares. Instead, we want to find out how many threes there are in 180. These two quite different justifications for division are both used frequently. When explaining the reason for dividing, we must choose between these two interpretations of division (and possibly others). Usually, only one will make sense.

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Exercise 9.3

A certain piece of metal has a mass of 125 g and a volume of 25 cm^3. Answer the following questions about this metal. Explain your reasoning at each step. In your explanations, be sure to tell why you multiply or divide.

A. What is the mass of 1 cm^3 of this metal?

B. What is the volume of 80 g of this metal?

C. What is the mass of 40 cm^3 of this metal?

✔ Check your reasoning with a staff member.

In this course, detailed explanations of reasoning are required. It is most important to explain why to divide or multiply. The key parts of such explanations are given below.

Multiplication

An explanation of why to multiply should contain something equivalent to the following:

We need to add up 120 of these 3 g packages.

A quick way to do this is to multiply 120 × 3.

Division

At this stage, we have encountered two explanations of why to divide. One is:

5 cm^3 has a mass of 15 g. We can find

the mass of just 1 cm^3 by dividing 15/5.

The other kind of explanation is this:

We want to find out how many packages of 3 g

fit into 60 g. We can do this by dividing 60/3.

Later, we will encounter other explanations as well. In most problems, only one explanation will make sense.

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When the numbers involved in a calculation are not whole numbers, the reasoning is exactly the same although it sometimes appears more difficult. If thinking about problems involving non-whole numbers seems difficult, just pretend that the numbers are whole. Then follow the same procedure you would use if they really were whole.

Sample problem

A block of a particular kind of wax has a mass of 12.2 g and a volume of 16.4 cm^3. Explain your reasoning thoroughly in each of the following questions.

A. What is the mass of 1 cm^3 of this kind of wax?

B. What is the mass of 4.12 cm^3 of this kind of wax?

C. What is the volume of 0.0086 g of this kind of wax?

Sample solution

A. We want the mass of 1 cm^3 of wax but we are given the mass of 16.4 cm^3 of wax instead. What we need to do is to split up the 12.2 grams into 16.4 equal shares, one share for each one cubic centimeter. This can be done by dividing 12.2 by 16.4: 12.2/16.4 = 0.744. Thus, each 1 cm^3 piece has a share of the mass equal to 0.744 g.

B. Each cubic centimeter has a mass of 0.744 g, and we want to know the mass of 4.12 cm^3. We can calculate the mass of 4.12 cm^3 of wax by adding the mass of one cubic centimeter 4.12 times. This can be done quickly by multiplying: $4.12 \times 0.744 = 3.06$ g.

C. We want to find the volume of 0.0086 g of wax. Volume means the number of 1 cm^3 packages that fit inside the piece of wax. Each of these packages has a mass of 0.744 g. We can find out how many 0.744 g packages fit in 0.0086 g by dividing: 0.0086/0.744 = 0.012. Since each of the packages has a volume of 1 cm^3, the total volume is 0.012 cm^3.

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Exercise 9.4

Explain every step in your reasoning for the following problem. A piece of clay has a mass of 14.1 g and a volume of 12.2 cm^3.

A. What is the mass of 1 cm^3 of this clay? Draw a diagram that illustrates the thinking involved in the calculation.

B. What is the mass of 15.2 cm^3 of the same type of clay?

C. Suppose we have a 68.8 g piece of the same clay. By how much would the mass increase if we added a lump of clay with a volume of 3 cm^3?

✔ Write your responses and check them with a staff member.

Exercise 9.5

In this problem, the calculating has been done for you. The questions ask about the reasoning involved at each step.

Problem: 15.0 cm^3 of iron has a mass of 118 g. What is the volume of a piece of iron that has a mass of 413 g?

$$\frac{118}{15} = 7.87 \quad \text{(step 1)}$$

$$\frac{413}{7.87} = 52.5 \quad \text{(step 2)}$$

A. How do you interpret the number *7.87?*

B. Use a diagram to explain why the division is done in step 1.

C. How do you interpret the number *52.5?*

D. Use a diagram to explain why the division is done in step 2.

✔ Check your responses with a staff member.

Exercise 9.6

Refer to Experiment 9.1. In part A of that experiment, you were asked to write a clear explanation of what you thought the number you obtain by dividing an object's mass by its volume represents.

Discuss your response to that experiment in light of your work in this section. Do you believe your answer would be clear to someone with no background in science?

How would you respond to the question in that experiment now?

✔ Check your answer with a staff member.

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Section 10. Density

The result obtained when two numbers are divided is called the *ratio* of the two numbers. Thus, 3.1/5.4 is called the ratio of 3.1 to 5.4. In the sciences there is a preferred notation for a division or ratio:

Preferred: $\frac{3.1}{5.4}$

Rarely used: $3.1 \div 5.4$, $5.4\overline{)3.1}$

In the preceding section, we worked several problems using the ratio of mass to volume. We have found that mass and volume are proportional for a given homogeneous substance. Therefore, the ratio of mass to volume is the same for all samples of the substance. *Density* is the name given to this ratio.

Mass, volume, and density are all properties of matter. However, mass and volume tell us something about a specific object. In contrast, the density contains information about a class of objects: under the same conditions, all objects made of the same material have the same density. (This is not true for mass and volume.) We use the term *characteristic property* to refer to properties such as density that characterize a particular material or substance. Characteristic properties can be used to help identify the material out of which a given object is made.

Exercise 10.1

Write an operational definition of density.

✔ Check your answer with a staff member.

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The operational definition of density does not tell us how the number is used. For that we need an interpretation of what this number tells us. In Section 9, the density number was interpreted as the mass of one unit of volume of the material. Note that this interpretation of density makes sense only if all the units of volume share the mass equally.

The units in which the mass and volume are measured have to be known before the density number can be of any use. A density of 15 is very large if the mass is measured in grams and the volume in cubic centimeters. However, a density of 15 is very small if the mass is measured in milligrams and the volume is measured in cubic meters. Usually the mass and volume units are written right after the density number. A slash between the mass and volume units indicates which quantity was on top in the division. Some examples are:

$$5\text{g} / \text{cm}^3,\ 5\frac{\text{g}}{\text{cm}^3},\ 5000\text{kg} / \text{m}^3$$

where 5 g/cm^3 is read "five grams *per* cubic centimeter."

Per means *for each* and indicates that division has been used in computing the number. Thus, 5 g/cm^3 could be read as "five grams *for each* cubic centimeter."

Interpretation of calculated numbers

The meaning of a measured number like mass is usually clear. When a number is calculated, however, it must be interpreted. An interpretation is a statement of what the number tells us.

Sample problem

A block of wood has a mass of 18 g and a volume of 25 cm^3. How would you interpret the number 18/25?

Sample solution

18/25, or 0.72, is the mass in grams of 1 cm^3 of that kind of wood, or the number of grams in each cubic centimeter of that kind of wood.

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Comments

This is not a difficult problem. The only trouble might arise from a misunderstanding of what counts as an interpretation. For example, the following two responses are not acceptable interpretations of the number 18/25.

Inadequate response 1:

> "18 gm is the mass of the block and 25 cm^3 is the volume." Although this is certainly correct, it is an interpretation of *two numbers:* 18 and 25. The problem asks for the interpretation of 18/25. 18/25 is *one number:* 0.72.

Inadequate response 2:

> "18/25 is the density of the wood." Again, this is true, but it only tells the name of the number. It is not an interpretation of what the number means.

Not all calculated numbers have clear meaning. For example, aluminum has a density of 2.7 g/cm^3. Suppose we have a 5.4 cm^3 piece of aluminum. Consider the number obtained by dividing the second quantity by the first, 5.4/2.7, in other words, 2. It is difficult to imagine any reason for doing this calculation, and indeed the number *2* has no simple physical interpretation in this instance.

Exercise 10.2

A piece of plastic has a mass of 650 g and a volume of 720 cm^3. Give an interpretation, if there is one, of each of the following numbers.

A. $\frac{720}{650}$ B. $\frac{650}{720}$

C. 720×650 D. $720 + 650$

✔ Check your results with a staff member.

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Exercise 10.3

Suppose we have a piece of iron with density 7.87 g/cm^3 and volume 6.25 cm^3. Give an interpretation, if there is one, of each of the following numbers.

A. $\frac{7.87}{6.25}$ B. $\frac{6.25}{7.87}$

C. 7.87×6.25 D. 6.25×7.87

Exercise 10.4

Suppose we have a piece of tin with a density 5.75 g/cm^3 and mass 8.31 g. Give an interpretation, if there is one, of each of the following numbers.

A. $\frac{8.31}{5.75}$ B. $\frac{5.75}{8.31}$ C. 8.31×5.75

Exercise 10.5

Each pound of bananas costs 30 cents. We bought 4.3 pounds of bananas. Give an interpretation, if there is one, of each of the following numbers.

A. $\frac{4.3}{30}$ B. $\frac{30}{4.3}$

C. 4.3×30 D. $4.3 + 30$

Exercise 10.6

A man wants to buy a $24 pair of shoes. He earns $8 a day at his job. Give an interpretation, if there is one, of each of the following numbers.

A. $\frac{24}{8}$ B. $\frac{8}{24}$

Exercise 10.7

Suppose there are two tanks. One has a large volume. It is partially filled and contains 72 liters of wine. The other has a volume of 68 liters and is empty. The winery crew has to transfer the wine to the smaller tank and gets to keep the excess. Give an interpretation, if there is one, of each of the following numbers.

A. 72×68 B. $72 + 68$ C. $72 - 68$

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Exercise 11.2

Suppose some measurements are made on two homogeneous stones to find out whether they are made of the same kind of rock. The mass and volume measurements and their uncertainties are listed below.

	Mass	Volume
Stone 1	58.0 ± 1 g	20.0 ± 0.5 cm^3
Stone 2	50.0 ± 1 g	15.0 ± 0.5 cm^3

What would you conclude from these data? Are the stones made of the same kind of rock? Explain your reasoning.

Experiment 11.3

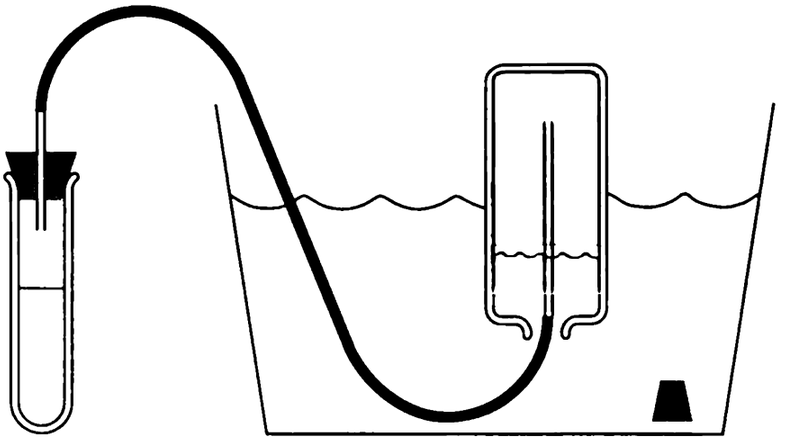
Measure the density of water. Make the most accurate measurement of the mass and volume as possible. (The most accurate container for measuring volume may not be suitable for the mass measurement.)

Give your measured value for the density and your highest and lowest reasonable values.

Experiment 11.4

In this experiment you will measure the density of a gas. You will need a test tube, a one-hole stopper, a short glass tube, a hose, a bucket, an Alka-Seltzer tablet, and a plastic bottle and stopper. See the diagram below.

Here is what should happen: Gas is produced when the tablet is dropped into water in the test tube. The gas travels through the hose attached to the test tube and emerges into a bottle that is initially full of water. The gas pushes the water out of the bottle and is trapped inside. The mass and volume of the gas in the bottle can then be determined and the density can be computed. Explicit instructions are in parts A–E.



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A. The indirect method of measuring the mass of gas produced in Experiment 7.4 will also work in this experiment. Insert the glass tube in the top of the one-hole stopper. Put about 10 mL of water in the test tube and plug it with the stopper-and-glass-tube. Lay the test tube on the balance in such a way that the water does not spill. Break an Alka-Seltzer tablet into pieces that will fit in the tube and put them on the balance with the test tube. Find and record the mass of the system.

B. Connect the hose to the glass tube in the stopper. Fill a bucket with water and completely submerge a plastic bottle in it. Make sure the bottle has no air in it, then hold it upside down with its mouth under the water. Run the hose up into the bottle. When gas comes out of the hose, it will be trapped in the bottle.

C. Hold the test tube vertically, either by hand or in a test tube rack. Drop the tablet into the test tube and immediately cap the test tube with the stopper-and-hose. Gas should begin to enter the bottle.

D. When the reaction is over, or going very slowly, do the following steps in order:

   (1) Pull the hose out of the bottle and entirely out of the water.

   (2) Put a stopper in the bottle and remove it from the water.

   (3) Remove the hose from the glass tube.

E. Find the mass of the test tube and its contents. Take care that nothing spills out of the glass tube. Record the mass.

   (1) Find the mass of the gas produced. Explain why your method is valid.

   (2) Find the volume of the gas produced.

   (3) Compute the density of the gas.

In addition to errors in measurement discussed earlier, this experiment has other sources of error. Some gas escapes. Water and Alka-Seltzer tablet may be forced up the hose. The gas dissolves readily in the water in the collection bottle, so more gas is produced than we may think. With all of these sources of error, this experiment can only be considered to yield an estimate for the density of the gas. In practice, the results are usually between half and twice the accepted value for the density. Although the error may be large, the measurement allows us to contrast the density of the gas to the density of water.

> How does the density of the gas compare to the density of water? Compare your measured density for the gas to the densities listed in the table of densities in the Appendix at the back of this book. Is your result consistent with the tabulated densities?

✔ Explain your reasoning to a staff member.

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Section 12. Sinking and floating

Experiment 12.1

Obtain a set of objects from the staff. Over the next few days you will need to use this same set of objects. Mark your set in some way so that you can tell your objects from those of other students in the class.

A. Predict whether each object in your set will sink or float. Make a record of your predictions with a brief explanation of your reasoning. After you have made your predictions, test each of the objects.

   How did you test whether an object sank or floated? In other words, what are your operational definitions for *sink* and *float?* Record your operational definition in your notebook.

   Can you make any of the objects that were "sinkers" float? Can you make any "floaters" sink?

B. If, in part A, you were able to turn a sinker into a floater, then your operational definition is ambiguous and should be modified. Modify it so that it always gives the same answer for whether a particular object sinks or floats.

   How does your operational definition compare with the everyday usage of the words *sink* and *float?*

C. Separate the objects that sank from the objects that floated. What observable similarities are there among the objects that floated? What differences can you observe?

   What similarities and differences are there among the objects that sank?

Often a word in everyday usage has a different meaning in science. The words *sink* and *float*, as used above, are examples. You could have created new words for the operational definitions you developed in the experiment above, but the ideas described by those definitions are very similar to the everyday meanings of the words *sink* and *float.* In this case, it would have been more confusing to have different words for those ideas. However, you must remember that you have changed the meanings of the words *sink* and *float* for this class and must be careful to use those words in the same way in the future.

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Other words you see in science texts may also have meanings that differ subtly from what you expect. In part, that is the reason why operational definitions are so important. They help you to know what is meant by the words other people use.

Experiment 12.2

A. On the basis of your observations in the preceding experiment, make a list of the things that you believe can influence whether an object sinks or whether it floats. Include in your list some things that someone who had not performed any experiments in sinking or floating might believe would influence whether an object sinks or floats.

   Make another list of a few things that you believe *do not* influence whether an object sinks or floats.

B. Earlier in this module, you worked with several properties of objects, one of which was mass. Do you think an object's mass influences whether or not it floats? Why or why not?

   Plan and execute an experiment with your set of objects that will show whether or not mass can affect sinking or floating. Note that clay is available if you need to make additional objects for your experiment.

   Describe your plan and the results of your experiment in your notebook to refer to later. Include in your notebook a description of the objects that you used.

C. Refer to Exercise 2.4, part A. In light of the discussion between the two students in that exercise, reconsider the experiment you performed in part B above. Could another student criticize *your* experiment as student 2 criticized the experiment of student 1?

   If not, explain why not; otherwise, redesign your experiment so that student 2 can no longer criticize your experiment. Then perform your new experiment.

D. In part A of this experiment, you listed a number of different things that you or someone else might think influence the sinking or floating behavior of an object. Test each of the items you listed and decide whether they do or do not affect sinking and floating. Be sure to write out your experiments and record the objects that you use.

   Test also the things that you did not think would influence sinking and floating behavior.

   On the basis of your experiments, which properties appear to influence sinking and floating?

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Exercise 12.3

A. Review the experiments *and the reasoning* that you used to decide whether the *volume* of an object affects whether it sinks or floats. Review the experiments *and the reasoning* that you used to decide whether the *shape* of an object affects whether it sinks or floats.

How did the results of the two experiments differ?

Describe how the results in each case led you to your conclusions about whether the property that was being tested affected sinking and floating.

B. Three students have performed an experiment in which they tested the sinking and floating behavior of three objects, each of the same volume but of different mass. They found that two of the objects sank while the third object floated. Consider the following discussion among them.

Student 1: *"We held the volume constant and changed only the mass. Since we saw a difference in behavior, it must be due to the change in mass. So mass does affect sinking and floating."*

Student 2: *"In our experiment, we didn't hold everything fixed; the mass and the density changed. We can't say the mass was solely responsible for the change in behavior, but we can say that when the volume is kept the same, then changing both the mass and the density of the object affects sinking and floating."*

Student 3: *"That's right, we didn't change only one thing, we changed two. We didn't do the experiment correctly. We should have kept everything but the mass constant. Since we didn't, our experiment is useless; it doesn't tell us a thing."*

Discuss this dialog with your partner. Do you agree with any of the students? If so, why? If not, explain why not.

C. In your answer to part A of this exercise, you may have assumed that it was possible to investigate the role of volume by changing the volume and holding all other variables constant. However, as you saw in part B, the interpretation of the results of an experiment may be more difficult if several of the variables are related in some way, as are the mass, volume, and density of an object.

Review your answer to part A. If you had assumed in your answer that all the variables but volume could be held constant, then modify your answer so that you agree with it now.

✔ Check your reasoning for this and the first few exercises with a staff member.

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Exercise 12.4

A. Imagine you are given two objects, each of the same mass. One of the objects sinks. What can you say about the sinking and floating behavior of the other object?

> Are you able to predict the sinking and floating behavior of an object if you only know the mass of that object? Explain your reasoning. Refer explicitly to experiments that you have conducted to answer this question.

B. Imagine you are given two objects with the same volume. One of the objects sinks. What can you say about the sinking and floating behavior of the other object?

> Are you able to predict the sinking and floating behavior of an object if you know only its volume? Explain. Be sure to refer to any experiments you have conducted that allow you to answer this question.

Until now we have used the idea of control of variables to design experiments to test whether a property of an object (e.g., volume) affects the sinking or floating behavior of that object. Through those experiments, you have seen that both mass and volume influence sinking and floating behavior. However, as the previous exercise illustrates, knowing either the mass alone or the volume alone is not enough to predict whether an object will sink or float.

In order to decide whether a variable can be used to *predict* sinking and floating behavior, we must perform experiments different from those we use to decide whether that variable *influences* sinking and floating behavior. Each type of experiment involves the idea of control of variables in a different way. The following exercise illustrates the way in which control of variables is used to decide whether some variable can be used to predict sinking and floating behavior.

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Exercise 12.5

A. Two students are discussing an experiment they have performed. Their experiment consisted of placing four cylinders of the same volume but different mass into beakers of water; two cylinders sank and two floated.

Student 1: *"This experiment tells us something about mass. The behavior changed when we kept the volume the same and changed the mass of the objects. So changing mass while keeping volume constant does affect sinking and floating."*

Student 2: *"This experiment tells us something about volume. We can tell that volume is not enough to predict sinking and floating behavior, since the behavior of each cylinder was different."*

Discuss the remarks of the two students with your partner. With which student do you agree? Explain your reasoning.

B. Compare the reasoning used by the students above with the reasoning you used in answering parts A and B of the previous exercise. Do you still agree with your answers and explanations to those questions? If not, why not?

C. Describe the experiments you need to perform in order to decide whether a certain variable can be used to predict the behavior of a system. As part of your description, include an explanation of how you would use the results of the experiments to decide.

✔ Check your reasoning with a staff member.

In the following experiments, we will begin to look for a property that does allow us to predict the sinking and floating behavior of an object.

Experiment 12.6

A. Find (by measurement) the densities of the objects that you have been using. Measure the largest object that you have of each substance.

Why is it a good idea to measure the largest objects?

B. Compare the sinking and floating behavior of objects having the same volume but different densities.

Compare the sinking and floating behavior of objects having the same mass but different densities.

What do the results of these experiments indicate? Explain.

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C. Compare the sinking and floating behavior of objects having the same density but different volumes.

Compare the sinking and floating behavior of objects having the same density but different masses.

What do you conclude from these experiments? Explain.

Experiment 12.7

A. Imagine that you are given two objects of the same density. One of the objects sinks. On the basis of your experiments, what can you say about the sinking and floating behavior of the other object? Explain. Describe the experiments that you used to decide your answer.

Is there any one property of an object that you can use to predict the sinking and floating behavior of the object? Explain your answer.

B. Some objects float high in the water, others float low in the water. What properties of an object are associated with its level (high or low) in the water? (You may need to perform some experiments in order to answer this question.) Explain your reasoning.

✔ Discuss your answers to this exercise and the preceding experiment with a staff member.

Experiment 12.8

A. Obtain some alcohol and some very salty water from the staff. The alcohol has a density 0.79 g/cm^3 and the salt water has a density of 1.20 g/cm^3. Do not throw away these liquids when you are done. Return them to the staff.

Test the behavior of each of the objects used in Experiment 12.6 in each of the liquids.

B. Compare the level at which these objects float in water, alcohol, and salty water. Wash off and dry the objects after taking them out of each of the liquids and when you are finished with the experiment.

What variable is being investigated in this experiment?

Does that variable influence the sinking and floating behavior of an object? Explain.

Can that variable be used to predict the sinking and floating behavior of an object? Explain.

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C. How do the results of this experiment affect your previous answer to the question:

> What do you need to know in order to be able to predict the sinking and floating behavior of an object?

✔ Explain your reasoning.

Exercise 12.9

A. Formulate a concise statement of what information you need in order to predict whether an object will sink or float in a liquid.

B. Review the reasoning leading from your observations to your conclusion.

Exercise 12.10

Some students find that two objects (labeled A and B) balance when placed in opposite pans of a balance. They recall from their explorations with water that one floated and one sank. However, they cannot remember which object floated. They know from previous investigations that the volume of object A is greater than the volume of object B.

Which object floats? Justify your answer.

The experiments thus far in this section have been chosen to lead to a particular way of understanding sinking and floating. There are often several ways of understanding the same physical behavior, and this is the case with floating. The following experiment gives a brief view of another way of thinking about the physics of floating.

Experiment 12.11

Obtain a set of wood rods from the staff and measure the mass of each.

A. Float the rods in a narrow graduated cylinder of water. Measure the volume of water displaced while the object is floating (not submerged). Also find the mass of the displaced water.

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B. Repeat the experiment using salt water and then again using alcohol. Then fill in the following table:

Wood rods	Fresh water displaced		Salt water displaced		Alcohol displaced	
Mass	Mass	Volume	Mass	Volume	Mass	Volume

C. State a rule for predicting how much liquid will be displaced by a floating object.

Experiment 12.12

Obtain a test tube, a stopper, a number of paper clips, and a bucket of water. Place the stopper in the test tube and place the tube into the water. What do you observe?

Use your balance and a graduated cylinder to take the measurements necessary to *predict* the minimum number of paper clips you must put inside the stoppered test tube in order for the test tube to sink. Perform the experiment and check your prediction.

Exercise 12.13

A 100 mL graduated cylinder is filled with water *to the very top* of the cylinder. A small object is then lowered slowly into the graduated cylinder but not pushed down into the water. The volume and mass of the water that runs over the side of the tank are then measured and found to be 10.2 mL and 10.2 g respectively.

What do you know about the mass and volume of the object if (1) the object sinks, and (2) the object floats.

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Part C: Scientific representations

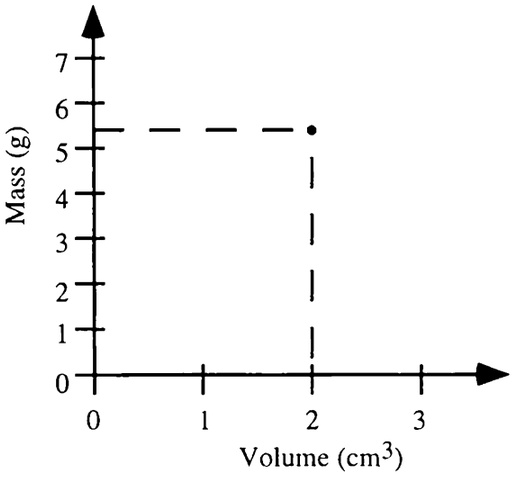
Physical concepts and the relationships between them can be represented in many ways (e.g., graphs, equations, diagrams). In this part of the module, we examine graphical and algebraic representations for the concepts of mass, volume, and density.

Section 13. Graphing mass and volume

Much of the information contained in a graph cannot be extracted at a glance but requires skill and care to obtain. The material in this section is an introduction to some of these graph interpretation skills. There are many kinds of graphs, but only one type is discussed here: graphs that represent a visual image of a set of number pairs—for example, the masses and volumes of a set of objects.

Points on a graph

Each point on a graph represents a pair of numbers. These two numbers are represented by the location of a point with respect to two perpendicular lines called *axes.* On the graph at the right, the *vertical axis* has a scale to represent mass, and the *horizontal axis* has a scale to represent volume. The single point on the graph represents the mass and volume of a particular piece of aluminum with a mass of 5.4 g and a volume of 2.0 cm^3. The position of the point is determined in this way: find 2.0 on the horizontal scale; go straight up, directly above 2.0, until even with 5.4 on the vertical scale.



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In the language of graphs, the number 2.0 is called the *horizontal coordinate,* or *abscissa.* The number 5.4 is called the *vertical coordinate,* or *ordinate.* This graph would be described as a graph of mass versus volume or mass vs. volume.

The first rule of graph interpretation is to know what a point on the graph represents. In this case, the point means that 2.0 cm^3 of aluminum has a mass of 5.4 g.

Scales

Whenever a graph is plotted, it is necessary to choose scales for the axes. In this course, we will always have equal distances along the axis represent equal amounts of a quantity.

Two guidelines to keep in mind when choosing graph scales are:

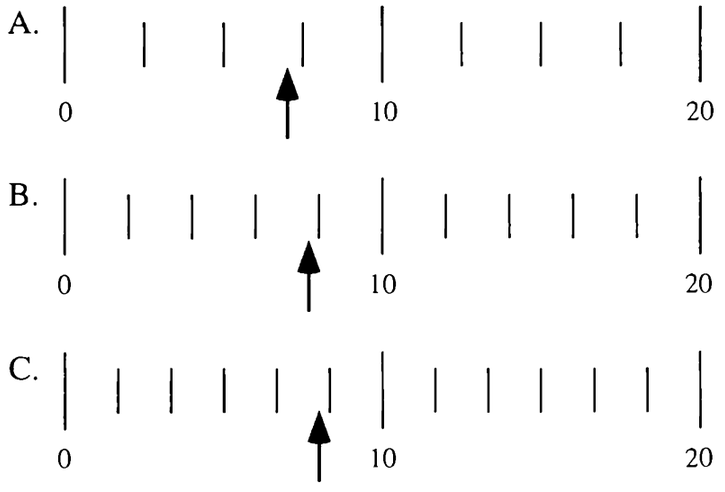
(1) Take care to number the lines on the graph so that all of your data will fit on the graph.

(2) Number the lines so that it is easy to read values between them.

The following exercise gives examples of easy- and hard-to-read scales.

Exercise 13.1

Read the values indicated by the arrows on the following scale:



✔ Check your answers with a staff member.

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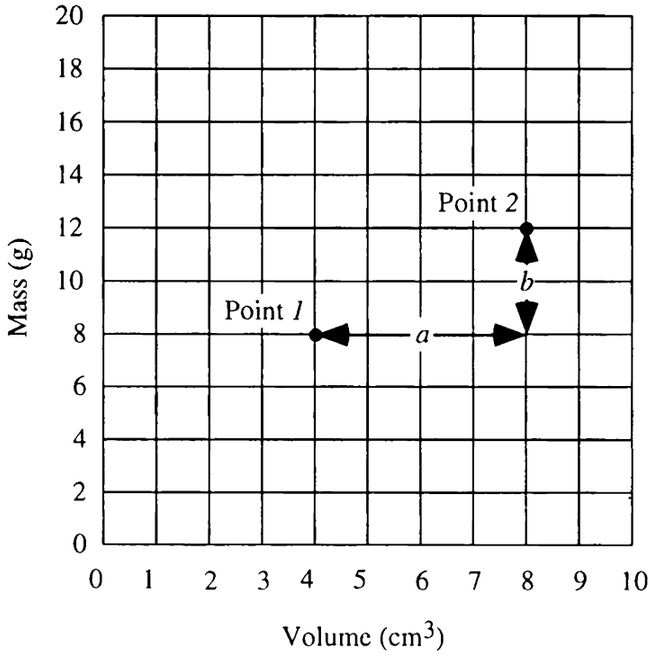
A scale is easy to read if it is easy to estimate the correct reading when the pointer falls between two lines. This is usually the case if the smallest division represents 1, 2, or 5, or some multiple of 10 of these numbers, such as 0.1, 0.2, 0.5, 10, 20, 50, 100, and so on. It is much harder to read scales in multiples of other numbers such as 3, 7, and so on.

Sets of points

A graph gives information not only about each point but also about relations between points. Vertical and horizontal lengths on a graph often have important interpretations. These lengths are always measured by reading the scales on the axes, never with a ruler. On the following graph, the vertical length, *b,* represents a number of grams. The horizontal length, *a,* represents a number of cubic centimeters. It might seem appropriate to measure the direct, slanted distance between the two points on the graph and label it *c.* However, the length of this line does not represent a number of anything. The direct length between these two points is meaningless.



In the diagram above, the vertical length between the two points is the *rise* from the first point to the second. The horizontal length is the *run.*

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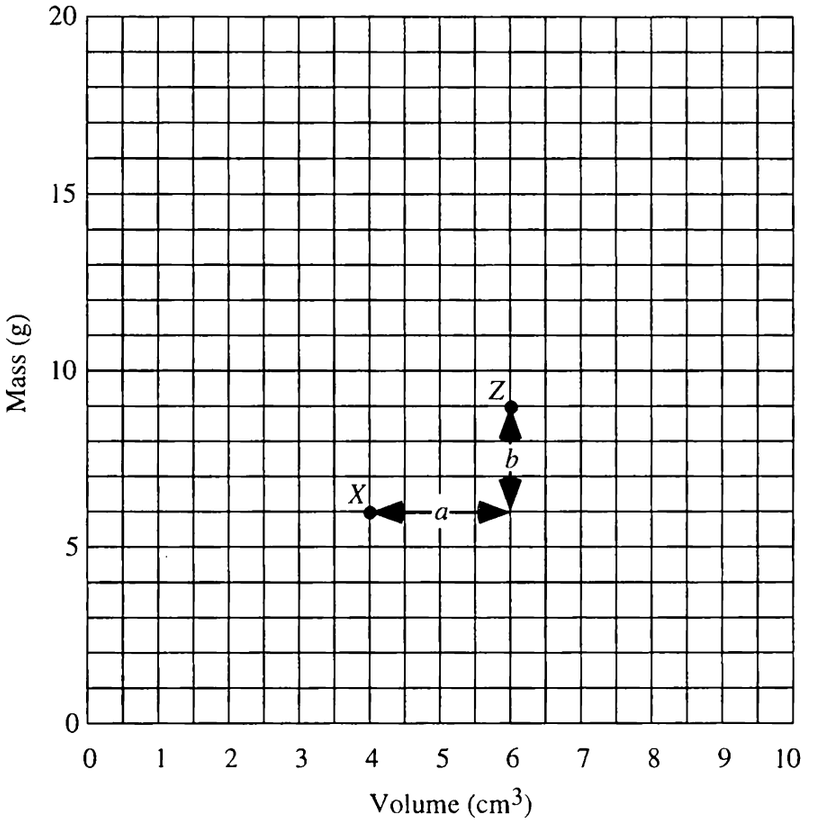
Exercise 13.2

Suppose you have a lump of clay represented on the graph by point *X*. A second lump, lump *Y*, has not been measured. Lump *Y* is added to lump *X*, and they are joined to form a new lump. The new lump is represented by point *Z*.

A. What does the length labeled *a* represent? Can the number represented by *a* be interpreted as the mass or volume of a particular piece of clay? If so, identify the piece of clay. If not, explain why not.

B. What does the length labeled *b* represent? Can the number represented by *b* be interpreted as the mass or volume of a particular piece of clay? If so, identify the piece of clay. If not, explain why not.



✔ Check your reasoning with a staff member.

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Exercise 13.3

A. Fill in the following table of masses and volumes for ice, aluminum, and iron. Use values from the table of densities in the Appendix to calculate the missing values.

Ice		Aluminum		Iron	
Mass (g)	Volume (cm^3)	Mass (g)	Volume (cm^3)	Mass (g)	Volume (cm^3)
_____	0.0	_____	0.0	0	_____
_____	1.0	_____	1.0	10	_____
_____	2.0	_____	2.0	20	_____
_____	3.0	_____	3.0	30	_____
_____	4.0	_____	4.0	40	_____

B. Plot these points on a *single* mass versus volume graph.

C. Draw a line through the ice points. Do the same for the other two sets of points.

Is there any interpretation you can give to the *line* drawn through the points you plotted for ice? for aluminum? for iron?

Lines on graphs

In the preceding exercise, if more points were plotted for aluminum, ice, or iron, the new points would also lie on the three lines. The lines can be used to make predictions. For example, using the aluminum line on the graph above, we can find the mass of 3.6 cm^3 of aluminum. Even though 3.6 cm^3 was not one of the original volume numbers, we can read the mass off the vertical axis: 9.7 g. Thus we would predict that 3.6 cm^3 of aluminum has a mass of 9.7 g.

Exercise 13.4

A. Based on your graph for ice in Exercise 13.3, predict the volume of 6.1 g of ice.

B. Calculate the volume of 6.1 g of ice using the density of ice. How does the result compare with your answer for part A?

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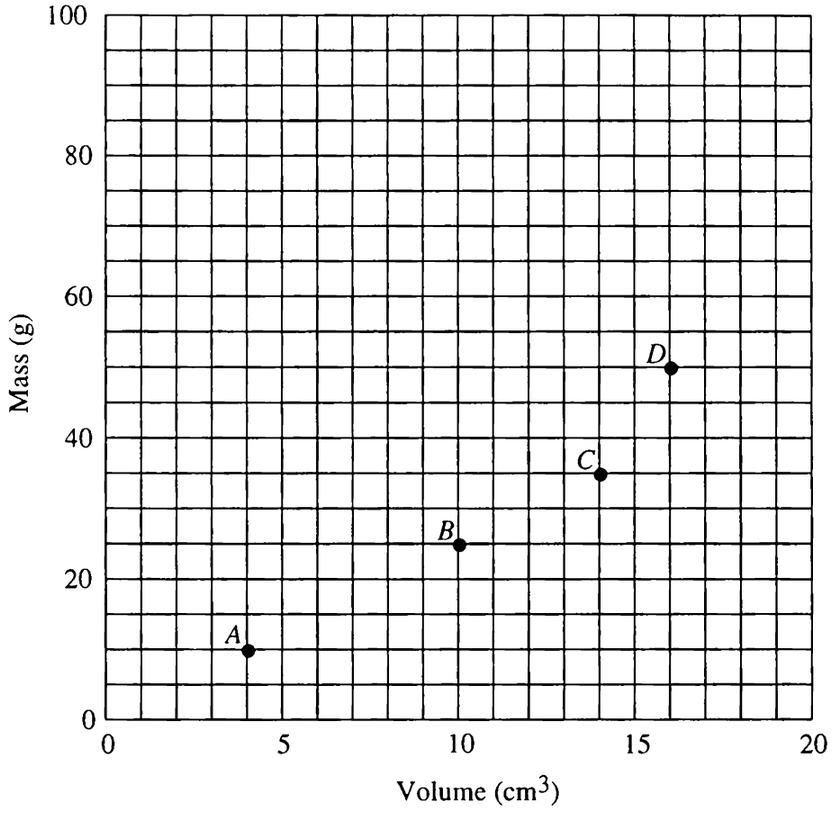
Exercise 13.5

Four points are labeled on the graph below. Calculate the ratio of rise to run for each of the following pairs of points:



A. From *A* to *B* From *A* to *C*

From *B* to *C* From *C* to *D*

B. How can you account for the fact that the numbers you calculated in part A are not all the same?

✔ Check your reasoning with a staff member.

Slope

The ratio of rise to run, $\frac{\text{rise}}{\text{run}}$, is constant for straight-line graphs. It does not matter which points along the line are chosen to compute the ratio. For a straight-line graph, the ratio of rise to run is called the *slope* of the graph. This one number applies to the entire line. The slope of a straight-line graph usually has an important interpretation in terms of the physical system represented on the graph.

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If the graph is not a straight line, however, the ratio of rise to run is not constant. Conversely, if the ratio is not constant, the graph is not straight.

Exercise 13.6

A. Find the slopes of the mass versus volume graphs for aluminum, ice, and iron plotted in Exercise 13.3.

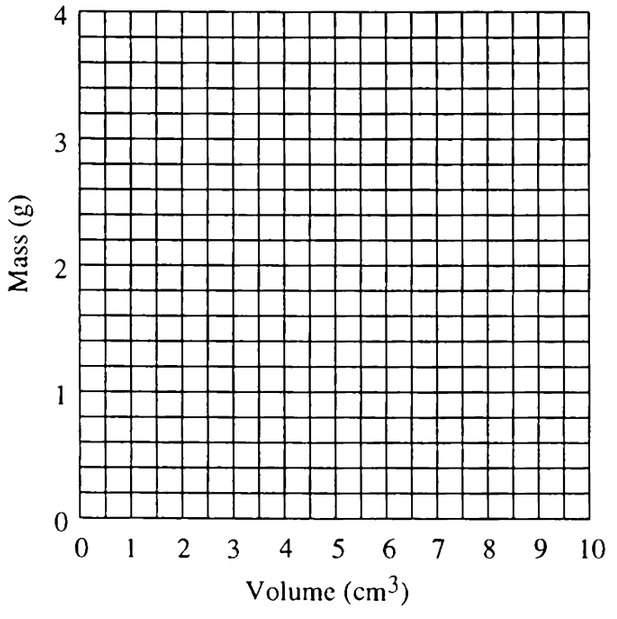
B. What does the slope number tell you about each substance?

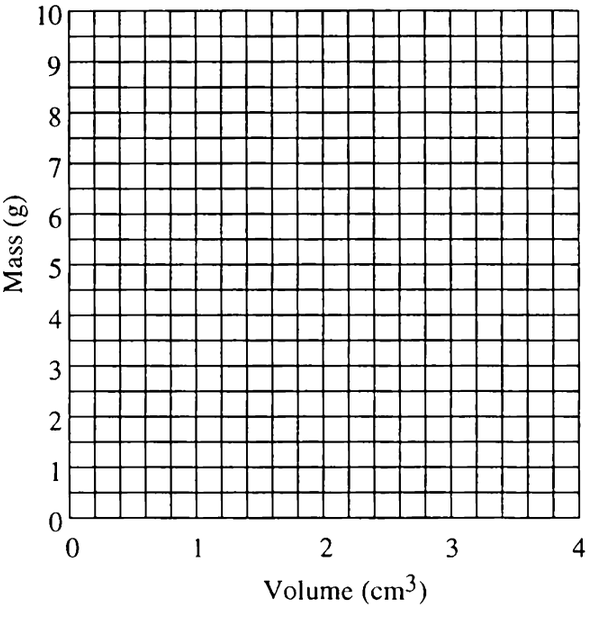
Exercise 13.7

A. Plot your data for ice from Exercise 13.3 on both of the axes below.

B. How do the slope numbers of the two graphs compare? How do you explain this?

✔ Discuss Exercise 13.6 and Exercise 13.7 with a staff member.





Slope or height?

Both the slope of a graph and the coordinates contain information. Each time information is to be extracted from a graph, the choice must be made whether to use the coordinates, or the slope, or other features.

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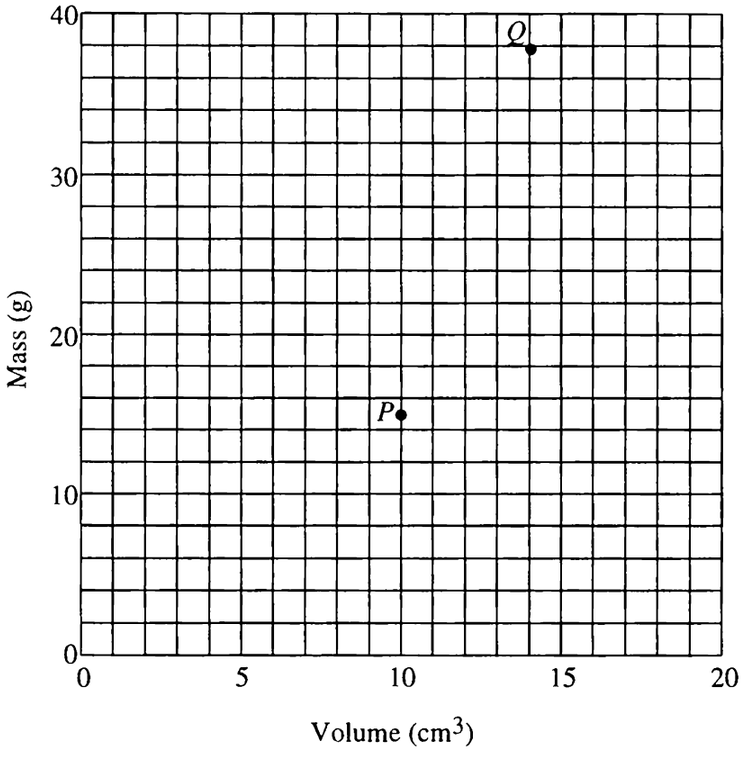
Exercise 13.8

Suppose we have a lump of clay represented by point P on the mass versus volume graph below. Enough mystery metal is mixed with this lump of clay so that the resulting ball of clay and metal is represented by point Q.

A. What is the density of pure clay (without the metal)? Explain your reasoning.

B. What is the density of the mystery metal? Explain your reasoning.

C. Identify the mystery metal.



✔ Check your reasoning with a staff member.

Exercise 13.9

In Section 12, you found the densities of a number of different materials. On a single mass versus volume graph, plot lines that represent objects made out of each of the substances that you used in that section. Include on your graph a line that represents water.

What does it mean to say that a "line represents water" on a mass versus volume graph?

Imagine that you are given an object made out of a material different from those you have plotted and that the line that represents that material is drawn on your mass versus volume graph. Explain how you could tell by looking at your graph (without doing any calculations) whether that material would sink or float in water.

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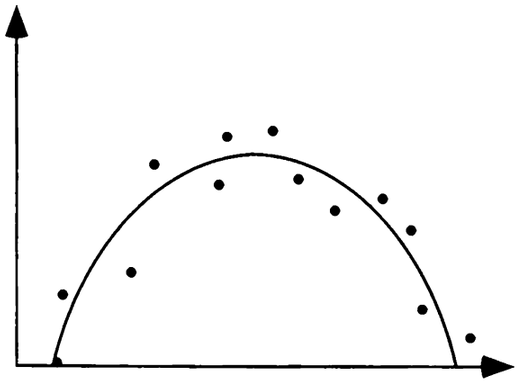
Graphs of experimental results

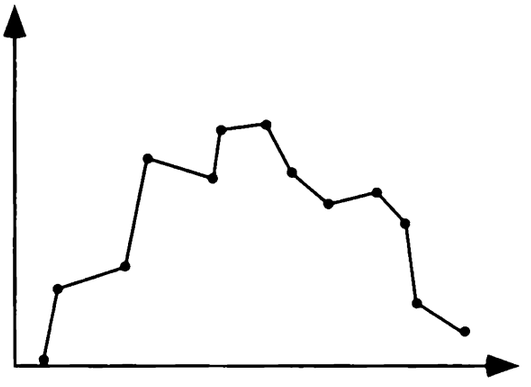
Because of uncertainty, experimental results never make a perfect pattern on a graph. In Exercise 13.3 we used a given value of the density of aluminum to calculate the masses for given volumes. The plotted points lay in exactly a straight line. However, if we had measured the masses and volumes of pieces of aluminum instead, the points on the mass versus volume graph probably would not lie exactly on the line we drew.

When the points do not lie exactly in a neat pattern, it is reasonable to assume that half the experimental measurements are too high and half too low. The usual practice is to draw a smooth line through the middle. The smoothness of the line reflects our belief that there is a simple functional relationship between the quantities being graphed.

Never draw a connect-the-dots, zigzag line through data points. Drawing a zigzag line would imply that you thought that the quantities were related in a very irregular way and that the measured data were exact with no uncertainty.



Reasonable



Unreasonable

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Experiment 13.10

A. Obtain a set of circular objects from the staff. Measure the circumference (distance around) and the diameter (distance across) of these objects.

B. Plot a graph of circumference versus diameter for your objects.

C. Should the point *(0,0)* be included on the graph? To help answer this, ask yourself what the point *(0,0)* represents and then ask whether it represents information known to be true.

D. Draw a smooth line through the middle of your points. A straight line should fit the data well.

E. Compute the slope of the line. In doing this computation, use the line you drew rather than the data points. Do not use the actual data points unless they happen to lie on the line. Always calculate the slope by choosing two points that are far apart; for example, at opposite ends of the line. If the points chosen are close together, errors in reading the graph can result in the calculation of an incorrect value for the slope.

F. The slope of the line is a special number for circles. How do you interpret this number?

   The number obtained as the slope of the graph is encountered so frequently that it is given its own name, π (pi).

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Section 14. Interpreting algebraic expressions

An algebraic problem solution can be divided into two phases. The first phase is the scientific reasoning that goes into writing the algebraic equation. The second phase is using the rules of algebra to solve the equation. The material presented here will concentrate on the first phase: translating scientific reasoning into algebraic equations.

Equations are composed of three parts: the right side, the left side, and the equal sign. The sides are called *algebraic expressions.* We will first study expressions, then the equal sign.

Expressions stand for numbers. Expressions do not have equal signs in them, only symbols for mathematical operations, such as + and –, that are to be performed on numbers and letters. An expression tells what mathematical operations are to be done to the numbers in it.

Before working with expressions involving algebraic variables (letters like x), we will first give a few examples of expressions involving only numbers.

Sample problem

An object has a mass of 108 g and a density of 2.7 g/cm^3. Write an expression for its volume in cm^3.

Sample solution

In order to find the volume, we need to find out how many packages of 2.7 g fit into 108 g. The expression for the volume is thus:

$$\frac{108}{2.7}$$

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Comments

It is essential to think of an expression like $\frac{108}{2.7}$ as *one number,* not two numbers. Think "this number is the volume of the object." By dividing 108 by 2.7, we find that the number is 40. Both 40 and $\frac{108}{2.7}$ mean exactly the same thing: 40. The pattern of ink on paper that looks like this:

$$\frac{108}{2.7}$$

represents a single number, the volume of the object.

Sample problem

Suppose we have 50 g of a substance with density 3 g/cm^3. We then remove 20 g of this substance. Write an expression for the volume of the remaining piece.

Sample solution

First, we should find the mass of the remaining piece. Since we started with 50 g and removed 20 g, the remaining mass is 50 – 20. If we want to find the volume of the remaining substance, we must find out how many 3 g packages fit into the remaining mass. We do this by dividing the remaining mass by 3:

$$\frac{(50-20)}{3}$$

Comment

This expression should be thought of as one number: the volume of the object. A little calculating shows that the symbol $\frac{(50-20)}{3}$ stands for 10. It would be equally correct to say "the volume is 10 cm^3" or "the volume is $\frac{(50-20)}{3}$ cm^3."

Exercise 14.1

Suppose we have 80 g of aluminum and we take away a volume of 15 cm^3 of aluminum. Write an expression for the mass of the remaining aluminum. (The density of aluminum is 2.7 g/cm^3.)

Do not actually do any calculations.

✔ Check your reasoning with a staff member.

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Exercise 14.2

Suppose we have 40 cm^3 of a certain liquid. If we add 20 cm^3 more liquid, the total mass is 80 g. Write an expression for the density of the liquid. Do not actually do any calculations.

Exercise 14.3

Suppose we have 60 g of iron. We then remove 4 cm^3 of the iron. Write an expression for the volume of the remaining iron. (The density of iron is 7.9 g/cm^3.) Do not actually do any calculations.

✔ Check your reasoning for Exercises 14.1–14.3 with a staff member.

The difference between algebra and arithmetic is the use of variables—that is, letters like *x*—instead of known numbers. The letters, however, just represent numbers, so one should always think of *x, m,* etc., as just some numbers that have letters taking their places for the time being. Here are some examples of expressions involving variables:

$x + 5$ This expression stands for the number we get by adding the number *x* and the number *5*.

$4y + 3$ First we multiply 4 and *y*. We then add 3 to the result. The expression stands for this sum.

$\frac{8}{(1+z)}$ First we add 1 and *z*. We then divide 8 by the result. The expression stands for the number we get by this division.

A major portion of the scientific reasoning in an algebraic solution goes into writing the expressions on each side of the equation. The following example illustrates the thinking involved in writing an expression.

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Sample problem

Suppose we have 15 g of gold, and we sell m grams of it. Write an expression for the mass of gold we have left.

Sample solution

If we start out with 15 grams of gold and sell m grams, then we will have m grams less than the 15 we started with. Thus, the answer is m less than 15. Another way to say this is to say that we have 15 grams minus m grams. Written in symbols, this is:

$$15 - m.$$

Comments

Caution must always be observed when dealing with a variable like m, which is so often used to stand for mass. Here, for example, there are three masses in the problem:

the initial mass of gold,

the mass of gold sold,

the mass of gold left.

The letter m stands for only the second of these. It is important to state clearly what a variable stands for. Telling what a variable stands for is called *defining the variable.* In this case, the definition of the variable m is contained in the phrase "we sell m grams of it" in the statement of the problem. These few words tell us that m stands for the number of grams of gold sold.

If it seems difficult to write an expression like the one above, it may help to work with an actual number first, rather than with the variable. In the above problem, for example, we might pretend that 3.2 grams of gold are sold. In that case we would have 15 grams minus 3.2 grams of gold left. In symbols, this would be:

$$15 - 3.2$$

which is 11.8 grams. Going through the same reasoning and following the same steps with m grams rather than 3.2, we get

$$15 - m$$

as an expression for the mass of gold left.

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Exercise 14.4

Write an expression for the mass of a piece of tin that has volume V. The density of tin is 5.75 g/cm^3.

Exercise 14.5

Suppose we have an 18 g sample of a certain metal. Let D be the density of this metal. Write an expression for the volume of the sample.

Algebraic expressions can become quite complicated. Complex expressions can be written in small steps, with each step being simple. Here is an example.

Sample problem

A certain sample of gas had a mass of 1.2 g and a volume of 600 cm^3. The gas was compressed to a volume of 40 cm^3, but in the process some of the gas escaped. If the density after compression (in g/cm^3) was D, write an expression for the mass of gas that escaped.

Sample solution

First, realize that the mass that escaped is just the difference between the mass still present after the compression and the initial mass. The mass still present after the compression can be obtained from the final density and volume.

(1) After compression, the mass in grams of each cubic centimeter was D. There were 40 such cubic centimeters. Thus the mass in grams after compression was $40D$.

(2) The initial mass was 1.2 g, so the difference is $1.2 - 40D$. Thus the mass in grams of the gas that escaped was

$$1.2 - 40D.$$

Comments

Each step in this solution is similar to a simple problem worked in the last section. The only difference is that this problem required several steps and a plan for deciding which steps should be taken to arrive at the answer. Such planning really has nothing to do with algebra; rather it is a matter of scientific reasoning. If this kind of planning seems difficult, remember that the letters just stand for numbers and think what you would do if you knew the actual numbers.

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Exercise 14.6

A sample of air contained in a balloon had a mass of 1.2 g and a volume of 920 cm^3 in the lab. When the balloon was taken up into the mountains, its volume increased by an additional volume V. Write an expression for the density of the air in the balloon up in the mountains.

✔ Check your reasoning with a staff member.

Exercise 14.7

Suppose we have a piece of iron with mass 800 g and density 7.9 g/cm^3. We then remove a mass m from the piece of iron. Write an expression for the volume of iron left.

Exercise 14.8

Suppose we have an object with a mass of 50 g and density D. We then take away 6 cm^3 of material. Write an expression for the remaining volume.

Exercise 14.9

Suppose we start with 75 g of a liquid with density D. We then add a volume V of the same liquid.

A. What is the final mass of liquid?

B. What is the final volume of liquid?

✔ Check your reasoning with a staff member.

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Section 15. Interpreting the equal sign

When applied to science, an equation is a statement about the physical world. The equal sign states that the numbers on the two sides of an equation are the same. For example, an equation might make the statement that two pieces of magnesium have the same density, as in the following example problem.

Sample problem

A 50 cm^3 piece of magnesium has a mass of 87 g. What is the mass of a 6 cm^3 piece of magnesium?

Sample solution

We must first realize that the only reason we can solve this problem is that the two objects both have the same density. That is why the large object gives us some information about the small one. To solve the problem algebraically, we write an equation which says that the densities are the same. First we write two expressions, one for the density of the large object and one for the density of the small object.

$\frac{87}{50}$ is the density of the large object in g/cm^3.

If we let m be the mass in grams of the small piece of magnesium, then

$\frac{m}{6}$ is the density of the small object in g/cm^3.

We now use the equal sign to make a statement about the real world, namely that these two numbers are the same.

$$\frac{87}{50} = \frac{m}{6}$$

Without the knowledge from physics that the density is the same for all pieces of magnesium, it would be impossible to set these two expressions equal. After the equation is written, no more physics enters the problem. We now just solve for the answer:

$$m = 10.4$$

Thus a 6 cm^3 piece of magnesium has a mass of 10.4 g.

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Suggested format for problem solutions

Problem solutions should be presented in a clear and organized manner. When a solution is presented in an orderly fashion, it is easy to check for errors. Also, a grader can follow the steps and often give partial credit.

Following is a suggested format for presenting the solution to the preceding sample problem. In solving the rest of the problems in this course, the use of a similar format is strongly recommended.

Given: Mass of larger object is 87 g.
Volume of larger object is 50 cm^3.
Volume of smaller object is 6 cm^3.

Find: Mass of smaller object.

Definition of variable: Let m be the mass in grams of the smaller object.

Justification of equation: The density of the first object in g/cm^3 is 87/50. The density of the second object in g/cm^3 is $m/6$. The densities are equal since the objects are made of the same homogeneous material. Thus:

$$\frac{m}{6} = \frac{87}{50}$$

Calculations:

$$m = \frac{(87 \times 6)}{50}$$

$$m = 10.4$$

Answer: The mass of the smaller object is 10.4 g.

Note that each successive line in the calculations is below the previous one, not off to the side. Notice also that the final answer has units and is boxed.

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Sample problem

Suppose we want to add some alcohol, with a density of 0.80 g/cm^3, to 50 g of water in a beaker to get a mixture with a total mass of 66 g. What volume of alcohol should be poured into the beaker?

Sample solution

We can solve this problem because we know that when alcohol is added to water, the total mass will just be the mass of the water plus the mass of the alcohol, in compliance with the law of conservation of mass in a closed system. We therefore write expressions for the mass before and after mixing. Let V be the volume of the alcohol added in cm^3. Then adding a volume, V, of alcohol can be thought of as adding 1 cm^3 of alcohol V times. Each cm^3 of alcohol added corresponds to a mass of 0.80 g of alcohol. Adding 0.80 g of alcohol V times results in a total mass of alcohol added that is $0.80V$. Thus,

$50 + 0.80V$ is the total mass in grams before mixing;

66 is the total mass in grams after mixing.

Because mass is conserved, these two numbers will be exactly the same. Therefore, we are entitled to set them equal:

$$50 + 0.80V = 66.$$

This has the solution:

$$V = 20.$$

Hence 20 cm^3 of alcohol should be added.

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Sample format

Given: Density of alcohol is 0.80 g/cm^3.
50 g of water initially in beaker.
66 g of mixture in beaker at the end.

Find: Volume of alcohol added.

Definition of variable: Let V be the volume of alcohol added in cm^3.

Justification of equation: We know, by the law of conservation of mass, that the combined mass of the liquids before they are mixed is equal to the final mass of the mixture. The mass of mixture at the end is given as 66 g. The mass of the water plus the mass of the added alcohol is the combined mass before the two liquids are mixed. We are given the mass of the water, but we are not given the mass of the added alcohol, so we must first make a side calculation to find an expression for the mass of the added alcohol.

Aside: We know the volume in cm^3 of the added alcohol is V, and the mass of one cm^3 of alcohol is 0.80 g. Therefore the mass in grams of the added alcohol is $0.80V$.

We now can write that the mass of the water plus the mass of the added alcohol is $50 + 0.80V$. We use the law of conservation of mass to set this equal to the mass of the mixture at the end. Thus:

$$50 + 0.80V = 66$$

Calculations:

$$0.80V = 66 - 50$$

$$V = \frac{16}{0.80}$$

$$V = 20$$

Answer: The volume of alcohol added is 20 cm^3.

Sometimes an equation makes the especially simple statement that two expressions for the same thing are equal. The following problem illustrates this kind of statement.

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Sample problem

What is the volume of 6 g of aluminum? (Aluminum has a density of 2.7 g/cm^3.)

Sample solution

This problem can be solved by writing two expressions for the mass of the piece of aluminum in question. If we let V be the volume of the piece in cm^3, then:

$2.7V$ is an expression for the mass of the piece.

6 is also an expression for the piece's mass.

Now we can write the equation:

$$6 = 2.7V$$

The reason for setting the two sides equal is that they are expressions for exactly the same number: the mass of the piece of aluminum. The solution is:

$$V = 2.2.$$

The volume of the aluminum is 2.2 cm^3.

Sample format

Given:	Mass of piece of aluminum is 6 g. Density of aluminum is 2.7 g/cm^3.
Find:	Volume of piece of aluminum.
Definition of variable:	Let V be the volume of piece in cm^3.
Justification of equation:	The mass in grams of a volume, V, of aluminum is $2.7V$. The mass in grams of that piece of aluminum is also equal to 6 g. Thus we can set both expression equal to each other: $6 = 2.7V$
Calculations:	$\frac{6}{2.7} = V$ $2.2 = V$
Answer:	The volume of 6 g of aluminum is 2.2 cm^3.

Notice that in the sample format above, we did not have an aside in which it was explained why the expression $2.7V$ represents the mass of a piece of aluminum with a

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volume V. If the reasoning is not obvious to you, you should go back and put in the aside as was done in the previous example. Often, relatively straightforward steps in the reasoning are left out when solving a problem, although this can make it difficult for a grader to give partial credit. However, the justification for setting two sides of an equation equal to each other should never be left out.

Expressions may also be set equal to fulfill a special condition stated in the problem, as illustrated in the following problem.

Sample problem

Suppose we have a 200 cm^3 sample of water and a 700 cm^3 sample of water. If we add equal volumes of water to each sample, how much must be added to each to make one sample twice as large as the other?

Sample solution

We begin by writing expressions for the volumes of the two samples. Let V be the volume in cm^3 of water added to each of the samples.

$200 + V$ is an expression for the volume in cm^3 of the small sample.

$700 + V$ is an expression for the volume in cm^3 of the large sample.

We can now write the equation:

$$700 + V = 2\,(\,200 + V\,)$$

The reason we are justified in setting these expressions equal is that the problem stipulates (requires) that the volume of the large sample should be two times as large as the volume of the small sample. The solution of the equation is:

$$V = 300$$

Thus 300 cm^3 of water must be added to each sample. Note that this satisfies the requirement of the problem since the volume of the small sample will be 500 cm^3 and the volume of the large sample will be 1000 cm^3.

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Sample format

Given :	Volume of first sample is 200 cm^3. Volume of second sample is 700 cm^3.
Find:	Volume of water added to each sample.
Definition of variable:	Let V be the volume in cm^3 of water added to each sample.
Justification of equation:	The volume in cm^3 of the first sample after the water has been added is $700 + V$. The volume in cm^3 of the second sample after the water has been added is $200 + V$. We want twice the volume of the second sample to be equal to the volume of the first sample, thus we can write: $700 + V = 2(200 + V)$
Calculations:	$700 + V = 400 + 2V$ $700 - 400 = 2V - V$ $300 = V$
Answer:	The volume of water added to each sample is 300 cm^3.

Exercise 15.1

If 10 cm^3 of a liquid, with a density of 0.80 g/cm^3, is added to an original volume of the same liquid, the resulting mass is 60 g. What is the original volume? Follow the sample format above in your answer.

Exercise 15.2

Equal volumes of aluminum and iron are used to make a machine. If the total mass of the machine is 6 kg, what volume of each metal is used? Follow the sample format in answering this question.

Exercise 15.3

Initially, 500 cm^3 of air is in a balloon. Then 0.35 g of air is pumped into the balloon, and the balloon expands to a new volume of 700 cm^3. If the final density of the air inside the balloon is 0.0015 g/cm^3, what was the density of the air in the balloon initially? Follow the sample format in answering this question.

✔ Check your reasoning with a staff member.

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Misuse of the equal sign

A common mistake made in manipulating algebraic equations is to misuse the equal sign by using it to mean "and the next step is..." The equal sign does not mean this. It should only be used between two expressions that stand for the same number. In the following example, a student solves the equation $\frac{72}{10} = \frac{90}{V}$. The student uses the equal sign both correctly and incorrectly, as discussed in detail below.

$$\frac{72}{10} = \frac{90}{V} = \frac{72}{10}V = 90 = \frac{90 \times 10}{72} = V = 12.5$$

Incorrect

Discussion

- The first use of the equal sign is correct because the two expressions on opposite sides really do stand for the same number. (This is the original equation.)
- The second equal sign is used incorrectly because $\frac{90}{V}$ is not the same number as $\frac{72V}{10}$. The student is trying to say that multiplying by V is the next step, but it is wrong to use the equal sign for this purpose.
- The third equal sign is used correctly. $\frac{72V}{10}$ is the same number as 90. The equation $\frac{72V}{10} = 90$ is the same as the original, but both sides have been multiplied by V.
- The fourth equal sign is used incorrectly, again to mean "and the next step is..." Clearly 90 is not the same number as $\frac{90 \times 10}{72}$, so the equal sign should not appear between them.
- The fifth and sixth equal signs are used correctly because $\frac{90 \times 10}{72}$, V, and 12.5 are all exactly the same number.

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Section 16. Reasoning by analogy

An *analogy* is a correspondence between two situations, physical systems, processes, etc. If a topic is already familiar, an analogous one can often be quickly understood. To make an analogy clear, it is necessary to indicate how corresponding parts of the analogy are alike. The statement "A is like B" can be interpreted in many ways. It is important not only to specify the correspondences between the objects or substances in the two systems but also to specify the similarities between the processes in the two systems.

Sample problem

Make an analogy between measuring the area of a floor and the volume of a box.

Sample solution

Before making an analogy between the two quantities, we must define them carefully.

Area: Fit standard squares on the floor and count, estimating the number that fit. The resulting number is the area of the floor.

Volume: Fit standard cubes inside the box and count, estimating the number that fit. The resulting number is the volume of the box.

The following correspondences can be made:

Area	Volume
Physical objects	
floor	box
standard squares	standard cubes
Measurement process	
fitting inside	fitting inside
counting	counting

Now we must explain how these parts are alike:

- The floor and the box correspond in that each is measured by fitting standard units inside.
- Fitting standard units inside an object is the procedure used in finding both area and volume.
- The standard squares correspond to the standard cubes. Both are fitted inside the object that is being measured and both are counted.
- Counting standard units gives a measurement for both area and volume.

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The sample problem above illustrates an analogy between two quantities with similar operational definitions. Sometimes analogies are drawn between two very different physical systems.

Exercise 16.1

We often hear that heat rises. A more precise way of expressing this idea is to say that hot air rises. The situation this statement describes is analogous to the situation of a piece of wood in water.

A. Explain this analogy in detail. First identify what corresponds to the wood and what corresponds to the water. Then tell how these elements are alike. Note that there are several ways that these elements correspond.

B. Is the situation of the rising hot air analogous to that of a piece of wood floating on the surface of the water or that of a piece of wood released below the surface (underwater)? Explain your reasoning.

✔ Check your reasoning with a staff member.

By making an analogy between the hot air–cold air system and the wood-in-water system, we are able to arrive at an understanding of rising and falling air without having to do a series of experiments like the sinking and floating experiments in Section 12. By making analogies, we can avoid starting from scratch each time we study a new situation.

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Part D: Solutions of solids in water

In Part D of *Properties of Matter,* we study *solutions.* Some common examples are salty water, tea, and coffee. When two substances combine to form a solution, one is usually called the *solute* and the other is called the *solvent.* For a solution like salt water, the solid (salt) is the solute and the liquid (water) is the solvent. We say that the salt *dissolves in* the water or that the water *dissolves* the salt.

Dissolving involves two substances, the solute and the solvent. Note that dissolving is not the same as melting. In melting, there is only one substance involved, as when ice turns into water in a warm room.

Section 17. Dissolving in water

Experiment 17.1

The purpose of this experiment is to determine how much of a particular substance can be dissolved in 10 mL of water. In trying to determine how much of the substance will dissolve, you will make several tries at dissolving various amounts of solute in the 10 mL of water. Each time you will add solute until you overshoot and not all of the solute dissolves.

A. Obtain a sample of substance to use as a solute, a scoop, and a weighing dish from the staff. Accurately pour 10.0 mL of water into a test tube.

B. Whenever you add solute in this experiment, you should first place it in a weighing dish and find its mass. The mass of the weighing dish must thus be subtracted from each measurement. Find the mass of the weighing dish.

C. First try to dissolve one gram of the substance. If it all dissolves, then try to dissolve two more grams. If all this powder dissolves, try to dissolve four more grams. Continue this way, doubling the amount added, until not all of the added solute dissolves.

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D. Now start over again. First add as much solute as you added in the previous trial, except for the last (excess) addition. Instead of adding the same excess amount, add half what you added last time. If all of it dissolves, then try half that much additional solute, then half that much, and so forth. When some of the solute remains undissolved, start over again.

E. Continue the process of closing in on the exact amount that can dissolve. Stop when the last addition (the addition that does not completely dissolve) has a mass of 0.032 g or less.

F. Record your value for the maximum mass of your substance that can dissolve in 10.0 mL of water. Compare your results with those of your classmates. How do you interpret the variation in results?

✔ Check your results with a staff member.

Evidently there is a limit to the amount of these solids that will dissolve in 10 mL of water. A solution that will not dissolve any more solute is said to be *saturated.* Thus, a saturated solution contains the maximum amount of solute that can be dissolved in the solvent at hand.

Exercise 17.2

A. What happens when additional solute is put into a saturated solution? an unsaturated solution?

B. Modify this statement to make it correct: any amount of a substance, a little or a lot, can be dissolved in a given volume of water.

Experiment 17.3

✔ Find a staff member to assist you; then do the saltwater tasting experiment.

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Section 18. Concentration of solutions

An important quantity used in describing a solution is the number obtained by dividing the amount of dissolved solute by the amount of solvent. This number is called the *concentration* of the solution. It can be expressed in various forms depending on how the amounts are measured. The form we will use most often is the number obtained by dividing the mass of solute by the volume of solvent.

The concept of concentration is useful because solutions are homogeneous. Since solutions are homogeneous, the total mass of dissolved solute is divided equally among all equal volumes of solvent. In a saltwater solution, for example, the total mass of dissolved salt is shared equally by all of the 1 cm^3 packages of water in the solution. The concentration number can therefore be interpreted as the mass of solute dissolved in each one unit of volume of solvent (e.g., the mass of salt dissolved in each one cubic centimeter of water).

Many properties of a solution are closely related to its concentration. For example, the sea contains much more salt than a spoonful of sea water, but the concentration is the same and so is the taste.

Solution concentration is described qualitatively by terms like concentrated, strong, weak, dilute. These terms do not have precise definitions; their meanings can vary from situation to situation. In this sense, they are similar to terms like tall and short. Two meters is short when applied to a tree but tall when applied to grass. Similarly, in medicine, 0.0001 g/mL may be dilute when speaking of salt water but concentrated when talking about a potent drug. The way these terms are used with a particular solution depends on what concentrations are normally encountered.

Exercise 18.1

Tell whether any of the following are characteristic properties. (See Section 10.) Explain your reasoning.

A. mass of solute B. volume of solvent C. concentration

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Exercise 18.2

A beaker has 50 g of potassium chloride dissolved in 250 mL of water. In answering parts A–D below, explain your reasoning. Do not use algebra.

A. What is the concentration of this solution?

B. What would be the concentration of the remaining solution if we poured off 50 mL of the solution?

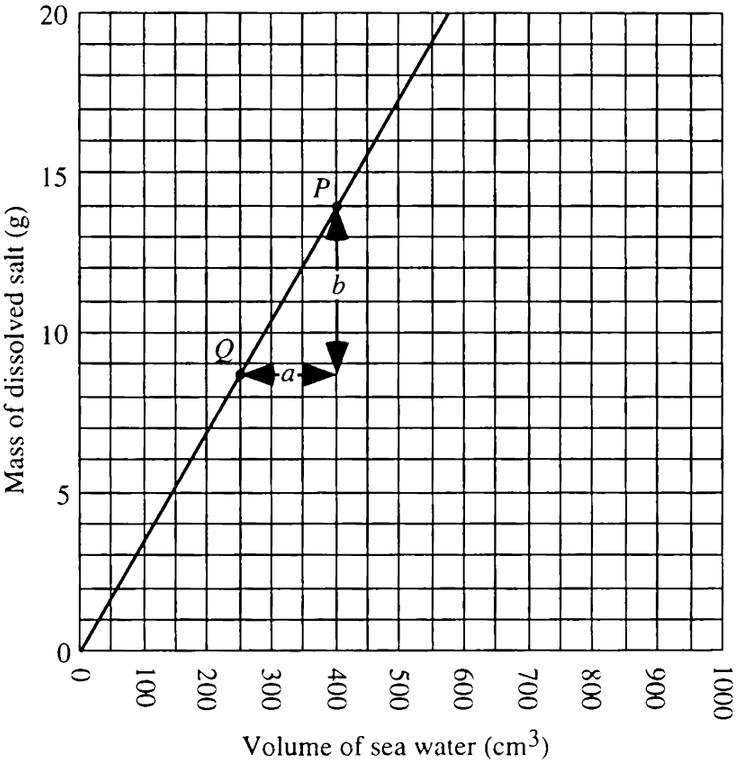
C. What would be the concentration of the solution if, instead, we added 250 mL of water?

D. What would be the concentration of the solution if, instead, we added 12.5 g of potassium chloride?

✔ Check your reasoning with a staff member.

Exercise 18.3

At right is a graph of mass of dissolved salt versus volume of sea water.



A. (1) What is the numerical value of the slope of this graph?

(2) What does this number tell us about sea water?

(3) What can you conclude from the fact that this graph is straight?

B. Suppose we have a beaker containing a sample of sea water represented by point *P*. What would you do to the sample in the beaker in order to obtain a sample of sea water represented by point *Q*?

C. What does the length *a* represent? Can the number represented by *a* be interpreted as belonging to a particular sample of sea water? If so, tell which sample sea water *a* describes. If not, explain why not.

D. What does the length *b* represent? Can the number represented by *b* be interpreted as belonging to a particular sample of sea water? If so, tell which sample sea water *b* describes. If not, explain why not.

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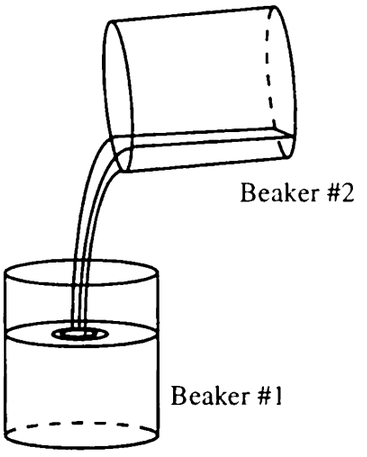
Exercise 18.4



A student obtained a beaker with some pure water in it, beaker 1; and a beaker with a strong salt solution in it, beaker 2. He began adding salt water from beaker 2, a little at a time, to the liquid in beaker 1, as shown on the right. He added solution from beaker 2 three times in all.

The graph below represents the contents of beaker 1. It is a graph of mass of dissolved salt versus volume of water for beaker 1. Points *P*, *Q*, and *R* represent the contents of beaker 1 after each addition of strong solution from beaker 2.

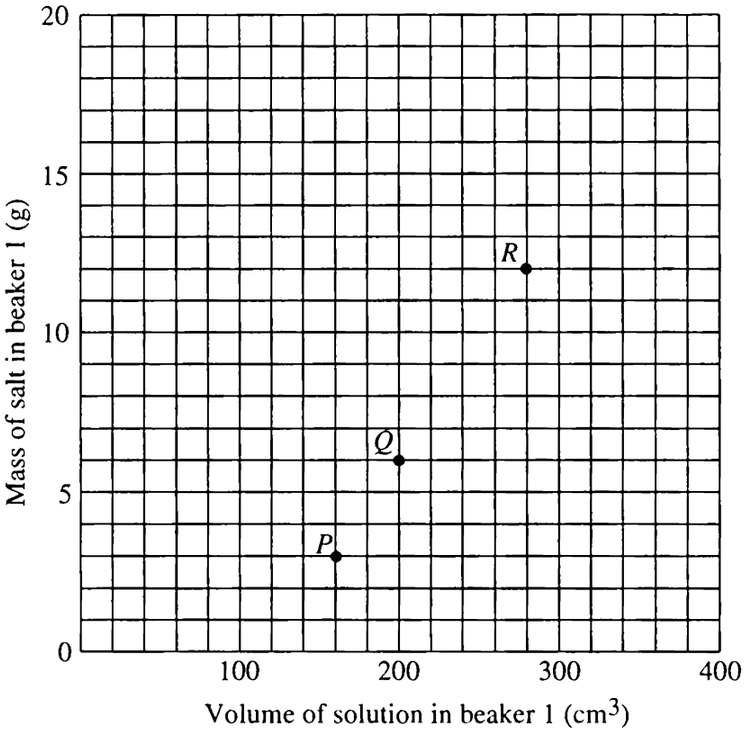
A. What volume of strong solution did the student add the last time?

B. What was the concentration of the solution in beaker 1 after the last addition?

C. What was the concentration of the strong solution in beaker 2?

D. How much water was in beaker 1 at the beginning?

✔ Check your work with a staff member.



Exercise 18.5

How much water is needed to make a solution with 85 g of solute with concentration *C*? Answer by writing an algebraic expression. Explain your reasoning.

Exercise 18.6

Suppose we have a solution of 350 cm^3 of water with 18 g of salt dissolved in it. Then a mass *m* of salt is added to this solution. Write an expression for the new concentration after the additional salt is added. Explain your reasoning.

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It is important to remember that the concentration is *one* number and contains only one piece of information. For example, if someone says that the concentration of a solution is 13 g/100 mL, this does not give any information about the total amount of solution. It definitely does not mean that there actually is 100 mL of solvent in the solution, nor that there is 13 g of solute. There could be any amount of solution from a drop to an entire ocean. The concentration gives no hint as to how much of a particular solution there is—it merely tells the strength of the solution, however much of it there is.

Because concentrations are sometimes expressed in terms of a large volume, such as 100 mL or 1 liter, an interesting puzzle arises in interpreting the concentration number. Here is the puzzle:

Sample problem

Suppose we say that in a beaker there is 15 mL of solution with a concentration of 5 g/100 mL. What does the number *5* tell us about the solution in the beaker?

Sample solution

There are many answers. One is: the number *5* tells the mass of solute that would be dissolved in the solution if we had 100 mL of the same strength solution.

Exercise 18.7

Apply the interpretation of the concentration number given above to resolve the disagreement between the two students quoted below.

Student 1: *"I have 25 mL of solution in this test tube. It has a concentration of 3.1 g/100 mL."*

Student 2: *"That doesn't make sense. First you say that you have 25 mL of solution, then you say you have 100 mL."*

Exercise 18.8

If new students were admitted to the class, each student would receive less personal attention from the staff during class time. If such a thing were to happen, it would in some ways be analogous to diluting a saltwater solution. Explain this analogy by identifying the corresponding parts and telling how they are alike.

✔ Check your reasoning with a staff member.

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Section 19. Changes in concentration

The experiments in this section are typical of the manipulations performed on solutions in many laboratories.

Caution: Cupric acetate is poisonous and is a skin irritant. Wash off any that gets on your skin.

Experiment 19.1

In separate test tubes, mix and save the solutions described below. Use masking tape to label the test tubes A, B, and so on, to H.

In each case below, measure the mass of the powder and the volume of water carefully. Use a paper weighing-dish to hold the powder while you are measuring its mass. Plug the test tubes and shake thoroughly to dissolve the powder.

Calculate the concentration of each solution as you make it.

Solution A: 280 mg of cupric acetate dissolved in 20 mL of water.

Solution B: 90 mg of cupric acetate dissolved in 15 mL of water.

Solution C: 140 mg of cupric acetate dissolved in 10 mL of water.

Solution D: 120 mg of cupric acetate dissolved in 20 mL of water.

How are the concentration numbers related to what you can observe about the actual solutions in the test tube?

In each of the following experiments, first reason through the question, then actually do the experiment. Solve the problems by any method you choose, but be sure to write explanations of your reasoning.

Experiment 19.2

How much cupric acetate must you add to solution B to get a solution with the same concentration as solution A? Add the necessary powder, then put the resulting solution in the test tube labeled E.

Experiment 19.3

Dilute solution C so that it has the same concentration as solution D. How much water must you add? Put the resulting solution in test tube F.

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Experiment 19.4

Pour enough of solution D into the test tube labeled G so that there will be 100 mg of cupric acetate in solution G. What is the volume of solution G?

Experiment 19.5

Pour enough of solution A into the test tube labeled H so that there will be 100 mg of cupric acetate in solution H. What is the volume of solution H?

Experiment 19.6

What do solution G and solution H have in common? Can you detect this similarity by looking at the solutions?

Exercise 19.7

At various times during this experiment, you have had four solutions with the same concentration as solution A and four with the same concentration as solution D (including solution A and solution D). On a sheet of graph paper, plot a graph of mass of dissolved solute versus volume of solvent for these eight solutions.

A. Calculate the slopes of the straight lines that appear. How do you interpret these numbers?

B. What do solutions represented by points on the same line have in common?

Exercise 19.8

Draw arrows on the graph that represent the changes that took place in Experiments 19.2 and 19.3. How do the lengths of these arrows represent what you actually did to the solutions?

Experiment 19.9

Add 8 mL of solution G to solution E. Plot the resulting solution on the graph and call the point *X*. Draw a line connecting points *E* and *X*.

Call the origin of the graph *O*. This is the point *(0,0)*. What are the slopes of the lines *OE, OX,* and *EX?* What does each of these numbers tell us about what happens when solution G is added to solution E?

✔ Check your work in this section with a staff member.

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Section 20. Measurements of concentration

It was mentioned earlier that there are many ways to express concentration. Until now, we have been using the number of grams of solute dissolved in each milliliter of solvent. It is possible, however, to use a different unit for the volume of solvent. For example, the concentration number may tell the mass of solute dissolved in 100 mL of solvent. Exactly the same reasoning is used when the concentration is expressed in other units. When in doubt one can always convert the concentration into familiar units.

Exercise 20.1

The concentration of a certain solution is 6.2 g/100 mL. How much solute is dissolved in 13 mL of solvent? Explain. Do not use algebra.

Exercise 20.2

Suppose we have 50 mL of a solution with a concentration of 0.10 mole/liter. How much water must be added to dilute the solution to a concentration of 0.0075 mole/liter? Explain. Do not use algebra. (A mole is a measure of quantity used in chemistry. It is not necessary to know exactly what a mole is to solve this problem.)

✔ Check your reasoning with a staff member.

Concentrations are often described in terms of the mass of solute dissolved in a standard mass of solution. For example, suppose a saltwater solution contains 3 g of salt dissolved in 47 g of water. The total mass of the solution would be 50 g. The concentration of this solution could be described as 3 g/50 g.

It is customary when describing concentrations in terms of mass of solute dissolved in a certain mass of solution to choose 100 g of solution as the standard amount. The concentration of the example solution in the previous paragraph would be given as 6 g/100 g of solution. This concentration might also be expressed as "6 percent," meaning that if we had 100 g of this solution, 6 g of its mass would be solute and the other 94 g solvent.

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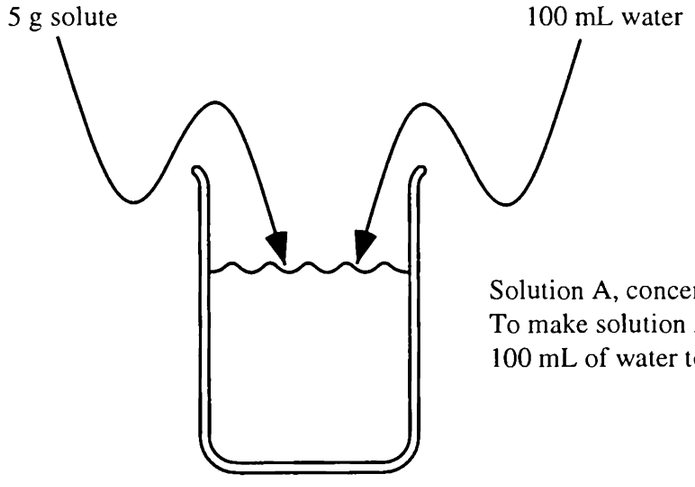
Exercise 20.3

What is the percent concentration of a solution made from 1.2 g of solute and 75 g of water? Explain your reasoning.

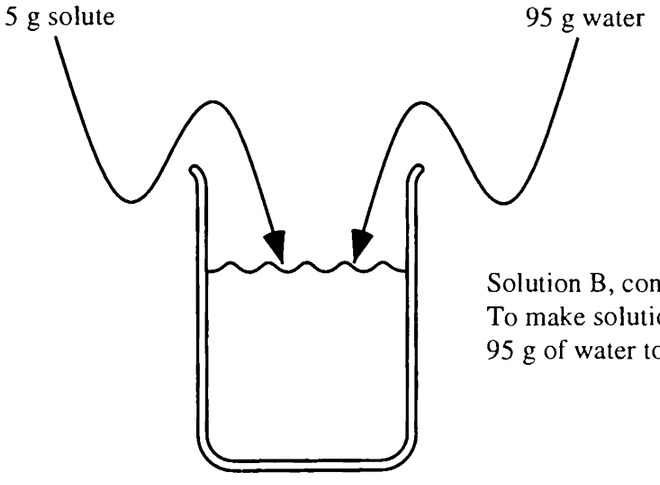
Exercise 20.4

How much 7% solution must we pour into an empty beaker in order to have 21 g of solute present in the beaker?

We have now expressed solution concentration as the mass of solute (e.g., salt) dissolved in 100 mL of solvent (e.g., water) and as the mass of solute dissolved in 100 g of solution (e.g., salt water). The diagrams below show how solutions with concentration 5 g/100 mL water and 5 g/100 g solution could be made in the laboratory.



Solution A, concentration 5 g/100 mL water.
To make solution A, add 5 g of solute and
100 mL of water to an empty beaker.



Solution B, concentration 5 g/100 g solution.
To make solution B, add 5 g of solute and
95 g of water to an empty beaker.

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Both of the above solutions have 5 g of solute, but solution B is slightly more concentrated than solution A. For dilute solutions, there is very little difference between the two forms of the concentration; 0.01 g/100 mL water is very nearly the same concentration as 0.01 g/100 g solution.

In many practical applications, the solutions involved are dilute solutions with water as the solvent. Under these circumstances, the difference between the volume of solvent and the volume of solution is neglected. This means that although we realize that there is a difference, we also realize that the difference is too small to matter. For dilute water solutions, 100 mL of water, 100 mL of solution, and 100 g of solution all refer to nearly the same amount of solution. When this is the situation, the term "percent" may be used to refer any of these three standard amounts.

Examples of concentrations in dilute solutions

3 g of solute in 100 mL of water is a 3% solution.

3 g of solute in 100 mL of solution is a 3% solution.

9 g of solute in 1000 g of solution is a 0.9% solution.

A solution with concentration 0.023 g/mL water is a 2.3% solution.

Exercise 20.5

A very common practical problem is to take a concentrated solution and dilute it to a desired concentration. Suppose we have 10 mL of a 15% solution.

A. How much water must be added to get a 5% solution? Explain your reasoning.

B. How much water must be added to 10 mL of a 15% solution to obtain a 10% solution? Explain your reasoning.

✔ Check your reasoning with a staff member.

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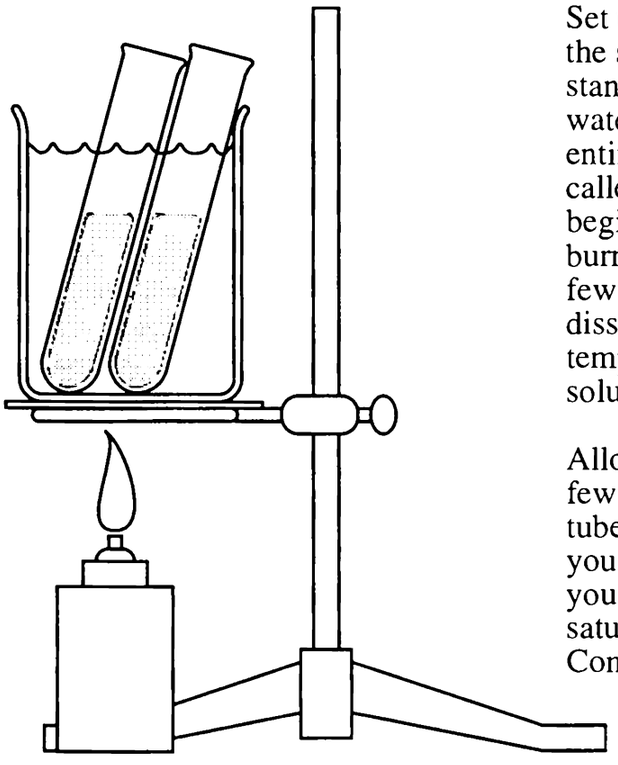
Section 21. Saturation of solutions

The subject of this section is the effect of temperature on the saturation of solutions. We begin with an experiment.

Experiment 21.1

Caution: Use safety goggles when performing this experiment.

Measure out 5.0 ± 0.1 g of sodium chloride. That is, obtain 5.0 g of salt, but do not try to measure exactly 5.000 g. You need only come within 100 mg of 5.000 g. Dissolve this salt in 10 mL of water in a test tube. In another test tube, dissolve 5.0 ± 0.1 g of potassium chloride (KCl) in 10 mL of water. Plug and shake the tubes until no further dissolving takes place.



Set up the following arrangement for heating the solutions: Obtain a small beaker and stand the two test tubes in it. Add enough water to the beaker so that the solutions are entirely below the water level. This setup is called a water bath. Remove the plugs and begin to heat the water bath with an alcohol burner. Stir the solutions thoroughly every few minutes and note whether more dissolving takes place at higher temperatures. Continue to heat and stir the solutions until the water bath is boiling.

Allow the water bath to boil gently for a few minutes while you observe the test tubes and draw conclusions about what you have seen. How do you interpret what you have seen? Are the solutions saturated when the bath is boiling? Continue with the next experiment.

Experiment 21.2

Transfer the two test tubes to a room temperature water bath and watch closely what happens in the tubes. Describe what happens in the potassium chloride solution.

What do you conclude about the effect of temperature on the maximum mass of KCl that 10 mL of water will dissolve?

How does the sodium chloride solution compare in this respect?

✔ Discuss Experiment 21.1 and Experiment 21.2 with a staff member.

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The technical term for what happened to the potassium chloride as it cooled is *precipitation.* The solid is said to *precipitate* as the solution cools. The solid itself is called the *precipitate* in this situation.

When a solution is saturated, the concentration has its highest possible value because no more solute can dissolve in a saturated solution at a given temperature. This maximum value of the concentration is called the *solubility.*

Thus, if we find that the concentration of a solution is less than the solubility, we know that the solution is not saturated. Also, if a solution is saturated, we know that the concentration must have its maximum value, the solubility. Like density, solubility is a property of substances that can be used to distinguish one substance from another.

Exercise 21.3

A. Was the potassium chloride solution saturated at the highest temperature in Experiments 21.1 and 21.2? Explain your reasoning.

B. Describe how the solubility number for potassium chloride changed as the temperature was raised in Experiment 21.1. Explain your reasoning.

C. Describe how the solubility number for potassium chloride changed as the temperature dropped in Experiment 21.2. Explain your reasoning.

D. How must the concentration and solubility numbers be related in order for precipitation to occur?

Exercise 21.4

Consider the following two statements about numbers that describe solutions. Are the statements true or false? Explain your reasoning.

A. Every solubility is a concentration.

B. Every concentration is a solubility.

✔ Check your reasoning with a staff member.

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Exercise 21.5

The solubility of salt in water is 36 g/100 mL water at room temperature. If 25 g of salt is mixed with 65 mL of water at room temperature, what will be the concentration of the solution?

The following table of solubilities of potassium chloride in water is to be used in the next three problems.

Temp. (°C)	Solubility (g/100 mL)	Temp. (°C)	Solubility (g/100 mL)
0	27	60	46
10	30	70	48
20	33	80	51
30	37	90	53
40	40	100	56
50	43		

Exercise 21.6

15 mL of saturated solution of potassium chloride in water at 50°C is raised to 70°C. How much more potassium chloride will dissolve?

Exercise 21.7

Suppose we have 175 mL of saturated potassium chloride solution at 20°C. If the solution is cooled to 0°C, what will be the final concentration of the solution?

Exercise 21.8

In a test tube, 40 g of potassium chloride is dissolved in 130 mL of water at 60°C. Describe what will happen in the test tube if the solution is cooled to 30°C.

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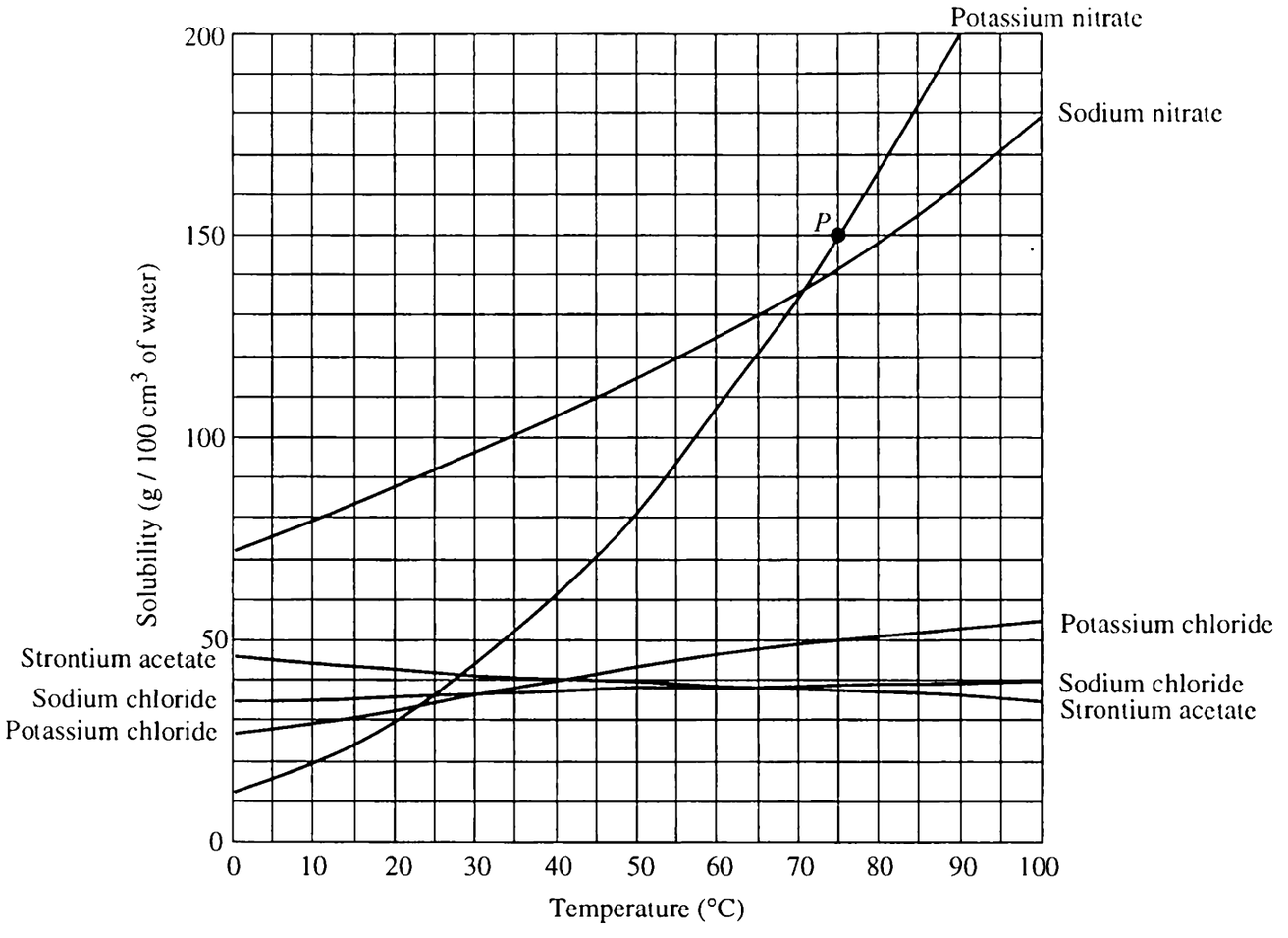
Temperature affects the solubility of various substances in different ways. The graph below shows the temperature dependence of the solubilities of several solids in water.



Graphs are deceptively simple in appearance but hold a wealth of information. An essential first step in graph reading is to be certain of what quantities are being plotted against each other. One way to do this is to ask: "What does just one point on the graph represent?"

Exercise 21.9

A. What does the one point *P* on the preceding graph represent?

B. What general similarity do you see among the substances described in the graph? Which substance shows behavior contrary to normal?

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Exercise 21.10

Suppose we have 36 mL of saturated sodium chloride solution and 72 mL of saturated potassium chloride solution. Both solutions are at the same temperature, but the potassium chloride solution has twice the mass of dissolved solute. What is the temperature?

Exercise 21.11

A mass of 22 g of potassium nitrate is dissolved in 25 mL of water at 80°C. This solution is gradually cooled to 20°C. Fill in the following table:

Temperature	Mass of solute remaining in solution	Mass of solute precipitated
80°C	_____	_____
60°C	_____	_____
40°C	_____	_____
20°C	_____	_____

✔ Discuss your reasoning with a staff member.

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Section 22. Applications of analogies

Section 16 presented examples of analogies between similar concepts such as area and volume and between similar physical systems such as hot air rising and wood floating. There is another important class of analogies: mathematical analogies. In these analogies, two sets of quantities have the same mathematical relationship.

In a mathematical analogy the corresponding quantities may describe completely different concepts, but they play the same roles mathematically.

Sample problem

At the hardware store, wire is sold "by the foot." This means that the buyer must pay the same price for every foot of wire purchased. The total price of a piece of wire and the length are thus mathematically related. Make a mathematical analogy between the price and length of wire and the mass and volume of aluminum.

Sample solution

The price and length of sections of wire are mathematically related to each other in the same way that the mass and volume of pieces of aluminum are. In order to draw this analogy, we must identify the corresponding quantities and tell how they are alike.

The price of one unit length of wire is analogous to density. Both of these numbers are ratios—that is, they are both obtained by division. Both have the same value, no matter how large or small the quantities in the ratio are. The price per foot of wire is the same regardless of the length of wire bought; the mass per cubic centimeter is the same regardless of how much aluminum there is.

Once the correspondence of density and price per foot is established, we can examine these two ratios and see how other quantities correspond mathematically: Total price corresponds to mass. Both are on top in the ratios. Length of wire corresponds to volume. Both are on the bottom in the ratios.

The analogy is illustrated in the following table.

Aluminum	Wire
density	price per foot
mass	total price
volume	length

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Comment

Note that there is no physical similarity between mass and price. The similarity lies only in the mathematical roles these quantities play. In drawing this analogy, it is critical to focus on mathematical relationships when telling how the various quantities correspond.

Exercise 22.1

Make an analogy between mass, volume, and density and the following quantities involving solutions: mass of dissolved solute, volume of solvent, and concentration.

✔ Check your reasoning with a staff member.

Exercise 22.2

Make an analogy between the circumference and diameter of circles and the mass and volume of pieces of tin.

Exercise 22.3

Two shoppers are looking at a gift catalog from the country of Lauternstein. The prices are in Lauternstein francs, and one dollar is worth the same as two Lauternstein francs. They see a wristwatch for 100 francs.

Shopper 1 says: *"That watch costs 50 dollars."*

Shopper 2 says: *"No, it costs 200 dollars."*

A. With which shopper do you agree? Explain your reasoning.

B. Make an analogy between the important quantities in the problem above and the quantities involved with the concept of density. Identify what corresponds to density, to mass, and to volume; then tell how they are alike.

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Proportionality

The term *proportional* refers to a particular mathematical relationship between two quantities. The mass and volume of a substance are *proportional.* The circumference of a circle *is proportional* to its diameter. For samples of a particular solution, the mass of dissolved solute is proportional to the volume of solution. Each of these sets of quantities is mathematically analogous to the others, and the heart of the analogy is proportion. In all of these cases, the ratio of two quantities is the same no matter how large or small the quantities themselves are.

Exercise 22.4

Name two different quantities other than the ones above that are proportional.

Make an analogy between these quantities and the quantities of mass, volume, and density.

Many pairs of quantities in the sciences are proportional, but not all are. Therefore, treating two quantities as if they were proportional must be justified. For example, in the case of mass and volume of a particular substance, we reason as follows:

(1) If an object is made entirely out of one kind of material, all cubic centimeters of the object share the mass equally.

(2) Therefore, the mass divided by the volume will give the mass of each cubic centimeter. This number will be the same for all pieces of that material.

Exercise 22.5

In Experiment 14.10, how did you determine that the circumference and diameter of circles were proportional?

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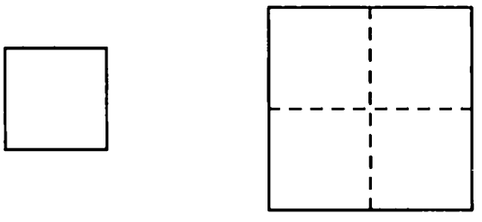
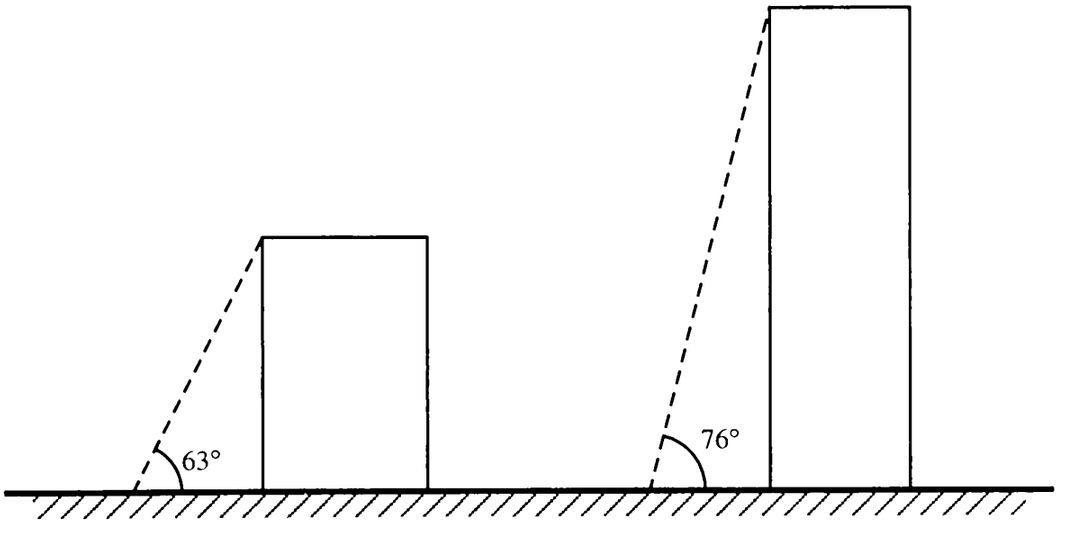
Examples of nonproportionality

One way to check whether two quantities are proportional is to check whether one quantity doubles if the other is doubled. Let us apply this procedure to the area and width of a square. If we double the width of a square as shown below, the new area will be four times as large as it was before. Therefore, the width and area of a square are not proportional. Neither, as we saw in Section 5, are the diagonal and area of a square.



As another example, consider the relation between heights and angles in triangles. Suppose we measure angles from the ground up to the tops of buildings from a fixed distance away, as shown in the diagram below. The angle and the height of the building are mathematically related, but they are not proportional. In the diagram below, the tall building is twice as tall as the short building. The larger angle, 76°, is *not* twice as much as the smaller angle, 63°.



Note that the taller the building is, the larger the angle, but this does not mean that the angle is proportional to the height of the building.

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Section 23. Applications of proportional reasoning

In most of the proportional reasoning encountered so far, we have focused on density or concentration. These quantities are both ratios and both have units that make it easy to remember their interpretations. For example, the units we have been using for density are g/cm^3, which reminds us of the interpretation of density: the mass of one cubic centimeter of material.

In this section we will work with ratios that have no units, such as the ratio of one mass to another, the ratio of two volumes, and so on. Ratios of similar quantities are called *dimensionless* ratios because they have no units, meaning the units in the numerator and denominator cancel out.

Use of dimensionless ratios involves an interpretation of the results of division different from the ones we have been using. Suppose we have two objects, one with mass 400 g and the other with mass 100 g. If we find the ratio of the masses, we get:

$$\frac{400\,g}{100\,g} = 4$$

The number *4* can be interpreted in the following way: the second object is four times as heavy as the first.

This kind of interpretation can also be made even if the ratio is quite small. For example, if we have two objects with the first one having a volume of 125 cm^3 and the second having a volume of 500 cm^3, the ratio of the volumes would be

$$\frac{125\,cm^3}{500\,cm^3} = 0.25$$

In this instance, we could interpret the number *0.25* by saying that the second object is 0.25 times as large as the first. Of course, even though the word "large" is used, this statement means that the second object is smaller than the first.

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Exercise 23.1

We have two objects with different volumes. The first object has a mass of 50 g and a density of 7.9 g/cm^3. The second object has a mass of 80 g and a density of 4.5 g/cm^3.

Give an interpretation, if there is one, of each of the following numbers.

A. $\frac{7.9}{4.5}$

B. $\frac{50}{80}$

C. $\frac{50}{7.9}$

D. $\frac{80}{7.9}$

Exercise 23.2

Two objects are made of the same material. The second object is 0.68 times as heavy as the first. The first object's mass is 80 g. Give an interpretation, if there is one, of each of the following numbers.

A. 0.68×80

B. $\frac{0.68}{80}$

C. 80×0.68

✔ Check your reasoning with a staff member.

Exercise 23.3

We have two identical full beakers containing solutions of salt in water. The solution in the second beaker is 75% as strong as the first. The first beaker contains 5 g of salt. Give an interpretation, if there is one, of each of the following numbers.

A. 0.75×5

B. $\frac{5}{0.75}$

C. $\frac{0.75}{5}$

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Exercise 23.4

A circle has a circumference of 31.4 cm and a diameter of 10.0 cm. Give an interpretation, if there is one, of each of the following numbers.

A. $\frac{31.4}{10.0}$

B. $\frac{10.0}{31.4}$

Exercise 23.5

We have two solutions of salt in water. The first has a concentration of 0.05 g/mL and a volume of 200 mL. The concentration of the second is 0.03 g/mL.

Give an interpretation, if there is one, of each of the following numbers.

A. $\frac{0.05}{0.03}$

B. $\frac{0.03}{0.05}$

C. 0.05×200

D. $\frac{0.05}{200}$

Exercise 23.6

1000 grams is the same mass as one kilogram. Suppose we have two objects with masses 6.3 kg and 4.9 kg.

A. What is the ratio of the masses measured in kilograms?

B. Convert the masses to grams—that is, express the masses as numbers of grams.

C. What is the ratio of the masses measured in grams?

D. Explain why the ratio is the same regardless of the units.

The "so many times as large as" interpretation of a ratio is valid only if the two quantities in the ratio are similar quantities, for example, two volumes or two prices. This interpretation cannot be made for the ratio of mass to volume. Suppose we have an object with a mass of 50 g and a volume of 10 cm^3. We cannot say that the mass is five times as large as the volume. Such a statement is meaningless. Mass and volume cannot be compared in this way.

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Before continuing the discussion of proportional reasoning with dimensionless ratios, let us review the use of the term *factor*. This word is used to describe a number that multiplies or divides another quantity.

For example, if the volume of water in a tank is doubled, we would say that the volume has increased by a factor of two. That means that the new volume is the old volume multiplied by two.

If the tank is drained so that only one-third of the water remains, we would say that the volume of water has decreased by a factor of three. In other words, the new volume is the old volume divided by three.

The term factor should never be used in connection with addition or subtraction. For example, if 2 gal of water is added to a tank with 5 gal of water in it, it would be incorrect to say that the volume of water has increased by a factor of two. If a quantity increases, its factor of increase is found by dividing the final quantity by the initial quantity. (Note that both quantities must be measured in the same units.) The factor of increase is therefore the dimensionless ratio of the final and initial quantities.

Exercise 23.7

If 2 gal of water is added to a tank with 5 gal of water in it, by what factor does the volume of water increase?

Solving problems using dimensionless ratios

Proportion problems can often be solved easily by using dimensionless ratios in connection with algebraic formulas as in the following sample problem.

Sample problem

Suppose we know that a piece of brass with a mass of 17 g has a volume of 2.0 cm^3. What would be the volume of a 51 g piece of brass?

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Sample solution

The basic equation relating mass and volume of the same substance is

$$D = \frac{m}{V}$$

where m is the mass, V is the volume, and D is the density. If we write this equation for both objects, we have:

$$\frac{m_2}{V_2} = D_2$$

$$\frac{m_1}{V_1} = D_1$$

We can now compare the two objects by dividing the two equations as shown below.

$$\frac{\frac{m_2}{V_2}}{\frac{m_1}{V_1}} = \frac{D_2}{D_1}$$

$$\frac{m_2 V_1}{m_1 V_2} = \frac{D_2}{D_1}$$

Since the materials are made of the same substance, the ratio of the densities, D_2/D_1, is equal to one. We therefore do not need to know or calculate the density of brass in this solution. When the densities are canceled, we have

$$\frac{m_2 V_1}{m_1 V_2} = 1$$

$$\frac{m_2}{m_1} = \frac{V_2}{V_1}$$

If we now solve for the volume of the second piece of brass, we obtain:

$$V_2 = \frac{m_2}{m_1} V_1 = 3(2\,\text{cm}^3) = 6\,\text{cm}^3$$

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Direct proportions

The relation between mass and volume as shown in the equation

$$\frac{m_2}{m_1} = \frac{V_2}{V_1}$$

is a classic example of a *direct proportion.* Mass and volume are said to be *directly proportional.* In a direct proportion, if one of the quantities is increased by a certain factor, the other quantity will also increase by exactly the same factor. In sample problem, the mass of the second object was three times as much as the mass of the first object. The volume of the second object (6 cm^3) was therefore also three times the volume of the first object (2 cm^3).

Direct proportions are easy to recognize in equations composed of dimensionless ratios. For example, we can see that mass and volume are directly proportional by looking at the subscripts in the equation. The subscripts in the dimensionless ratios are in the same order on both sides: *2* above *1* in the mass ratio and *2* above *1* in the volume ratio. The subscripts are always in the same order in a direct proportion.

Exercise 23.8

Two objects have the same volume, but the second object is four times as heavy as the first object. If the density of the first object is 2.7 g/cm^3, what is the density of the second?

Exercise 23.9

Suppose we have equal volumes of two solutions of the same solute and solvent. The first solution has a concentration of 0.05 g/mL and contains 3.6 times as much solute as the second. What is the concentration of the second solution?

✔ Check your reasoning with a staff member.

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In addition to direct proportions like those in the previous problems, another important class of problems involving proportions can be solved by using dimensionless ratios along with algebra. The following example illustrates one such problem.

Sample problem

A certain solution has a volume of 50 mL and a concentration of 0.18 g/mL. If this solution is to be diluted to a new concentration of 0.06 g/mL, what must be the new volume of solution?

Sample solution

The basic equation relating the quantities involved is:

$$C = \frac{m}{V}$$

where C is the concentration, m is the mass of the dissolved solute, and V is the volume of the solution. If we write an equation for each solution and divide them, we get an equation composed of dimensionless ratios.

$$C_2 = \frac{m_2}{V_2}$$

$$C_1 = \frac{m_1}{V_1}$$

$$\frac{C_2}{C_1} = \frac{\frac{m_2}{V_2}}{\frac{m_1}{V_1}}$$

Since the same amount of mass is used to prepare each solution, the ratio of the masses is one. Our equation then becomes

$$\frac{C_2}{C_1} = \frac{V_1}{V_2}$$

In this case, the concentration is to decrease by a factor of 0.18/0.06 = 3. The equation shows that the volume must *increase* by the same factor, 3. Solving for the volume of the dilute solution, we get:

$$V_2 = \frac{C_1}{C_2} V_1 = 3(50\text{ mL}) = 150\text{ mL}$$

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The relation between concentration and volume in this problem is called an *inverse proportion.* It is said that volume and concentration are *inversely proportional.* What this means is that if one quantity is increased by a certain factor, the other quantity is *decreased* by exactly the same factor. In this case, for example, the concentration decreased by a factor of three and the volume of solution increased by a factor of three.

Inverse proportions are easy to recognize in equations composed of dimensionless ratios. For example, the relation between concentration and volume of solutions containing the same amount of mass can be easily seen from looking at the equation. The subscripts on one side appear as *1* above *2,* while the other side has *2* above *1.* The subscripts are always reversed in an inverse proportion.

Exercise 23.10

The density of air in a closed balloon is 0.0015 g/cm^3. If the balloon is heated so that the volume of the air increases by a factor of 1.8, what will be the density of the air in the balloon?

Exercise 23.11

We have two solutions. The first solution is 1.2 times as concentrated as the second. They were both made from the same amount of solute. If the volume of the first solution is 100 mL, what is the volume of the second solution?

Reading proportions from an equation

Many problems involving dimensionless ratios can be solved without using algebra. In many cases, it is not necessary to solve the problem algebraically, but rather only to use an algebraic formula as a guide for proportional reasoning. In this way of solving problems, the equation is only used to determine whether two quantities are directly proportional or inversely proportional.

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It is easy to tell at a glance whether a proportion is direct or inverse by checking to see whether the two quantities increase together or decrease together, or whether one increases while the other decreases.

Sample problem

Two solutions are to be made from the same mass of solute. The first solution has a volume of 50 mL and a concentration of 100 mg/mL. If the second solution has a concentration of 130 mg/mL, what is its volume?

Sample solution

The basic formula for concentration is:

$$C = \frac{m}{V}$$

In this problem, we are asked for the volume, so we solve for *V:*

$$V = \frac{m}{C}$$

We now use the formula as a guide for proportional reasoning. When we use a formula as a guide for proportional reasoning, we do not actually plug numbers into the formula. For example, we do not substitute in either of the equations above. Although substituting values into a formula is the easiest way to solve some problems, there are many problems that can be solved more easily with just proportional reasoning.

The reasoning used in the solution presented below does not involve plugging values into any formula. Instead, the formula is only used in deciding whether quantities are directly or inversely proportional. The problem is then solved by proportional reasoning and no further use is made of the formula.

By examining the formula, we see that the volume is inversely proportional to the concentration of the solution. Since both solutions contain the same mass of solute, m is the same for both samples. The ratio of the concentrations is 130/100 = 1.3. The second solution is more concentrated than the first by a factor of 1.3. Since the volume of solvent is inversely proportional to the concentration, the volume of solvent in the second solution will be a factor of 1.3 smaller than that of the first:

$$V = \frac{50\,\text{mL}}{1.3} = 38\,\text{mL}$$

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Exercise 23.12

Two objects made of different kinds of metal have the same volume. The mass of the first object is 6.3 times as much as the second. The second object has a density of 3.0 g/cm^3. What is the density of the first object?

Solve using dimensionless ratios. Let a formula be your guide, but do not solve algebraically.

Dimensionless ratios without algebra

It is also possible to reason with dimensionless ratios without any recourse to algebra.

The following sample problem shows how this kind of reasoning can be carried out.

Sample problem

We know a 40 cm^3 piece of glass has a mass of 100 g. What is the mass of a 25 cm^3 piece of glass?

Sample solution

The ratio of the volumes is 25/40 = 5/8. We know that the mass and volume of samples of the same substance are directly proportional. Therefore the small piece must be 5/8 times as heavy as the large piece. The mass of the small piece is thus:

$$\frac{5}{8} \times 100\,\text{g} = 63\,\text{g}$$

Exercise 23.13

Supply the reasoning for each arithmetic step taken below. Interpret all calculated numbers.

Problem: A large block of wood has a mass of 8000 g and a volume of 12,000 cm^3. How much volume would 60 g of this wood occupy?

Solution:

$$\frac{60}{8000} = 0.0075$$

$$0.0075 \times 12{,}000 = 90$$

✔ Check your reasoning with a staff member.

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Part E: Solutions of solids, liquids, and gases

Thus far, we have studied solutions in which solids are dissolved in water. In Part E, we investigate solutions in which solids are dissolved in solvents other than water and solutions in which the solute is a liquid or gas dissolved in a liquid.

Section 24. Solutions of liquids in liquids

Two or more liquids can combine to form a solution. Usually when two liquids are mixed, either they completely dissolve in each other or they do not combine at all. Two liquids are called *miscible* if they dissolve each other and *immiscible* if they do not mix. For example, alcohol and water are miscible while oil and water are immiscible.

Experiment 24.1

Caution: The methanol (wood alcohol) used in this experiment is a lethal poison if ingested. Do not inhale large quantities of the vapor. Use safety goggles while performing this experiment.

Design a series of experiments to show that methanol and water are miscible in any proportion. Check your plan with a staff member, then carry out your experiments.

Experiment 24.2

In this experiment, methanol will be separated from a solution of methanol and water. Mix 5 mL of methanol and 10 mL of water in a test tube and shake thoroughly. Use a test tube that fits a two-hole stopper.

Make the following observations of the solution before beginning the experiment. Pour about 1 mL of the solution into an evaporating dish. Smell the small sample. Try to set fire to the liquid in the dish.

Does the liquid burn?

Caution: The flame, if any, will be pale blue, almost invisible. Check for a flame by cautiously putting your hand over the liquid.

Insert a thermometer through one hole of a two-hole stopper and a glass tube through the other. Plug the test tube. Adjust the thermometer so that it reaches almost to the bottom of the test tube and the glass tube so that it extends just below the stopper. Attach a hose to the glass tube and run the hose into the bottom of a collection test tube that is in a deep, cold water bath. The collection test tube should be kept away from the burner because the liquid that will be collected, the *distillate*, is flammable.

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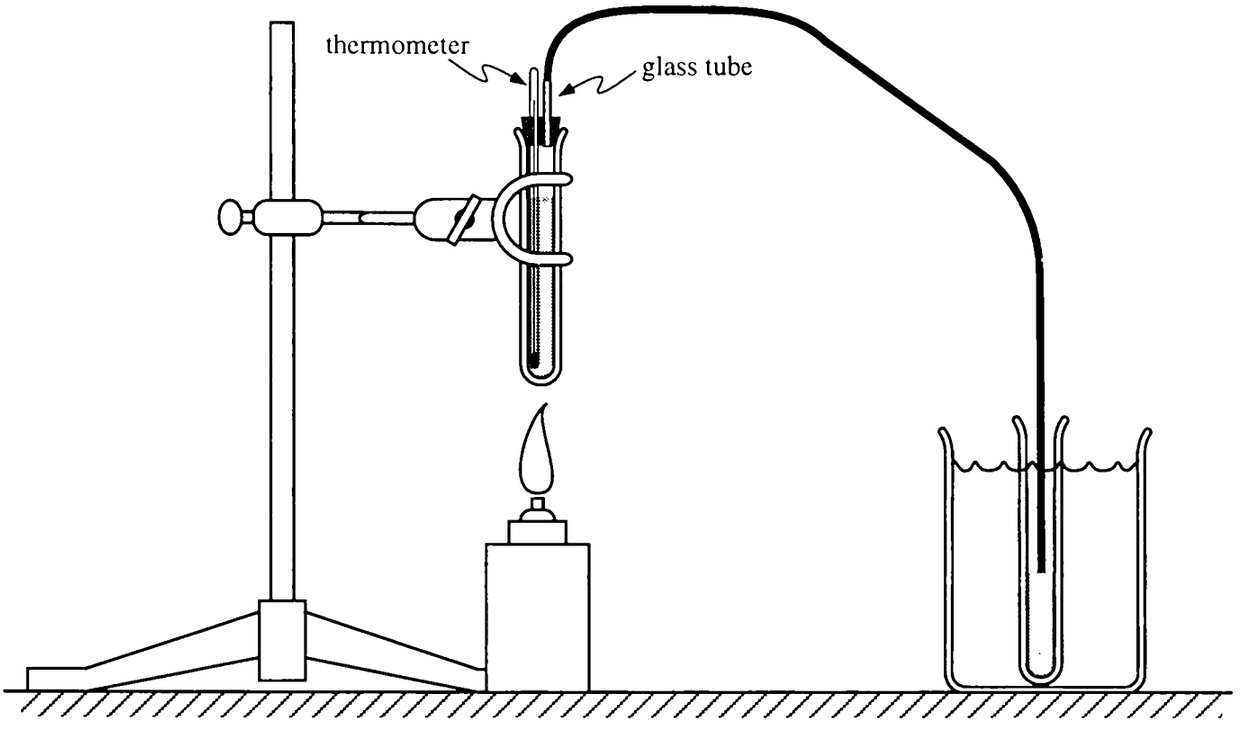
The assembled apparatus is shown in the following diagram.



Heat the solution. When it starts to boil, take care that the froth in the test tube does not rise high enough to enter the hose. Note the temperature at which the solution boils. Eventually the hose in the collection tube will stop bubbling vigorously and the temperature in the solution test tube will start to rise. When this happens, first remove the hose from the collection tube in the water bath and then extinguish the flame.

Now perform the same tests on the distilled liquid that you performed earlier on the solution—smelling and trying to set fire to the liquid. If you wish, you may also check the boiling temperature of the distilled liquid and compare it to the boiling temperatures of the original solution and of pure methanol. If you check the boiling temperature, use an apparatus identical to the distillation apparatus to trap the flammable vapor.

What do you suppose happens in the test tube during the boiling? in the hose? in the collection test tube? What is the purpose of the water bath?

✔ Discuss this experiment with a staff member.

The process you performed in the previous experiment is called *distillation.* Distillation is a commonly used procedure for separating substances in solutions. It is often very effective. For example, it is possible to remove almost all the salt from salt water by distillation. In other cases, however, it may not be possible to completely separate the components of a solution. Pure methanol cannot be obtained by distillation of a water solution because some water also evaporates when the solution is heated.

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Section 25. Solvents other than water

Thus far, we have only seen examples of solutions in which water is the solvent. Other liquids can also act as solvents. Many solvents are too dangerous to use without an exhaust hood to evacuate the vapors. Since the danger from such vapors is significant, we will restrict our investigation to water and methanol as solvents.

Experiment 25.1

Caution: The methanol (wood alcohol) used in this experiment is a lethal poison if ingested. Do not inhale large quantities of the vapor. Use safety goggles while performing this experiment.

Check whether the substances listed below will dissolve in water and methanol. Test for dissolving in two steps. First, add just a very few grains and see whether these will dissolve in about 5 mL of solvent. If they do not, list the solute as insoluble in the solvent. If the few grains do dissolve, then add a scoop of solute and see how much will dissolve. Then list the solute either as barely soluble or soluble in the solvent.

	Chalk	Sugar	Naphthalene	Citric acid
Water	_____	_____	_____	_____
Methanol	_____	_____	_____	_____

Experiment 25.2

Selective dissolving by various solvents is a useful way of separating substances that might otherwise be hopelessly mixed. From the staff obtain a mixture of sugar, chalk, and citric acid. Design an experiment to separate these three substances by using the solvents tested in Experiment 25.1. Then carry out your experiment.

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Section 26. Solutions of gases in liquids

Experiment 26.1

Set up a carbon dioxide generator using an Alka-Seltzer tablet as in Experiment 11.4. Let it bubble a few seconds to exhaust the air in the test tube and hose; then collect a test tube of carbon dioxide gas by water displacement.

Obtain a balloon and fill it with water, but do not put the water under pressure. Just put in as much water as the balloon will hold without stretching. Attach the balloon over the mouth of the test tube of carbon dioxide. Shake the system vigorously.

Record your observations.

What do you conclude from this experiment?

✔ Discuss this experiment with a staff member.

Exercise 26.2

What do you conclude about the effect of pressure on the solubility of carbon dioxide from the following information? (Carbon dioxide is the gas in carbonated beverages.)

- When a cola bottle is capped, no bubbles are seen inside the bottle.

- Bubbles appear in the cola as soon as the cap is removed.

Exercise 26.3

From your knowledge of carbonated beverages, what do you conclude about the effect of temperature on the solubility of carbon dioxide? Explain your reasoning.

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Gases dissolve to various extents in liquid solvents. The table at right lists the solubility of some gases in water at 0°C and atmospheric pressure of the gas. (Atmospheric pressure is approximately the pressure of air in a laboratory at sea level.)

Gas	Solubility g/100 mL
Oxygen	0.0069
Nitrogen	0.0029
Hydrogen	0.00019
Carbon dioxide	0.33
Carbon monoxide	0.0044
Sulfur dioxide	23.0

Exercise 26.4

A. Assume the water at the bottom of a long cascade or waterfall is saturated with oxygen. How much oxygen would be dissolved in 4 liters (about one gallon) of water at the bottom of the fall?

B. How much volume would this much oxygen gas occupy? (Use the density for oxygen listed in the table at the back of this book.)

C. Air is about one-fifth oxygen by volume. What volume of air contains the same mass of oxygen as 4 liters of saturated water at 0°C?

✔ Discuss this exercise with a staff member.

All gases are completely miscible with other gases at normal pressures. Gas mixtures are homogeneous, and in this way they resemble solutions a great deal. The wide variety of smells that may be detected in the air is evidence that many gases may be present in air.

Exercise 26.5

Make an analogy between water vapor in air and a solution of salt in water. Below is a list of substances, properties, numbers, conditions, and so forth, that apply to the water-vapor-in-air system. In making your analogy, identify what in the saltwater system corresponds to each of these and tell how they are alike.

A. water vapor B. air C. liquid water
D. evaporating E. raining F. humidity
G. 100% humidity

Exercise 26.6

Make an analogy between the bubbling that takes place in champagne when it is poured into a glass and the precipitation of potassium chloride in Experiment 23.2. Be sure to identify all the corresponding parts (substances, numbers, conditions, etc.) and tell how they are alike.

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Section 27. Extracting solutes from solutions

Experiment 27.1

Carefully measure about 1 g of salt and dissolve it in 5 mL of water. Also measure the mass of an empty evaporating dish. When all the salt is dissolved, pour the solution into the evaporating dish and heat the dish with an alcohol burner until all of the liquid is gone. Boil the solution slowly so that the liquid in the dish is not spattered out. After the dish has cooled, measure the mass of the dish and its contents.

Examine the residue. Does it appear to be the same or different compared to the original salt?

How does the mass of the residue compare with the initial mass of salt?

Test whether the residue will dissolve in water and in methanol.

Experiment 27.2

Caution: Acids are dangerous. Contact with the skin or eyes can cause injury. Immediately wash off any acid that spills on you with plenty of water. Acid will also damage clothes. *Wear safety goggles when boiling the acid!*

Obtain about 1 g of chalk dust, a graduated cylinder containing 5 mL of hydrochloric acid solution, an evaporating dish, and an Erlenmeyer flask. Find the combined mass of the chalk and evaporating dish carefully. Place the chalk in the Erlenmeyer flask and then add the acid. What happens as the chalk dissolves? How does this dissolving process compare with that of salt in water? When all of the chalk has dissolved, pour the solution into the evaporating dish. Heat with an alcohol burner as before, but *wear safety goggles when boiling the acid* to avoid eye damage in case the acid spatters. Find the mass of the dish and contents.

After the liquid is gone, is the residue that remains chalk? To test whether the residue is chalk, try to dissolve some of the residue in water and in hydrochloric acid solution. Is gas evolved when the residue is dissolved in the acid?

✔ Discuss this experiment with a staff member.

The term dissolving is applied to the processes occurring in both Experiments 27.1 and 27.2. The difference between them is indicated by saying there was a *chemical change* or a *chemical reaction* in Experiment 27.2 but not in Experiment 27.1.

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In Experiment 27.2, chalk was present initially, but afterward no chalk was present. The white solid remaining after the experiment is clearly a different substance, not chalk. Processes such as this one, in which different substances are present before and after, are called chemical changes.

In contrast, the salt remaining after Experiment 27.1 is the same substance as the initial salt. Salt dissolving in water is an example of dissolving without chemical change.

Exercise 27.3

"When salt dissolves in water, the salt still exists. The salt is still there in the solution. But when chalk dissolves in hydrochloric acid, it no longer exists. A different substance is present."

Support this statement in a brief essay, using as evidence the observations listed below (and any others you wish to add). Evidence:

- Your own observations of the nature of the dissolving process and the properties of the residue remaining when the liquid is boiled off.
- Measurements of the masses before dissolving and after the liquid has been boiled off.

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Supplementary problems for *Properties of Matter*

Problem 1.1

Two students decide to balance various objects against crayons from a box that has never been opened. They find that their class gerbil, Gerta, will balance 40 crayons. These students also found that Gerta would balance with 80 clothespins.

A. What is the mass of Gerta? Express your answer in two different ways.

B. Use the information given above to determine (1) the number of crayons that will balance one clothespin, and (2) the number of clothespins that will balance one crayon. Explain your reasoning in each case.

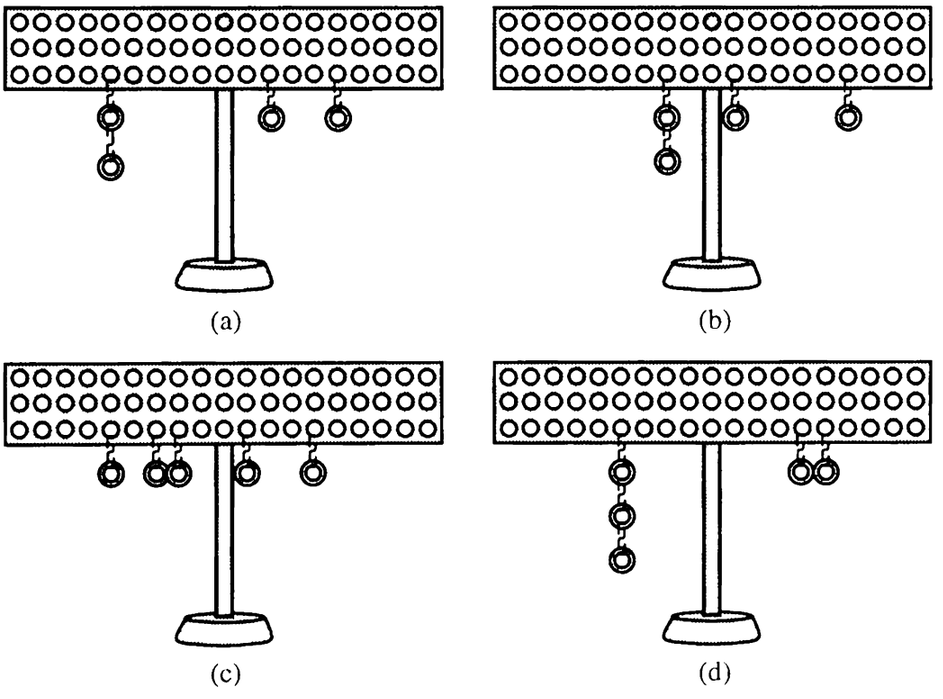
C. Would the students have been able to use the crayons if the crayons from the box had previously been used for coloring? Explain.

Problem 1.2

Terry lives in London and Joanne lives in Chicago. Each has an equal arm balance at home because they are studying *balancing* at school. In a phone conversation they discover that each has purchased a hand-held video game, but not from the same company. They want to find out which game has the greater mass. How can they do this by using their equal arm balances but without getting together?

Problem 2.1

Where could you hang a single square nut on each of the beams shown below in order to have it balance?



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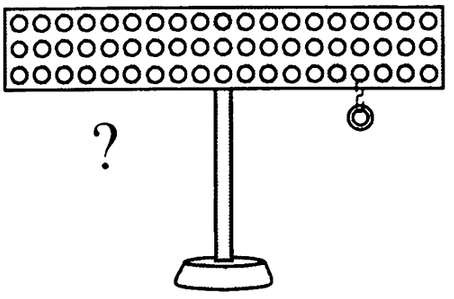
Problem 2.2

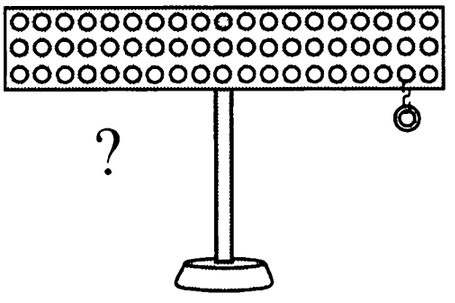


A. Where could you hang two square nuts, each from its own paper clip, on the left side of the balance shown, so that the two sides will balance? Can both square nuts be hung from the same hole? If so, where?

B. Suppose the square nut on the right side of the beam is moved one hole to the right. Could you still hang two square nuts from a single hole on the left side of the beam so that both sides balance? If so, where? If not, why not?

C. State a rule telling when a single square nut on one side of an equal arm balance can be balanced by two square nuts hung from a single same hole on the other side of the beam. Explain your reasoning.

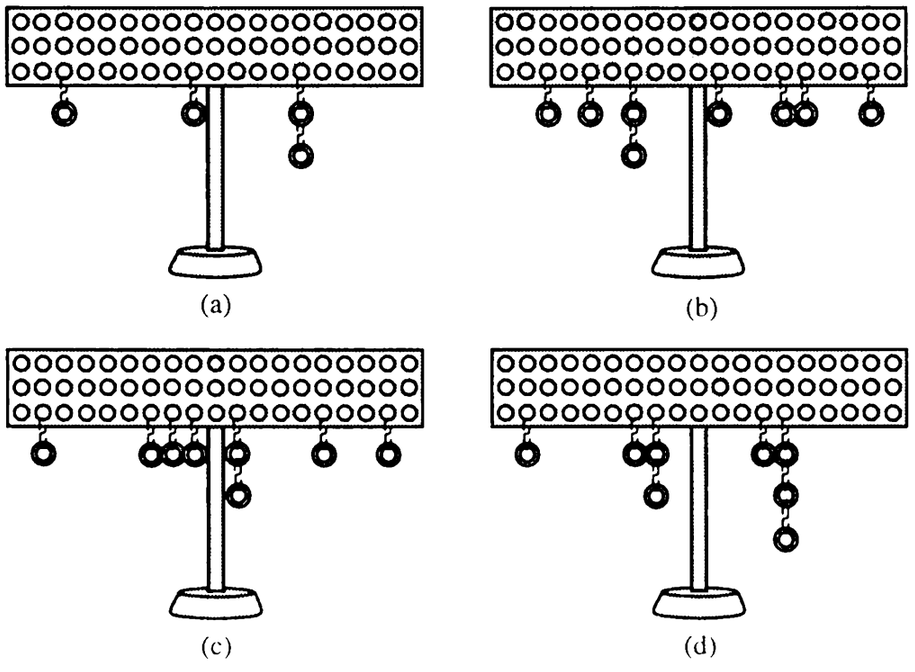
Problem 2.3



A. Where on the left side of the balance shown could you hang two square nuts from the same hole so that both sides will balance?

B. Suppose that one of the square nuts on the left side were moved one hole to the right. What must be done with the other square nut on the left side of the beam so that the beam remains balanced? What if the square nut that was moved to the right were moved another hole to the right?

C. On the basis of your answers to part B predict whether the sides of the beams below will balance. In each case, show the steps by which you arrived at your answer.



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Problem 2.4

Suppose you have six square nuts. Find three ways to balance the beam by placing one square nut on the left side and the other five on the right side.

Problem 2.5

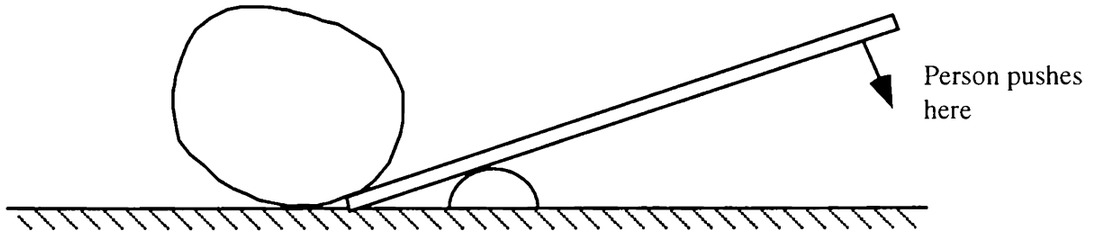
Suppose 200 square nuts are placed 9 cm from the fulcrum on one side of a balanced meter stick.

A. Where must an object with a mass of 75 square nuts be placed to balance the stick?

B. If an object of mass m is added to the 200 square nuts at 9 cm, the object with a mass of 75 square nuts must be moved away from the fulcrum to maintain the balance. Call the distance the object must be moved d. Solve for d in terms of m.

Problem 2.6

A. Make an analogy between the system of masses hanging on a balance and a person lifting a heavy stone with a lever as shown in the diagram.



B. What is the point of using a lever? Answer in terms of *turning effects.*

Problem 2.7

Two masses are hanging on a meter stick suspended at its center. On the right side, an object with a mass of 200 square nuts is 10 cm from the fulcrum, and on the left side, an object that balances 100 square nuts is 20 cm from the fulcrum.

A. Is the system balanced?

B. Now the mass of the 100 square nut object is decreased by an amount m. How much farther must the diminished object be moved from the fulcrum to maintain the balance?

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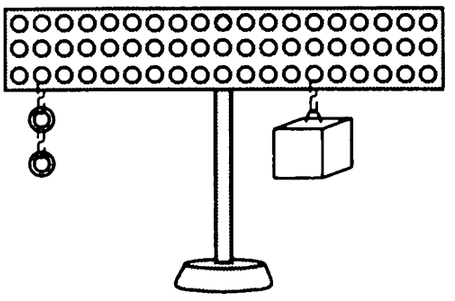
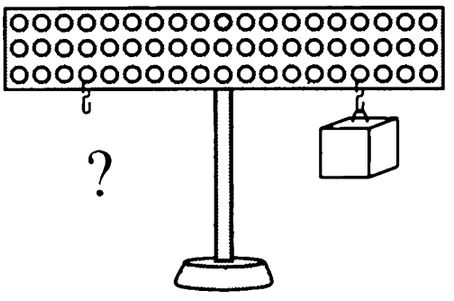
Problem 2.8



A. Consider the arrangement of objects shown at right.

If the box were removed, how many square nuts would you need to hang from that hole on the right for the beam to balance?



B. How many square nuts (and paper clips) would you need to hang from the paper clip on the left in order for the beam to balance? (The box is the same one as used in part A.)

What is the mass of the box?

Problem 3.1

When first working with the balance, some students ask why we do not put calibrated scales on both arms of the balance. Explain in a short, clear paragraph why this is not necessary.

Problem 3.2

Discuss the function of the rider on a balance in light of your answer to Experiment 2.16. Why do you think you were justified in dividing the rider scale into 20 (or 10) equal parts when you calibrated your balance? Explain your reasoning.

Problem 4.1

Suppose your evil friend asks to borrow your ruby. You suspect your friend wants to steal your ruby and replace it with a worthless fake, so you find its mass on *your balance* and obtain 30.639 g.

When your friend returns what he says is your ruby, you find its mass and obtain 30.660 g.

On the basis of this information, can you conclude that your friend has stolen your ruby? Explain your reasoning.

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Problem 4.2

You and your partner each find the mass of a piece of aluminum on your balance. You obtain 10.612 g, and your partner obtains 10.638 g.

Your partner says: *"Well, one of us definitely did something incorrectly."*

Do you agree or disagree? Explain your reasoning.

Problem 4.3

Suppose you find the mass of the battery of a transistor radio on your balance and obtain 40.513 g. You then use the battery for one day and find its mass afterward, obtaining 40.513 g.

Your partner says: *"From this information we can conclude that the mass of the battery remained exactly the same."*

Do you agree or disagree? Explain your reasoning.

Problem 5.1

Write an operational definition for the "turning effect" discussed in Section 2. If your operational definition includes concepts that you have previously operationally defined, you need not write out those operational definitions in this problem.

Problem 6.1

Suppose we want to find the volume of a cylinder. The area of one circular end of the cylinder is 6 cm^2 and the cylinder is 8 cm long from end to end.

A. Explain how you could find the volume of this cylinder arithmetically from the information given.

B. Explain why the number you obtain by your method is the same as you would get by counting the number of one-centimeter cubes that fit inside.

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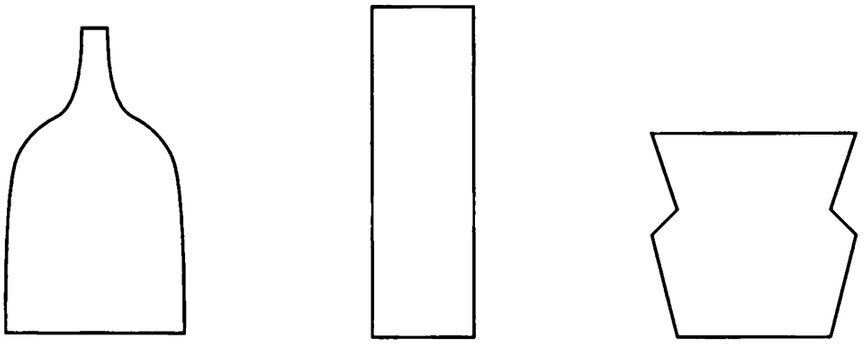
Problem 6.2

Two students are discussing their methods for calibrating the water containers shown below.

Student 1: *"You have to first put in 10 mL of water from another container and put a mark on the side of the container. Then put in a second 10 mL and make another mark. You keep doing this until you have finished. Then you can read the amount of water in the container to the nearest 10 mL."*

Student 2: *"That's the slow way of doing it. Just put in 100 mL of water in the container, then divide the distance between the 100 mL mark and the bottom of the container into 10 equal parts. Each of those parts will be 10 mL."*

Will both students' methods allow them to calibrate the containers shown below? Explain your reasoning.



(a) (b) (c)

Problem 6.3

Suppose that you taught your niece Tara about balancing and measuring mass. For standard objects she used some small plastic cubes. Now you are teaching her about volume from Section 6 of the module. One day, Tara comes to you and shows how she can build a model of a large block using some small plastic cubes.

She says: *"Since it takes 64 plastic cubes to make a block the same size as the big block, I know that the volume of the big block is 64. But since we used the same plastic cubes to measure mass several weeks ago, wouldn't 64 be the mass of the big block as well?"*

Write a dialog between you and your niece that will lead her to the understanding that this is not the case.

Important points to cover are (1) the operational definitions of mass and volume, and (2) the role that the balance plays in determining the mass.

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Problem 6.4

How could you measure the volume of something using the water displacement method if that object does not fit into a graduated cylinder?

Discuss how the uncertainty of your method compares with the uncertainty in using a graduated cylinder to measure volume.

Problem 6.5

Diagrams alone are not sufficient to answer this question, they must be accompanied by clear explanations.

Explain why determining the volume of an object by the method of water displacement gives the same result as counting the number of standard cubes that fit inside the object. Do this by working the following steps:

(1) Draw a diagram of the object (a stone, say) and the water *before* the object is put into the water. Divide up the water into cubes and divide up the stone into cubes.

(2) Draw a diagram of the stone submerged in the water. Divide up the entire system into cubes.

(3) Are there the same number of cubes of stone in the second situation as there are in the first? Why do you say so?

(4) Are there the same number of cubes of water in the second situation as there are in the first? Why do you say so?

(5) Identify the *displaced water* in your diagram.

Explain why the volume (that is, the number of standard cubes) of displaced water is the same as the volume of the stone.

Problem 7.1

Suppose a graduated cylinder is filled up to the 50 mL mark with dry sand. Then suppose that 30 mL of water is poured into the cylinder and the water level stands at the 62 mL mark.

A. Why does the water not stand at the 80 mL mark?

B. What is the volume of just the solid sand in the cylinder?

Explain your reasoning.

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Problem 7.2

In the popular press, one often finds statements such as: "Scientists have proved that..." Through our experiences with uncertainty and with the law of conservation of mass, we have seen that experimental tests of physical laws do not always lead to a simple conclusion that the law is true or false. Explain why not and discuss the kinds of conclusions that can be drawn about the truth of a physical law.

To answer this question, write a clear, concise essay (about 200–300 words). Your work will be graded on both content and English usage.

Problem 9.1

Explain your reasoning for every step in the following problem. A stick of butter has a volume of 120 cm^3 and a mass of 110 g.

A. What is the volume of 1 g of butter? Draw a diagram that illustrates the thinking involved in the calculation.

B. What is the volume of 6 g of butter?

C. What is the mass of 50 cm^3 of butter? Draw a diagram that illustrates the thinking involved in the calculation.

Problem 10.1

Explain your reasoning thoroughly in answering the questions below. Give a justification each time you multiply or divide. Do not use algebra. The density of silver is 10.5 g/cm^3.

A. What is the mass of 7 cm^3 of silver?

B. What is the volume of 64 g of silver?

C. How many 1 cm^3 pieces of silver must be combined to have a total mass of 64 g?

D. How are questions B and C related?

Problem 10.2

One cubic centimeter of copper has a mass of 8.9 g. What is the volume of a piece of copper with a mass of 1.23 kg?

Explain your reasoning. Illustrate your calculation with a diagram. Do not use algebra.

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Problem 10.3

Suppose we have a piece of lead with a mass of 70 g. How much will the volume of lead increase if we add 30 g more lead to this piece? Lead has a density of 11.3 g/cm^3.

Explain your reasoning. Do not use algebra.

Problem 10.4

In this problem, the calculating has been done for you. The questions ask about the reasoning involved at each step.

Problem: A sample of liquid has a mass of 1.04 kg and a volume of 1300 mL. If the mass of the sample is to be increased by 70 g by adding more of the same kind of liquid, what volume of liquid must be added?

Solution: (1) $\frac{1040}{1300} = 0.80$ (2) $\frac{70}{0.80} = 88$

A. How do you interpret the number *0.80?*

B. Explain why the division is done in step 1.

C. How do you interpret the number *88?*

D. Explain why the division is done in step 2.

Problem 10.5

In this problem, the calculating has been done for you. The questions ask about the reasoning involved at each step.

Problem: Two blocks of metal have the same mass. The first block has a volume of 827 cm^3 and a density of 4.50 g/cm^3. The second block has a volume of 193 cm^3. What is the density of the second block?

Solution: (1) $4.50 \times 827 = 3720$ (2) $\frac{3720}{193} = 19.3$

A. How do you interpret the number *3720?*

B. Explain why the multiplication is done in step 1.

C. How do you interpret the number *19.3?*

D. Explain why the division is done in step 2.

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Problem 11.1

An instructor gives a painted piece of metal to two students and says: "This is one of the metals listed in the table of densities. What metal do you think it is?"

The students measure the object and find that the mass is 139.2 ± 0.1 g, and the volume is 16.0 ± 0.5 cm^3.

Student 1 says: *"It must be nickel."*

Student 2 says: *"Don't forget uncertainty. It might be silver."*

A. What would you conclude from the data?

B. Do you agree with student 1, student 2, or neither?

Problem 11.2

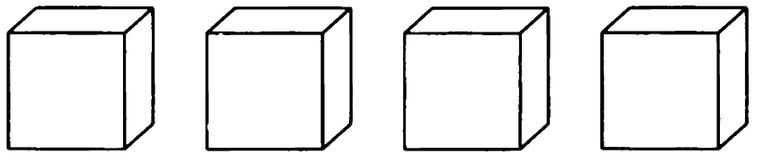
Suppose we have a sample of water with volume measured to be 1535 ± 10 mL, 10 mL being the uncertainty of the measurement. This water is to be poured into a rectangular tank with the following measured dimensions: 15.0 ± 0.1 cm long, 10.0 ± 0.1 cm wide, 10.0 ± 0.1 cm deep. If we attempt to pour all of the water into the tank, some of it may not fit and may spill out.

A. Can you be sure some of the water will spill out? Explain your reasoning.

B. Can you be sure no water will spill out? Explain your reasoning.

Problem 12.1

Two students perform an experiment to see if mass affects sinking and floating. They place the four objects shown below—which appear identical except for mass—into water and find that two of them sink and two of them float.



Mass = 21 g Mass = 5 g Mass = 8 g Mass = 14 g

The students then discuss their observations:

Student 1: *"The mass influenced the sinking and floating. When the mass changed, the behavior changed."*

Student 2: *"The mass doesn't influence sinking and floating because they didn't all sink or all float. If the mass influenced their behavior, they should all behave the same.*

With which student do you agree? Explain your reasoning.

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Problem 12.2

Decide whether the variable on the left side of each line below influences the behavior listed to the right. If so, state whether the variable can be used to predict the behavior. Explain your answer.

A. the temperature inside the freezer	whether the water in the ice cube tray will freeze
B. the temperature outside	whether it will snow on a particular day
C. the time of year	how early the sun will rise in Chicago
D. the time of year	the price of airline tickets

Problem 12.3

A current theory of the structure of the earth states that the continents are floating on a fluid interior. If this were true, what could you conclude about the fluid inside? Explain.

Problem 12.4

Discuss what is meant by the phrase "control of variables." Include in your written response specific examples from Section 12.

Explain why the method of control of variables is important to follow in performing an experiment.

Problem 12.5

Answer the questions below about the objects listed in the following table. Explain your reasoning. Do not use algebra.

Objects	Mass	Volume	Density
X	54.0 g	20.00 cm^3	2.70 g/cm^3
Y	23.2 g	20.00 cm^3	1.16 g/cm^3
Z	19.0 g	20.00 cm^3	0.95 g/cm^3

A. Suppose each of these objects is carefully lowered (but not pushed down) into a completely full beaker of alcohol (density 0.80 g/mL). When this happens, alcohol is spilled out of the beaker. Give the volume of alcohol spilled for each object.

B. Suppose each object is carefully lowered (but not pushed down) into a completely full beaker of salty water (density 1.19 g/mL). Give the volume of salty water spilled for each object.

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Problem 12.6

This problem requires reasoning similar to that used in some of the experiments on sinking and floating.

Suppose an object is hanging on a spring and bobbing up and down. Some experiments are performed to find out whether the length of the spring and the mass of the object influence the bobbing rate. The results of three experiments are shown below.

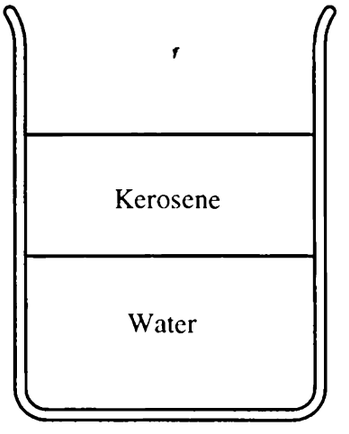
Variables	Exp. 1	Exp. 2	Exp. 3
Length of spring	12 in	27 in	12 in
Mass	50 g	72 g	72 g
Bobs in 10 sec	54	30	45

A. Does length of the spring *influence* the bobbing rate? Tell which experiments should be compared to answer this question and explain your reasoning.

B. Can the length of the spring be used as a predictor of the bobbing rate? Tell which experiments should be compared to answer this question and explain your reasoning.

Problem 12.7

Suppose we have a 5 cm^3 block of Lucite with a density of 1.16 g/cm^3 and a 3 cm^3 block of wood with a density of 0.65 g/cm^3. Suppose also that we have a beaker that has water in the bottom half and kerosene in the top half. The liquids do not mix but are in separate layers as shown in the diagram.



Water has a density of 1.0 g/mL; kerosene, a density of 0.82 g/mL.

A. Describe what would happen to the Lucite block if it were placed in the beaker. Where would the Lucite come to rest?

B. Describe what would happen to the wood block if it were placed in the beaker. Where would the wood come to rest?

C. Suppose we glue the wood and Lucite blocks together. Describe what would happen to this compound object if it were placed in the beaker. Where would it come to rest?

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Additional problems for Part A and Part B

Problem A.1

Here is a description of a familiar course of events: A glass bottle filled with water was placed in the freezer. Some time later it was found that the water had frozen and the glass bottle had cracked open.

In answering the following questions, cite evidence from the above description and experiments you have performed in the laboratory. Explain your reasoning.

A. Upon freezing, did the volume of the water increase, decrease, or remain the same?

B. Upon freezing, did the mass of the water increase, decrease, or remain the same?

C. Upon freezing, did the density of the water increase, decrease, or remain the same?

Problem A.2

Suppose the volume of a piece of glass is measured by displacement of water and then is measured again by displacement of kerosene.

How would the two measurements compare? Explain your reasoning.

Density of glass = 3.20 g/cm^3
Density of water = 1.00 g/cm^3
Density of kerosene = 0.82 g/cm^3

Problem A.3

Two objects, one red and one white, were put in a beaker of alcohol. The red object's density was 0.889 g/cm^3 and the white object's density was 0.891 g/cm^3. Both objects sank to the bottom.

The alcohol in the beaker had the following properties:

Mass	35.6 g
Volume	45.0 mL
Density	0.791 g/mL

A particular amount of water was added to the alcohol and mixed in. The properties of the water added were:

Mass	28.0 g
Volume	28.0 mL
Density	1.000 g/mL

When the water and alcohol were mixed, the white object remained on the bottom of the beaker, but the red object floated up to the surface.

What were the mass, volume, and density of the mixture of alcohol and water in the beaker? Explain your reasoning.

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Problem A.4

At very low temperatures (below –200° C), air becomes liquid. Suppose we have a balloon with 1000 cm^3 of air in it at room temperature and we cool it off so that the air inside is liquefied. What will be the volume of the liquid air in the balloon? Explain your reasoning.

Problem A.5

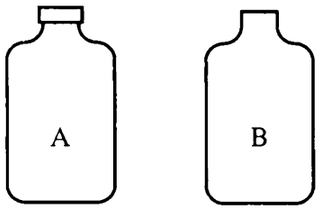
If 80 mL of alcohol and 80 mL of water are mixed, 1 cm^3 of the mixture has a mass of 0.91 g.

What are the mass, volume, and density of the whole mixture? Explain your reasoning.

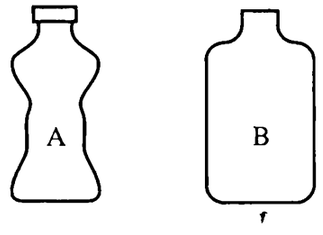
Problem S.6

Two identical empty plastic bottles were outside on a hot afternoon. They were initially uncapped. At about three o'clock, a physicist tightly capped one of the bottles, bottle A.

By midnight, the weather had turned very cold. When the physicist observed the bottles at midnight, she found that the sides of bottle A were caved in, but bottle B was the same shape it had been in the afternoon.



Bottles at 3 P.M.



Bottles at midnight

Based on the information given, answer the following questions. Explain your reasoning.

A. Did the mass of air inside bottle A increase, decrease, or remain the same from 3 P.M. to midnight?

B. Did the volume of air inside bottle A increase, decrease, or remain the same from 3 P.M. to midnight?

C. Did the density of air inside bottle A increase, decrease, or remain the same from 3 P.M. to midnight?

D. Did the mass of air inside bottle B increase, decrease, or remain the same from 3 P.M. to midnight?

E. Did the volume of air inside bottle B increase, decrease, or remain the same from 3 P.M. to midnight?

F. Did the density of air inside bottle B increase, decrease, or remain the same from 3 P.M. to midnight?

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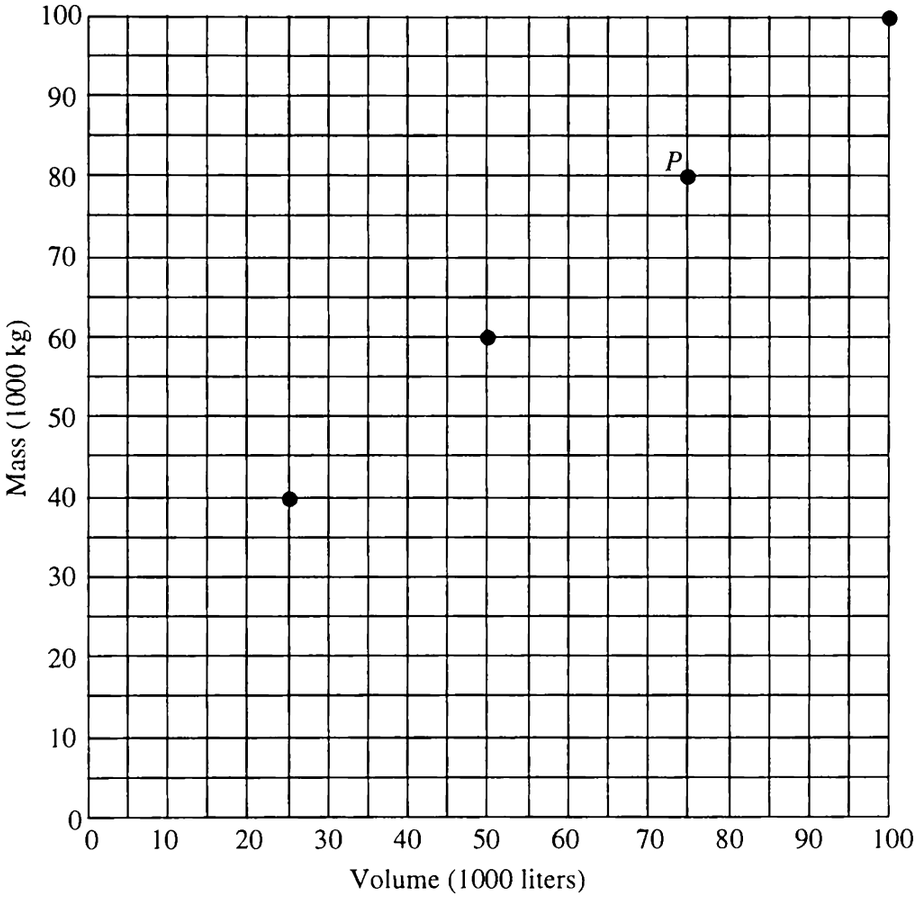
Problem 13.1

The circumference of the earth is about 25,000 miles. Suppose a rope is tied around the earth so that it fits tightly.

If 20 ft of rope were added, the rope would not quite fit tightly anymore. Suppose the rope were held up equally all the way around the earth. Which of the following could pass under the rope: an amoeba, a bumblebee, a cat, or a dinosaur? Explain.

Problem 13.2

A tank truck is used to haul a certain liquid. The truck has different masses depending on how full the tank is. The trucker is more concerned with the total mass of the truck and load than the mass of just the liquid alone. Below is a graph of the mass of the loaded truck versus volume of liquid in the tank. The points represent the tank being one-quarter full, half full, three-quarters full, and full.



A. What does the single point *P* tell us?

B. What is the density of the liquid when the truck is three-quarters full? Explain how you can tell.

C. What is the mass of the empty truck? Explain.

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Problem 14.1

Suppose 30 cm^3 of a particular stone has mass m. Write an expression in terms of m for the density of this kind of stone.

Problem 14.2

A beaker initially has 140 mL of water in it. A volume V of water is then added. Write an expression in terms of V for the total volume of water in the beaker.

Problem 14.3

Suppose we have a 130 g piece of aluminum. Aluminum has a density of 2.7 g/cm^3. Write an expression for the volume of aluminum that must be added to this piece to increase the mass to a total mass m.

Problem 14.4

Suppose we have a volume V of methanol, and we wish to increase the mass of methanol by an additional 24 g. Write an expression in terms of V for the volume of methanol that must be added. (Methanol has a density of 0.80 g/mL.)

Problem 14.5

A 400 gram piece of a certain substance is added to another piece of the same substance with an initial volume of 200 cm^3. After this addition, the new volume of substance is V. Write an expression in terms of V for the density of this substance.

Problem 14.6

What is the density of a layer of liquid 10 cm deep in a level-bottomed tank if a mass m of liquid covers each square centimeter of the tank bottom? Answer with an expression in terms of m.

Problem 14.7

A balloon contains 50 g of liquid. If the mass of liquid in the balloon is increased to m, the balloon swells by 18 mL. Write an expression in terms of m for the volume of one gram of this liquid.

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Problem 14.8

Two objects made of different substances are placed on a balance. Initially, the first object's mass is 50 g more than that of the second. If equal volumes V of each substance are removed, balance is achieved. The density of the first object is D. Write an expression for the density of the second object in terms of V and D.

Problem 14.9

A volume of 8000 mL of a liquid is poured into a tank containing 3000 mL of the liquid. The initial mass of the tank and liquid is 10 kg and the final mass is m. Write an expression in terms of m for the mass of the empty tank.

Problem 14.10

Suppose we have 68 g of aluminum. (The density of aluminum 2.7 g/cm^3). How much more mass is needed to make a total volume of 40 cm^3? Use algebra.

Include in your solution: a clear definition of any variables you define, explanations of the reasoning behind the operations in your expressions, interpretations of the meanings of the expressions that you set equal to each other, and an explanation of why the two sides of each equation are set equal.

Problem 14.11

Some air was released from a pressurized tire. The volume of air inside the tire, 450 cm^3, did not change. The mass of air released was 0.5 g, and the final density of the air in the tire was 0.0053 g/cm^3. What was the initial mass of air in the tire? Solve algebraically.

Include in your solution: a clear definition of any variables you define, explanations of the reasoning behind the operations in your expressions, interpretations of the meanings of the expressions that you set equal to each other, and an explanation of why the two sides of each equation are set equal.

Problem 14.12

Suppose we have equal volumes of aluminum and a mystery metal. The mystery metal has a mass of 80 g. What is the density of the mystery metal if the aluminum has a mass of 19 g and a density of 2.7 g/cm^3?

Include in your solution: a clear definition of any variables you define, explanations of the reasoning behind the operations in your expressions, interpretations of the meanings of the expressions that you set equal to each other, and an explanation of why the two sides of each equation are set equal.

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Problem 14.13

A certain statue must have just the right mass for use in an artistic display of clever balancing. The statue can be made of two substances that look the same but have different densities, 1.4 g/cm^3 and 1.6 g/cm^3. The first version of the statue is exactly the right size and shape but has 50 g too much mass. What volume of the material with a greater density must be replaced by the less dense material to achieve the correct mass?

Include in your solution: a clear definition of any variables you define, explanations of the reasoning behind the operations in your expressions, interpretations of the meanings of the expressions that you set equal to each other, and an explanation of why the two sides of each equation are set equal.

Problem 16.1

When alcohol and water are mixed, the final volume is less than the sum of the initial volumes. (Recall Experiment 7.1.) When water is poured into dry sand, the final volume i less than the sum of the initial volumes of dry sand and water.

Make an analogy between mixing alcohol and water and mixing water and dry sand. Be sure to identify as many corresponding parts as possible, then tell how they are alike.

Problem 18.1

A beaker contains 12 g of salt dissolved in 75 mL of water. Suppose we do the following:

(1) We add 50 mL of pure water to the beaker and stir up the contents.
(2) We pour 5 mL of the new solution from the beaker into an empty test tube.

How much salt will be in the test tube? Explain your reasoning. Do not use algebra.

Problem 18.2

A student dissolves 50 mg of sodium chloride in enough water to make a concentration of 0.32 mg/mL. The student then pours solution into a beaker until the beaker contains 8 mg of sodium chloride. How much water is in the beaker? Explain. Do not use algebra.

Problem 18.3

A certain lake in Utah is very salty. Assume it is a saturated solution of salt in water. In this problem, neglect the difference between volume of water and volume of solution. The solubility of salt at lake temperature is 35 g/100 mL.

A. What is the mass of salt dissolved in 100 mL of lake water?

B. What is the mass of 100 mL of pure water?

C. What is the density of lake water?

D. What is the concentration of lake water?

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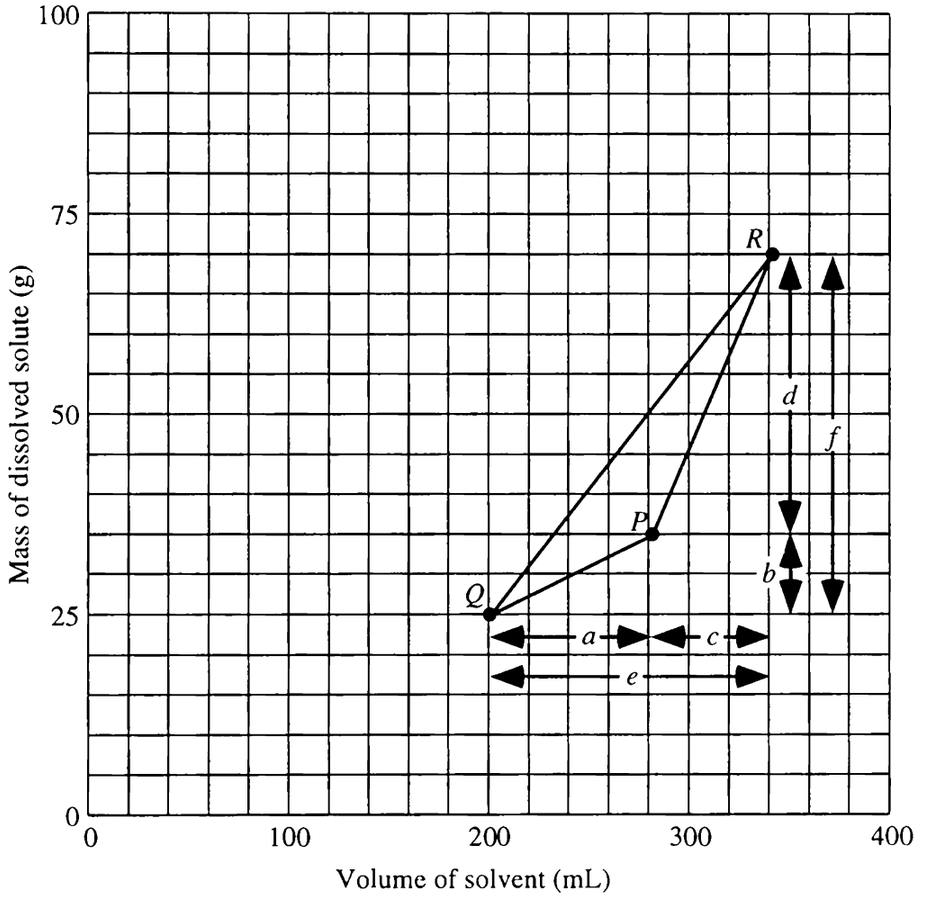
Problem 18.4

Imagine you have a solution of magnesium sulfate with a concentration of 4.2 g/100 mL to be used as medication. You have just prepared for a patient a two week supply of this solution containing 34.6 g of magnesium sulfate.

Suppose you want to give the patient 12.0 g more magnesium sulfate. What volume of solution would you have to add to the patient's supply? Explain. Do not use algebra.

Problem 18.5

Suppose we have a beaker of solution initially represented by point *P* on the graph below. Some is poured out leaving a solution in the beaker represented by point *Q*. A more concentrated solution is then added giving a solution in the beaker represented by point *R*.



A. Some of the lengths labeled *a, b, c, d, e,* and *f* represent the masses of dissolved solute or the volume of solvent in samples of solution that were involved in the problem. Which lengths describe samples of solution that actually existed, according to the problem? For each length that does, describe the sample of solution it describes and what property is described.

B. Do any of the three slanted lines connecting *P, Q,* and *R* have a slope that is the concentration of a solution that actually existed, according to the problem? For each such line, give the slope and identify the solution that has that concentration.

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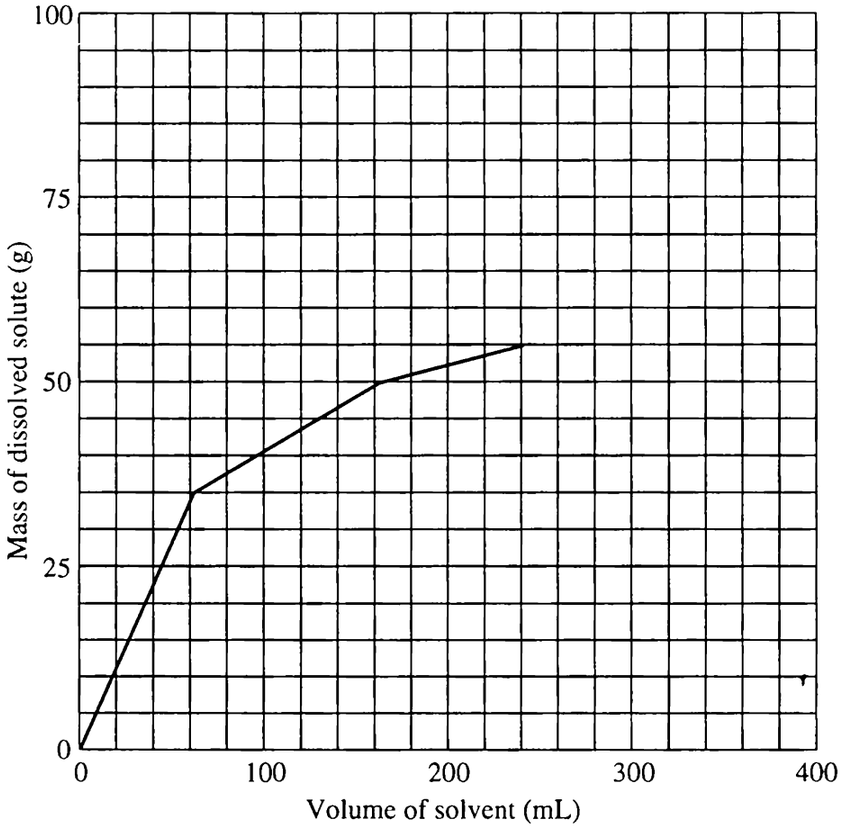
Problem 18.6

The following graph shows a plot of mass of dissolved solute versus volume of solvent for the contents of a beaker of solution. Three solutions were successively added to the beaker. There are many points plotted. Salt and water were continually added to the beaker and therefore the solution in the beaker had many masses of dissolved solute and many volumes of solvent as time went by.

The beaker was initially empty, then the three solutions were slowly poured into the beaker one after the other.



A. What was the concentration of the first solution added?

B. What was the concentration of the second solution added?

C. What was the concentration of the third solution added?

D. What was the total volume of solvent in the beaker after the second solution was completely added but before any of the third was added?

After all three solutions were added, the contents of the beaker were thoroughly stirred. The solution in the beaker was then slowly poured out.

E. What was the concentration of the contents of the beaker after stirring?

F. Plot a point that describes the contents of the beaker when half of the solution has been poured out.

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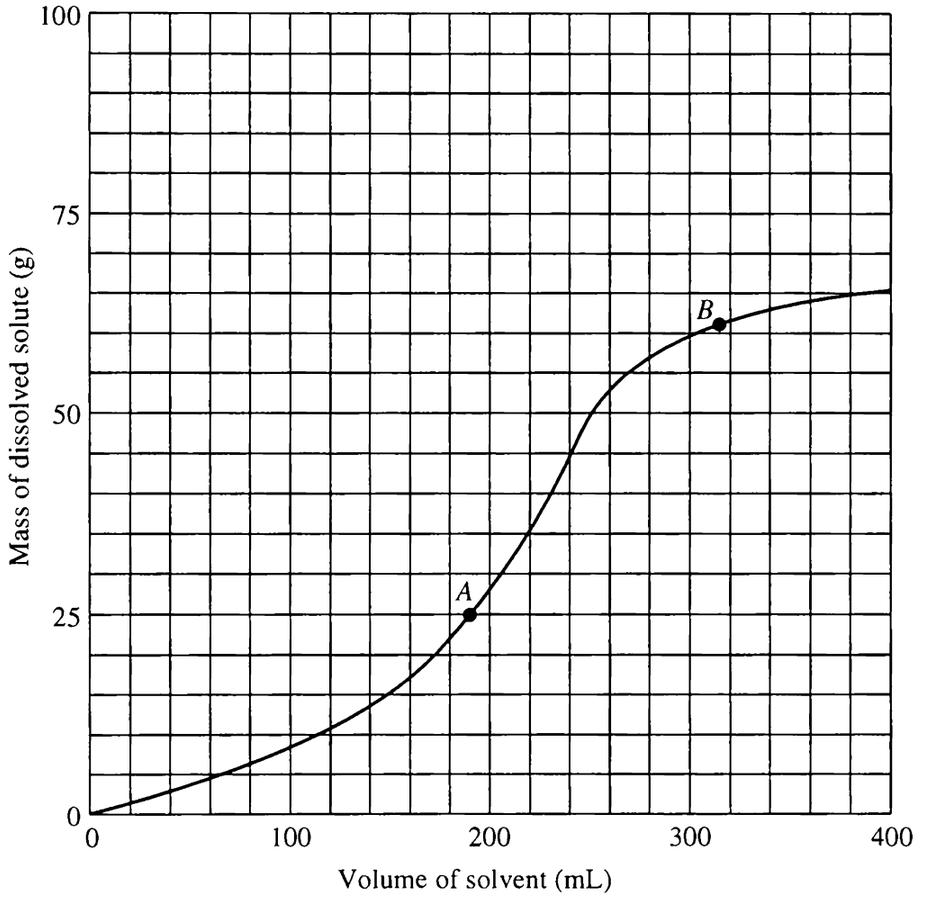
Problem 18.7

Consider the following two solutions of citric acid (a white crystalline solid) dissolved in water and methanol.

Solution A is 2.4 g dissolved in 80 mL of water.
Solution B is 2.0 g dissolved in 50 mL of methanol.

A certain object sinks in one of the solutions and floats in the other. In which solution does the object float?

Problem 18.8



The graph above shows a plot of mass of dissolved solute versus volume of solvent for the solution contained in a beaker. The line represents the succession of points that represented the contents of the beaker as time went by. The beaker started out empty, then solutions with varying concentrations were added.

A. Consider the points *A* and *B*. At which of these two points was the more concentrated solution being added to the beaker? Explain your reasoning carefully.

B. At which of these two points was the solution in the beaker more concentrated?

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Problem 18.9

Suppose we add 4 g of solute to 16 mL of a solution with concentration 1 g/100 mL. What will be the concentration of the resulting solution? Explain your reasoning.

Problem 18.10

Suppose we mix together 50 mL of a solution with concentration 9 g/100 mL and 5 mL of a solution with concentration 20 g/100 mL. What will be the concentration of the mixture? Explain your reasoning.

Problem 18.11

Suppose we initially have 75 mL of a saltwater solution with concentration 0.046 g/mL. This solution is heated until a volume V of water has been boiled off. Write an expression for the final volume of the solution. Explain your reasoning.

Problem 18.12

Suppose we have a volume V of solution with concentration 0.0015 g/mL. To this solution we add 50 g more solute. What is the total mass of solute? Answer by writing an algebraic expression. Explain your reasoning.

Problem 18.13

Suppose we have 370 mL of saltwater solution with concentration C. To this solution we add 18 g of salt, which dissolves. Write an expression for the total amount of dissolved salt in the solution after the addition. Explain your reasoning.

Problem 18.14

Suppose we have 400 mL of a mystery solution. To this is added a volume V of solvent. The resulting solution has a concentration of 0.075 g/mL.

A. What was the initial mass of dissolved solute before the addition? Answer by writing an algebraic expression. Explain your reasoning.

B. What was the initial concentration? Again write an expression and explain your reasoning.

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Problem 18.15

A beaker contains a solution of salt and water. Then more salt water of a different concentration is added to the beaker. The additional salt water contains 16 g of salt. After the addition, the total volume of solution in the beaker is 250 mL and the concentration is C. Write an expression for the initial mass of salt in the beaker before the addition. Explain your reasoning.

Problem 18.16

A 200 mL sample of a certain saltwater solution contains 7.0 g of salt. How much salt is needed to make 550 mL of salt water with the same concentration? Solve algebraically with the necessary explanations.

Problem 18.17

Suppose we have a solution of 16 g of solute dissolved in 70 mL of solution. How much solvent must we add to dilute the solution to a concentration of 0.17 g/mL? Solve this problem three ways as follows. In each case, let V be the volume of solvent added.

A. Write an expression in terms of V for the concentration of the new solution. Set this equal to the new concentration and solve the equation. Explain the reasoning behind your expression.

B. Write an expression in terms of V for the mass of solute dissolved in the new solution. Set this equal to 16 g (the mass of solute you know is dissolved in the new solution) and solve the equation. Explain the reasoning behind your expression.

C. Write an expression in terms of V for the volume of solvent in the new solution. Set this equal to the value that the volume must have if a mass of 16 g of solute yields a concentration of 0.17 g/mL. Explain the reasoning that is behind your expression and solve the equation.

Problem 18.18

Suppose we have a strong solution containing a total of 6.00 g of solute. We then dilute this solution by adding 140 mL of water, obtaining a diluted solution with concentration 0.03 g/mL.

What was the initial concentration of the strong solution? Explain your reasoning.

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Problem 18.19

Suppose you have two solutions of cupric acetate. Solution A is 240 mg of cupric acetate dissolved in 400 mL of water. Solution B is one liter of solution with concentration 14 mg/mL. How much of solution B must be added to solution A to yield a solution with concentration 9 mg/mL?

Solve this problem algebraically by the following steps:

(1) List the information given in the problem. Identify the quantity you are asked to find.

(2) Choose your variable and define it.

(3) Write an expression for the total mass of solute in the resulting solution.

(4) Write an expression for the total volume of solvent in the resulting solution.

(5) Write an equation. Be sure to interpret the expression on each side and explain why you set them equal.

(6) Answer the question.

Problem 18.20

50 mL of water is added to a certain amount of solution with concentration 0.080 g/mL. The final concentration is 0.065 g/mL. What was the initial volume of water in the solution before the addition was made? (*Hint:* What stays the same during the addition?) Explain your reasoning.

Problem 18.21

Suppose we have 100 mL of solution with concentration 0.0030 g/mL. To this is added 40 mL of a different solution. The resulting solution has concentration 0.0035 g/mL. What is the concentration of the added solution? Explain your reasoning.

Problem 18.22

Solution A has concentration C_A and solution B has concentration C_B. Solution A is twice as concentrated as solution B. Express this relation in an equation. Explain your reasoning.

Problem 18.23

Suppose you have a strong solution with toluene as the solvent. This solution has concentration 0.05 g/mL, but you need a diluted solution with concentration 0.02 g/mL. If there is only 50 mL of the solvent toluene available, what is the maximum mass of solute that can be diluted to the desired concentration? Explain your reasoning.

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Problem 19.1

Sea water is a 3.5% solution of salt in water.

A. Suppose we have 50 mL of sea water in a beaker. If we increase the volume of sea water in the beaker by 10 mL by adding more sea water, how much more salt will be in the beaker? Explain your reasoning.

B. Make up, state clearly, and solve an analogous problem involving the mass, volume, and density of aluminum. Be sure that what is given and what is asked correspond in both problems.

Problem 20.1

Beaker A contains 300 mL of a 3% solution of salt in water. Beaker B contains 100 mL of a 5% solution of salt in water. Beaker X contains 500 mL of an 8% saltwater solution. Beaker Y is initially empty.

Suppose the contents of beakers A and B are poured into beaker Y and mixed. Would the solution in beaker Y taste more salty, less salty, or just as salty compared to the solution in beaker X? Explain your reasoning.

Problem 20.2

Suppose you have, in a beaker, 400 mL of a 5% saltwater solution. What would you actually do in the lab to make this solution into a 4% solution? Explain your reasoning.

Problem 20.3

Suppose we mix 10 mL of a 5% solution and 25 mL of a 3% solution. How much solute will be dissolved in 12.5 mL of the mixture? Explain your reasoning.

Problem 20.4

A solution of potassium chloride is made with 200 mL of water. The entire solution has a mass of 215 g. What is the concentration of the solution?

Problem 20.5

Sometimes concentrations are referred to a standard mass of solvent. Suppose we have a solution of iodine dissolved in ethanol with a concentration of 0.01g/100 g solvent. How much solute would be dissolved in 50 milliliters of ethanol?

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Problem 20.6

"Normal saline solution" used in medicine is a saltwater solution with the same salt concentration as human blood. It is a 0.9% solution. What is the concentration of "half normal saline?"

Problem 20.7

Amounts of drugs are described in several different ways besides the mass in grams. For example, we do not speak of "number of grams of penicillin"; we speak of the "number of units of penicillin."

Penicillin comes in a very concentrated solution that must be diluted for use. A bottle containing 5,000,000 units of penicillin has a label that reads, "add 23 mL, 18 mL, 8 mL, or 3 mL diluent (such as water) to provide 200,000 u; 250,000 u; 500,000 u; or 1,000,000 u per mL, respectively."

A. Suppose you add 18 mL of water. How many mL of solution must you give a patient who is to receive 500,000 units?

B. Suppose instead you had added 3 mL to the bottle. Could you still administer 500,000 units? If so, how?

C. Suppose instead you had added 23 mL to the bottle. Could you still administer 500,000 units? If so, how?

Problem 20.8

Another amount used in the pharmaceutical business is the *milliequivalent.* (This is a measure that has its origin in chemistry.)

A commercially available bottle of potassium chloride solution contains 20 mEq dissolved in 10 mL of water.

What would you do if you wished to prepare an injection that consisted of 3 mEq of potassium dissolved in 30 mL of water? Explain your reasoning.

Problem 20.9

Yet another measure of concentration used occasionally is *ratio strength.* This is written:

grams of solute : mL of solution.

Thus, for example, a 1 : 500 solution contains 1 gram of solute in every 500 mL of solvent.

How much concentrated rat poison at 1 : 35 is needed to make 10 liters of spraying solution at 1 : 500? Explain your reasoning.

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Problem 21.1

Two students in the United States read the following report in a newspaper: "In the United States there were 3000 cases of purple plague reported last year but only 280 cases in the country of Lauternstein."

Student 1 says: *"We should move to Lauternstein; we would be safer from the plague."*

Student 2 says: *"No, we are safer here. Remember, the population of the United States is 250 million but there are only 4.8 million people in Lauternstein."*

A. With which student do you agree? Explain your reasoning. Why do you think the student who was wrong said what he did?

B. Make an analogy between the important quantities in the problem above and the quantities involved with the concept of density. First identify what corresponds to density, to mass, and to volume. Then tell how the corresponding parts are alike.

Problem 21.2

A. Let variables x, y, and z represent the quantities mass, volume, and density, respectively. Express the relationship among these quantities in an algebraic equation.

B. The quantities of mass, volume, and density are mathematically analogous to certain quantities describing circles. Identify the corresponding quantities from the two topics by filling in this table:

Material	Circles
mass	_____
volume	_____
density	_____

C. Now represent each quantity describing circles by the same variable that represented the analogous quantity from mass, volume, and density.

Variable	Circle quantity
mass	_____
volume	_____
density	_____

D. Using these variables, express the relationship among the circle quantities.

E. How does the equation written for circles in part D compare with the equation for mass, volume, and density written in part A? Explain any similarities or differences.

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Problem 22.1

Two objects are made of the same material. The larger object has a mass that is 1.15 times as much as the smaller object. The volume of the smaller object is 184 cm^3. How many cubic centimeters larger is the big object than the little one? Explain your reasoning.

Problem 22.2

Two objects made of different kinds of metal have the same volume. The first object has a mass that is 3.6 times as much as the second. The second object has a density of 4.5 g/cm^3. What is the density of the first object? Explain your reasoning.

Problem 22.3

Two objects made out of different kinds of wood have the same mass. The second object is 1.57 times as dense as the first. The first object has a density of 0.34 g/cm^3 and a volume of 400 cm^3. What is the volume of the second object? Explain your reasoning.

Problem 22.4

Explain what is wrong with the solution to the following problem:

Problem: A plastic object has a mass of 360 g and a volume of 400 cm^3. If 50 cm^3 of aluminum is glued to the plastic, how much will the mass of the object increase?

Incorrect solution:

The ratio of the volumes is 50/400, which is 0.125, so that the additional piece is 0.125 times as large as the original piece. Thus, the extra mass will also be 0.125 times as heavy as the original mass. The additional mass is therefore 0.125×360 g, which is 45 g.

Problem 22.5

Solve this problem by calculating the ratio of the volumes, interpreting the resulting number, and using it to answer the question. Check your reasoning with a staff member.

A concrete wall is being built. So far, 500 liters of concrete, with a mass 20,000 kg, has been used. How much mass will be added if an additional 400 liters of concrete is used?

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Problem 22.6

In this problem, the calculations have been done for you. The questions ask about the reasoning involved.

Problem: A certain object has a mass of 89.0 g. A second object made of the same material displaced 15% more water than the first. What is the mass of the second object?

Solution: $89.0 \times 1.15 = 102$

A. What does the 1.15 represent?

B. How do you interpret the number *102?*

C. Explain why the multiplication is performed.

Problem 22.7

In this problem, the calculations have been done for you. The questions ask about the reasoning involved.

Problem: A sample of liquid has a mass of 450 g and a volume of 300 mL. If 60 mL more of this liquid is added to the sample, how much will the mass increase?

Solution: (1) $\frac{60}{300} = 0.20$ (2) $0.20 \times 450 = 90$

A. How do you interpret the number *0.20?*

B. Explain why the division is done in step 1.

C. How do you interpret the number *90?*

D. Explain why the multiplication is done in step 2.

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Problem 22.8

In this problem, the calculations have been done for you. The questions ask about the reasoning involved.

Problem: A piece of oak has a mass of 3.4 kg and a volume of 4400 cm. What is the mass of 20 cm of this same kind of oak?

Solution: (1) $\frac{3.4}{4400} = 0.00077$ (2) $0.00077 \times 20 = 0.015$

A. How do you interpret the number *0.00077?*

B. Explain why the division is done in step 1.

C. How do you interpret the number *0.015?*

D. Explain why the multiplication is done in step 2.

Problem 22.9

In this problem, the calculating has been done for you. The questions ask about the reasoning involved at each step.

Problem: Two metal blocks have the same mass. The first has a volume of 62.4 cm^3 and a density of 2.7 g/cm^3. The second has a volume of 19.4 cm^3. What is the density of the second object?

Solution: (1) $\frac{62.4}{19.4} = 3.22$ (2) $3.22 \times 2.7 = 8.7$

A. How do you interpret the number *3.22?*

B. Explain why the division is done in step 1.

C. How do you interpret the number *8.7?*

D. Explain why the multiplication is done in step 2.

Problem 23.1

Sea water has a salt concentration of 0.035 g/mL. How much salt can be added to 500 mL of sea water before no more will dissolve? Explain your reasoning.

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Problem 23.2

A saturated solution of potassium sulfate is made with 50 mL of water at 20°C. The entire solution has a mass of 55.5 g. What is the solubility of potassium sulfate at 20°C? Explain your reasoning.

Problem 23.3

A certain saturated solution consists of a mass m of solute dissolved in 600 mL of solvent. Suppose we wanted to dissolve one more gram of this solute. Write an expression for the volume of solvent required to dissolve one gram of solute. Explain.

Problem 26.1

Using the similarity of water vapor in air to solutions discussed in Exercise 26.5, tell how you would explain what is going on in each of the following situations.

A. Water forms on the outside of a glass of cold beer.

B. Something is seen rising from a hot cup of tea or coffee.

C. Clean air in the mountains is sometimes hazy.

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PHYSICS BY INQUIRY

Heat and Temperature

In *Heat and Temperature,* we examine the behavior of objects when they are heated and cooled. On the basis of our observations, we develop the concepts of heat and temperature and investigate the thermal properties of matter.

Part A: Measurements of heat and temperature

As is often the case in science, common terms are used to represent very specific concepts. Heat and temperature are two examples. In Part A of this module we construct an operational definition for temperature that allows us to discuss the relationship between these concepts.

Section 1. Temperature

Any instrument used to measure temperature may be called a thermometer. In this class we will use either mercury or alcohol thermometers. These thermometers consist of a chamber or bulb of liquid with a long narrow tube attached, as shown below.

When the liquid inside becomes hotter, it expands and moves up the tube. When it becomes cooler, the liquid contracts and retreats down the tube. The level of the liquid in the narrow tube can therefore be used as a measure of the temperature. To measure the temperature of an object, we put the bulb inside the object and wait for the liquid to adjust to its new surroundings. We then read the level of the liquid in the tube.

Experiment 1.1

In a thermometer, almost all of the liquid is in the bulb. The liquid rises in the narrow tube because the volume of the liquid increases; the liquid expands as it becomes warmer. Using a narrow tube makes it easier to observe the small changes in volume of the liquid in the bulb.

Examine a thermometer. Approximately how many times as wide as the narrow tube is the bulb?

The volume change of the liquid is the same in a narrow tube and a wide tube. Explain why it is easier to observe this change in volume using a narrow tube than using a wide tube.

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In order to use a thermometer, we need to have marks along the narrow tube for reading the level of the liquid. But how can we decide where to put the marks and how to number them? In order to mark or *calibrate* the thermometer, it is helpful to be able to produce and recognize some standard temperatures. The following experiment illustrates how this can be done.

Experiment 1.2

For this experiment you will need about 75 mL of water. (Use de-ionized water if it is available.) Place the water in a beaker and record the temperature.

Heat the beaker over an alcohol burner or hot plate and record the temperature of the water every 30 seconds. Do not let the thermometer rest on the bottom of the beaker; instead place the thermometer about halfway into the water and stir gently. Do not let the thermometer touch the sides of the beaker.

Caution: When the water begins to boil, some may splash out of the beaker.

Record the time when the water starts to boil vigorously. (*Note:* Small bubbles of air come out of solution when water is heated. Do not mistake this for boiling, which involves large bubbles.) Stop the experiment after the water has been boiling for at least three minutes.

Make a graph of temperature versus time from your data. Indicate on the graph the time when the water began to boil.

Does the temperature of the water change in the same way before it starts to boil and after it has begun to boil? Explain how you can tell from your graph.

✔ Discuss this experiment with a staff member.

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The temperature at which water boils is called its *boiling point.* As you observed in the preceding experiment, the boiling point of water is easy to identify. Similarly, it is easy to recognize when water is freezing. Water begins to freeze at a certain temperature and then remains at that temperature until all of the water has solidified. Here again the temperature remains constant for a considerable time. In addition, there is both water and ice present as the water is in the process of freezing or melting.

On the *Celsius temperature scale* (formerly called the centigrade temperature scale), the freezing point of water is 0°C and the boiling point of water is 100°C. These temperatures define the Celsius scale. *In this module, unless otherwise stated, temperatures always will be given in degrees Celsius.*

Exercise 1.3

A. Give an operational definition of the temperature of an object or substance. (See Section 5 of *Properties of Matter* for what is meant by an operational definition.)

B. Discuss the steps necessary to calibrate a thermometer.

C. Find two thermometers of the same kind. Lay them side by side with their 100° marks lined up next to each other.

   Are the 0° marks on the two thermometers lined up?

   Do you think the thermometers were individually calibrated? Give the evidence supporting your conclusion. What difference(s) in the thermometers might make individual calibrations necessary?

✔ Discuss this exercise with a staff member.

The definition of temperature in terms of mercury thermometers (or any other kind of thermometer) is entirely adequate for our purposes. However, at sufficiently high or low temperatures mercury boils or freezes and cannot be used in a thermometer. More sophisticated definitions of temperature do not rely on a particular substance like mercury. These abstract definitions reflect more subtle aspects of the concept of temperature than we can cover in this module.

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The operational definition of temperature, like all operational definitions, does not describe the important uses of the concept. From common experience, it is clear that temperature is useful for knowing how the weather will feel outdoors or for setting an oven to cook. In the experiment below, we compare our sense of hot and cold with the temperature as defined by a thermometer.

Experiment 1.4

Obtain a block of wood and a block of aluminum that have been sitting out in the room for a while.

A. Measure the temperature of each object with a thermometer. Do not hold the objects while you are making your measurements. Leave them on the table.

   How do the temperatures compare?

B. Place the inside of your wrist against each object.

   Does one object feel colder than the other or do they feel the same?

C. According to your operational definition, do the two objects have the same temperature?

   Is it possible for two objects to feel different to the touch and yet have the same temperature?

   Does the feeling of hotness or coldness give a reliable indication of the temperature of an object?

Experiment 1.5

A. Place about 50 g of water in a beaker. (How can you use a graduated cylinder to measure 50 g of water?) Find the temperature of the water.

   Pour about half of the water into another beaker and record the temperature of the water in each beaker.

B. Suppose you were to divide a sample of water into many parts.

   How would the temperature of each part compare to the temperature of the original sample? Name another physical quantity that behaves in this same way.

   How would the mass of each part compare to the mass of the original sample? Name another physical quantity that behaves in this same way.

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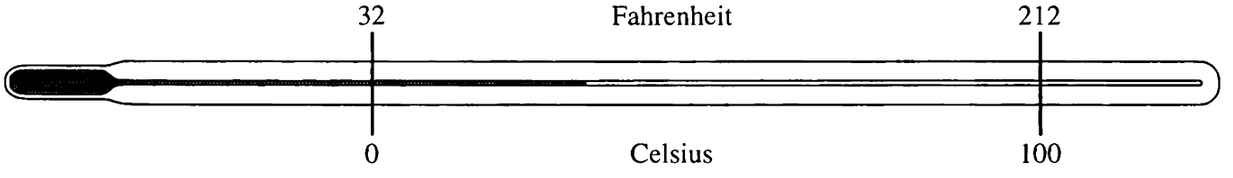
Exercise 1.6

A. Suppose a 50 g copper rod is heated so that the temperature of the rod rises by 14 Celsius degrees. By how much does the temperature of one gram of copper in the rod change? Explain your reasoning.

B. Suppose a greedy landlord owns an apartment building with seven apartments. The landlord turns down the heat so that the temperature in each apartment drops 3 Celsius degrees. By how much does the temperature of the whole building change? Explain your reasoning.

The *Fahrenheit temperature scale,* like the Celsius scale, is based on the freezing and boiling points of water. The freezing point is 32° on the Fahrenheit scale and the boiling point is 212°. The thermometer shown below is marked in both Celsius and Fahrenheit.



Exercise 1.7

A. Make a graph of Fahrenheit temperature versus Celsius temperature.

Measure the slope of the graph.

Give an interpretation for the slope.

B. Suppose a thermometer reads T degrees Celsius. Write an expression for how many Fahrenheit degrees there are between 0°C and T°C.

Your expression would give the Fahrenheit temperature if both temperatures had their 0° mark at the same place. However, when the Celsius scale reads 0°, the Fahrenheit scale reads 32°.

Write an equation relating the Fahrenheit temperature to the Celsius temperature. Define your variables clearly.

C. Which is larger, a Fahrenheit degree or a Celsius degree? That is, which is a larger temperature change: 1 Fahrenheit degree or 1 Celsius degree? Explain how you can tell from the graph in part A and from your equation in part B.

D. Suppose you had a thermometer with both Celsius and Fahrenheit scales on it. Is there any temperature at which you would read the same number off of both scales?

✔ Discuss this exercise with a staff member.

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Section 2. Changes in temperature

In this section and the next, we will investigate how temperatures change when hot water and cold water are mixed. In situations such as this, both the hot and the cold objects are affected: both change temperature. To reflect the fact that two objects participate, this process is called an *interaction;* specifically, a *thermal interaction*.

In most thermal interactions between two objects, the temperature of the hotter one decreases and the temperature of the colder one increases. Eventually, the two interacting objects arrive at the same intermediate temperature. The objects then maintain constant temperature unless affected by interactions with other objects. When two interacting objects have reached the same temperature, they are said to be in *thermal equilibrium.*

Since we are interested in the interaction between hot and cold water, we do not want any other interactions to be going on at the same time. That way, all of the temperature changes are due to just the interaction between the hot and cold water. Unfortunately, it is not possible to prevent all other thermal interactions entirely. However, by conducting experiments inside a Styrofoam cup it is possible to reduce the effects of other interactions. Styrofoam has the useful property of keeping hot things hot and cold things cold. In terms of thermal interactions, we would say that the plastic foam cup inhibits interactions between the surroundings and whatever is inside the cup.

In the following and all subsequent experiments, estimate temperatures to the nearest tenth of a degree Celsius (0.1 °C).

Experiment 2.1

In the next several experiments, water will be poured from one Styrofoam cup to another. This experiment concerns the effect that pouring has on the temperature of water.

A. Pour about 50 g of hot water (at 60°C to 70°C) from one cup to another. Measure the temperature before and after pouring.

How much does the temperature of the water change?

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B. Pour about 50 g of room temperature tap water from one cup to another.

How much does the temperature of the water change?

C. In which case above (if either) was the temperature change smaller?

D. Suppose you are interested in the thermal interaction between hot water and cold water and wish to minimize other interactions.

What does your answer to part C suggest about whether it is better to pour hot water into cold water or cold water into hot water? Explain.

It is important in the following experiments to know the temperatures of the two samples of water *immediately* before they are mixed. The following procedure for mixing the water should be followed.

Procedure for mixing water samples

Record the temperature of the tap water first, since its temperature will not be changing while it is in the cup. Hold the cup of tap water above the hot water as if you were about to pour the first drop. Just before you actually do pour, record the temperature of the hot water. Then immediately pour the room temperature water into the hot water. Quickly record the temperature of the resulting mixture.

Experiment 2.2

A. Mix equal masses of hot water and tap water. Use at least 25 g of water for each sample. Carefully note the temperature of each sample before mixing and then note the final temperature after mixing.

Use several initial temperatures for the hot water, but do not use temperatures higher than 60°C or the water will cool too fast in the cup. Continue experimenting until you have a rule for predicting the temperature of the mixture and data to support your rule.

Write a clear statement of your rule.

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B. When hot and cold water are mixed, the hot water cools off and the cold water warms up. The final temperature of both is the same. If we subtract the final temperature from the initial temperature of the cold water, the answer will tell us how much the temperature of the cold water changed. We can similarly calculate the temperature change of the hot water.

Calculate the temperature changes of the hot water and of the cold water in each of your experiments in part A.

How do the temperature changes of the hot and cold water compare?

✔ Discuss your results with a staff member.

It is conventional to distinguish between temperature readings and changes in temperature. If the mercury in a thermometer is next to the 20 mark on the Celsius scale, the *temperature* is said to be 20°C or 20 degrees Celsius. If later, the temperature is 40°C, the *temperature change* is said to be 20 C° or 20 Celsius degrees. Similarly, on the Fahrenheit scale temperatures are measured in degrees Fahrenheit, but temperature changes are measured in Fahrenheit degrees.

Experiment 2.3

In this experiment, we will investigate how the masses of the samples of water affect the final temperature of the mixture.

A. Mix various amounts of hot and room temperature water. Use at least 25 g of water for each sample. Do not use water hotter than 60°C. Use simple mass ratios such as 2 to 1, 1 to 2, 3 to 1, 2 to 3, and so on. Do five or six experiments. Enter your results in a table like the one below:

Hot water				Cold water			
Mass	Initial temp.	Final temp.	Temp. change	Mass	Initial temp.	Final temp.	Temp. change

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B. If two samples of water are mixed, it is possible to predict which sample will change temperature more. How? Support your answer with data from part A.

C. For each experiment in part A, calculate the following two quantities:

Quantity 1: the mass of the hot water times the temperature change of the hot water.

Quantity 2: the mass of the cold water times the temperature change of the cold water.

Plot a graph of quantity 1 versus quantity 2. That is, plot a graph of the *mass times temperature change* for the hot water versus the *mass times temperature change* for the cold water.

Should *(0,0)* be included as a data point?

D. The graph of quantity 1 versus quantity 2 should be nearly straight. Draw a single straight line through the middle of the points.

Note: When experimental data points do not lie exactly in a neat pattern, the usual practice is to draw a smooth curve or line through the points. *Do not draw a connect-the-dots, zigzag line.* Such a line implies that you think the measured data points are exact, with no uncertainty. A zigzag graph indicates that the quantities are related in a very irregular way; a smooth graph indicates a more regular relation.

Compute the slope of the graph.

What does your result for the slope imply about the relation between the mass times temperature change of the hot water and the mass times temperature change of the cold water?

✔ Discuss your results with a staff member.

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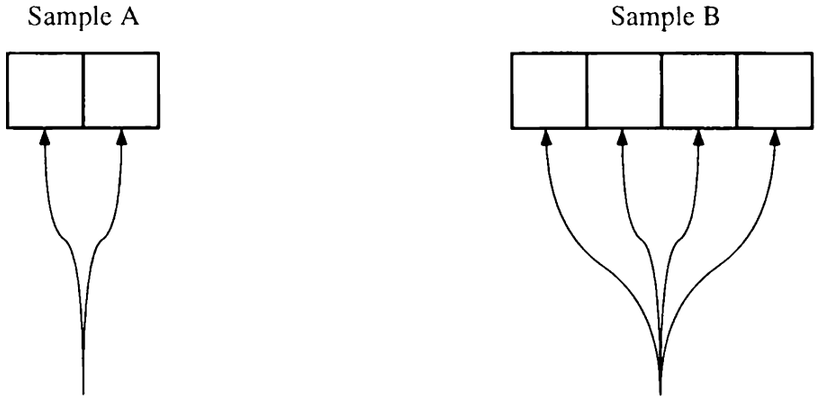
Section 3. Heat and heat transfer

In the previous section, we saw that when samples of hot and cold water are mixed, the masses of the samples play an important role in determining the equilibrium temperature of the mixture. Our observations are consistent with the following macroscopic model.

When two objects at different temperatures interact thermally, we say that there is a *transfer of heat* from the hot object to the cold object. In interactions like those in Experiments 2.2 and 2.3, the temperature of water changes when heat is transferred to or from the water. When samples of hot and cold water are mixed, we can think of heat as being transferred from each gram of the hot water to the cold water. As a result, the temperature of each gram of the hot water is lowered. To find the total heat transferred to the cold water, we add the heat transferred from each gram of the hot water. The heat is evenly divided among all the grams of cold water; each gram receives the same portion of the total. The temperature of each gram of cold water increases. If the temperature of a sample of water is raised or lowered by a certain amount, the temperature of each gram is raised or lowered by this same amount.

This model applies to substances other than water. As we shall see, it enables us to predict and explain a wide range of phenomena such as boiling, freezing, etc.

To see how this scheme takes masses into account, imagine transferring the same amount of heat to 2 g of cold water (sample A) and to 4 g of cold water (sample B).



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We may think about this problem in the following way: If the same amount of heat is transferred to each object, then the heat is spread more thinly throughout sample B, since there are more grams to share the heat. Each gram of sample A gets half of the heat transferred, but each gram of sample B gets only one-fourth of the heat transferred. Thus each gram of sample A gets twice as much heat as each gram of sample B. Therefore, each gram of sample A changes temperature by twice as much as each gram of sample B.

Note that we do *not* add up the temperature changes of the individual grams. The temperature change of the whole object is the same as that of one gram. The temperature change of sample A as a whole is twice as much as the temperature change of sample B as a whole. Sample A, which has half the mass, has twice the temperature change.

If we were to add heat to only one gram of sample A, only that gram would change its temperature. Then it would interact thermally with the other gram of sample A, the one whose temperature we did not change. One gram would transfer heat to the other, and the two grams would end up at an intermediate temperature.

Exercise 3.1

In Experiment 2.3 you mixed various amounts of hot and cold water. Consider the two cases described below. In each case, state whether the temperature changes of the hot and cold water are consistent with the idea of heat transfer described above. Explain your reasoning.

A. there is twice as much of the hot water as the cold water

B. there are equal amounts of the hot and cold water

Heat can be expressed in several units; for example, *calories* (cal), *joules* (J), or *British thermal units* (Btu). One calorie is defined as the amount of heat that will raise (or lower) the temperature of one gram of water by one Celsius degree. Calories are related to the joule and the British thermal unit in the following way:

$$1 \text{ cal} = 4.186 \text{ J} \quad \text{and} \quad 1 \text{ cal} = 0.003969 \text{ Btu}$$

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Sample problem

How much heat must be transferred to 20 g of water to raise its temperature by 4 C°?

Sample solution

Let us work this problem one gram at a time. Since the whole 20 g of water increases 4 C° in temperature, the temperature of each one-gram "package" of water will also increase by 4 C°. For this to happen, each gram of water must receive 4 calories, one for each degree of temperature increase. Since there are 20 one-gram packages of water in the sample, we must add up 20 of these 4-calorie allotments of heat: 20×4 cal, which is 80 cal of heat transferred in all.

Comment

If each gram has a 4 C° temperature increase, the whole sample also has a 4 C° temperature increase. The temperature changes are *not* added up. If each gram needs 4 calories of heat, then the whole sample needs 80 calories. The heat transfers *are* added up.

Exercise 3.2

Suppose we have a sample of 300 g of water initially at 30°C. If each gram gains 3.7 calories of heat, what is the final temperature of the whole sample and how much heat is gained by the whole sample?

Heat and temperature play quite different roles in thermal interactions. It is helpful to think of transfer of heat as the cause of the temperature change. When heat is transferred, both the object losing the heat and the object gaining the heat are affected. The hot object responds to the loss of heat by cooling off. When the cold object receives heat, it responds by warming up. In this way of thinking about thermal interactions, we would say that the *temperature changes are caused by the transfer of heat.*

One important rule must always be observed when applying the concept of heat to thermal interaction of two objects: the same amount of heat is lost by the hot object as is gained by the cold object. This statement forms the basis of everything that follows:

$$\textit{Heat lost} = \textit{Heat gained}$$

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Sample problem

If 30 g of water at 20°C are mixed with 20 g of water at 45°C, the resulting mixture has a temperature of 30°C. Explain this interaction using the concept of heat transfer.

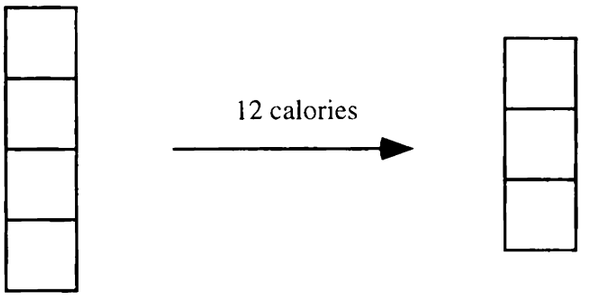
Sample solution

The entire sample of hot water cools by 15 C°. Therefore, each gram of hot water also cools by 15 C°. In order to decrease in temperature by 15 C°, each gram must give off 15 calories of heat: one for each degree of temperature change. Since there are 20 grams in all, the total heat given off by the entire sample of hot water is 20 × 15 cal, which is 300 cal.

These 300 calories spread out evenly among all of the 30 one-gram packages of cold water. Each one-gram package thus receives (300/30) = 10 calories of heat. The temperature of each package will thus increase by 10 degrees: one degree for each calorie of heat received. Since each one-gram package of cold water warms by 10 C°, the entire 30-gram package of cold water also warms by 10 C°. Thus, the equilibrium temperature is 20 + 10 = 30 or 30°C.

Exercise 3.3

Suppose we have 4 g of hot water and 3 g of cold water as shown in the diagram at right.



A. If 12 calories of heat are transferred from the hot water to the cold water:

(1) How much heat does each gram of hot water lose?

(2) How much does the temperature of each gram of hot water change?

(3) How much does the temperature of the entire sample of hot water change?

(4) How much heat does each gram of cold water gain?

(5) How much does the temperature of each gram of cold water change?

(6) How much does the temperature of the entire sample of cold water change?

B. By thinking about heat transfer, we are able to see that the temperature of the hot water changes less than the temperature of the cold water. This is because the mass of the hot water is larger than the mass of the cold water. Explain how the masses enter into the reasoning in part A.

✔ Check your results with a staff member.

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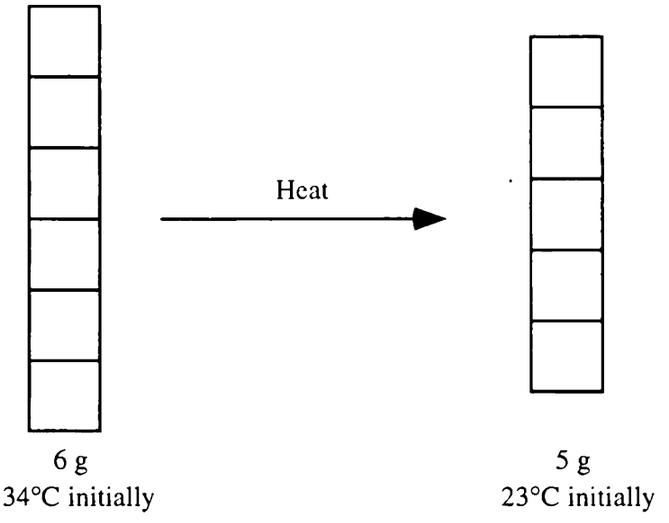
The following two exercises illustrate how the concept of heat transfer can be used to predict the equilibrium temperature of a mixture of hot and cold water.

Exercise 3.4

Suppose we mix 6 g of water at 34°C and 5 g of water at 23°C. The description of the interaction in terms of heat transfer is illustrated at right.



A. How much heat must be transferred for the hot water to cool by one degree?

In this case, how much heat does each gram of cold water receive?

What is the temperature change of the cold water as a whole when the hot water cools one degree?

B. Imagine that the same number of calories as in part A is transferred from the hot water to the cold water over and over again until equilibrium is reached. Show the results from each successive transfer of heat in a table like the following:

Heat transferred	Temp. of hot water	Temp. of cold water
0 cal	34°C	23°C
	33°C	

What is the final temperature?

What is the total heat transferred?

Exercise 3.5

Repeat the same type of analysis as that in part B of the preceding exercise for one of your experiments in Experiment 2.3. Choose one in which the numbers worked out well.

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Part B: Thermal properties of matter

In Part A of this module, we developed the concepts of heat and heat transfer to account for the changes in temperature that occur when hot and cold water are mixed. In Part B, we apply these concepts to thermal interactions that involve other substances. New concepts are developed that enable us to describe matter in terms of its thermal properties.

Section 4. Heat capacity and specific heat

Experiment 4.1

In this experiment, we will investigate the behavior of an aluminum cylinder in thermal interaction with water. The experiment consists of heating the cylinder to a high temperature, putting it into a cup of water, and noting the final temperature of the water.

A. Obtain an aluminum cylinder. Find its mass to the nearest 0.1 g. (You do not need to know the mass of the cylinder for this experiment, but it will be needed in Exercise 4.7.) Tie a thread around the cylinder. Leave about 15 cm of thread attached to use in handling the cylinder.

Pour enough room temperature water into a cup so that it would completely cover the cylinder. Record the initial temperature and mass of the water. Keep a thermometer in the water throughout this experiment.

B. The hot aluminum cylinder should be prepared as follows: Place the cylinder in about 200 mL of hot water at 40°C. Leave it in the water for at least thirty seconds. Keep a thermometer in the water during this time (not the same thermometer that is in the room temperature water). After about 30 seconds, the temperature of the hot water and the aluminum should be the same; that is, the water and aluminum should be in equilibrium with each other.

Record the temperature of the aluminum cylinder.

Immediately transfer the cylinder to the cup of room temperature water. Use the thread to handle the cylinder. To minimize the interaction with the air, transfer the cylinder to the water very quickly. Then gently stir the water with the thermometer for about 30 seconds.

Record the final temperature of the aluminum and water.

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C. Perform the experiment four times, with the aluminum at the following initial temperatures: 40°C, 60°C, 80°C, and 100°C. (In the last case, the water in the beaker must be boiling vigorously over a burner.)

How do you find the heat gained by the water?

How do you find the heat lost by the aluminum cylinder?

Make a table like the one below showing the heat gained and lost in each experiment:

Water			Aluminum		
Initial temp.	Final temp.	Heat gained	Initial temp.	Final temp.	Heat lost

D. Calculate the temperature change of the aluminum cylinder in each experiment. Make a graph of the heat lost by the aluminum cylinder versus the temperature change of the aluminum cylinder. *Use graph paper!* Should the graph go through *(0,0)?*

The graph should be nearly straight. Draw a single straight line through the middle of the points.

Determine the slope of the graph. How do you interpret this number?

✔ Check your results with a staff member

The slope of the graph above is called the *heat capacity* of the aluminum cylinder. For objects of almost any composition, whether homogeneous or inhomogeneous, the *same* amount of heat is required to change the temperature by one degree, regardless of the initial temperature. In other words, the heat capacity of most objects is a constant. The graph in Experiment 4.1 illustrates that this is the case for the aluminum cylinder.

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Exercise 4.2

What is the heat capacity of the aluminum cylinder used in Experiment 4.1? Explain your reasoning.

Which feature of the graph from Experiment 4.1 illustrates that the heat capacity of the aluminum cylinder is nearly a constant? Explain.

There are some objects for which the heat capacity is clearly different at every temperature. One example is a block of wax, for which the heat capacity changes as the wax softens. Actually, heat capacity is not *exactly* constant for any object. The table below shows the variation for a gram of water.

Temperature (degrees Celsius)	Heat capacity of one gram of water (calories/Celsius degree)
0	1.007
10	1.001
20	0.999
30	0.998
40	0.998
50	0.999
60	0.999
70	1.001
80	1.002
90	1.004
100	1.007

The most striking feature of the preceding table is not the differences but the almost perfect equality of the numbers. We will ignore the small differences, and in this module we will treat heat capacity as a constant.

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Exercise 4.3

A piece of iron at 300°C with heat capacity 50 cal/C° is placed in a tank of oil at 20°C with heat capacity 200 cal/C°. Find the final temperature by imagining that heat is transferred from the iron to the oil until equilibrium is reached. You may imagine sending heat in any amounts you want; you do not have to use 1 calorie at a time. Keep track of your calculations in a table like the following:

Heat transferred	Temp. of iron (°C)	Temp. of oil (°C)
0 cal	300	20

✔ Check your results with a staff member.

Exercise 4.4

A glass ball at −10°C is placed in 500 mL of water at 20°C. The final temperature is 15°C.

A. How much heat is lost by the water?

B. What is the heat capacity of the water?

C. How much heat is gained by the glass ball?

D. What is the heat capacity of the glass ball?

E. What is the ratio of the heat capacities of the water and glass ball?

F. What is the ratio of the temperature changes of the water and the glass ball?

G. How are the answers to E and F related?

Heat capacity and mass

The preceding experiments and exercises have illustrated the usefulness of the concept of heat capacity in analyzing heat transfer processes. The following experiment and exercises examine the relationship between the heat capacity of an object and the mass of the object.

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Experiment 4.5

In this experiment, hot iron washers interact thermally with water at room temperature. Different numbers of washers are used in order to see how the heat capacity of iron is affected by the mass. The procedure is similar to that of the last experiment except that it is not necessary to use hot water at four different temperatures: one will suffice.

Place 25 g of room temperature water in a Styrofoam cup and record its temperature. Loop thread through one washer for handling. Let the washer stand in hot water for a while, then record the initial temperature of the iron.

Quickly transfer the washer to the room temperature water. The washer should lie flat so it is completely under water. Record the final temperature.

Repeat the same procedure with sets of two, three, four, and five washers. For each trial, find the mass of each set of washers to the nearest 0.1 g.

A. Calculate the heat capacity of the set of washers used in each trial.

B. Make a graph of heat capacity versus mass for the iron. Should the graph go through the point *(0,0)?*

The graph should be approximately a straight line. Draw the best line you can through the points.

Determine the slope of the line.

Give an interpretation of the slope of the graph in terms of heat capacity. What does the fact that the graph is straight tell us?

C. Give an interpretation of the slope without mentioning heat capacity (i.e., an interpretation in terms of heat, temperature change, and mass).

The slope of the graph in the preceding experiment is called the *specific heat capacity* of iron or simply the *specific heat* of iron. A table of the specific heats for many materials is included in the Appendix.

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Exercise 4.6

Answer each of the questions below. In each case, explain your reasoning.

A. Which requires more heat to increase the temperature by 1 C°, a gram of water or a gram of iron?

B. How much iron has the same heat capacity as one gram of water?

C. The specific heat of iron is about 0.1 something. What are the units?

D. Give an interpretation of specific heat.

E. What is the specific heat of water?

F. Write an equation relating heat capacity to mass and specific heat.

✔ Check your results with a staff member.

Exercise 4.7

A. In Experiment 4.1 and Exercise 4.2 you found the heat capacity of an aluminum cylinder. Use the value you obtained in that experiment for the mass of the cylinder to find the specific heat capacity of aluminum.

   Compare the value you obtain for the specific heat with the table in the Appendix.

B. If equal masses of water and aluminum interact thermally, will the final temperature be closer to the initial temperature of the water or the initial temperature of the aluminum? Explain your reasoning.

Exercise 4.8

A. Suppose that you had a homogenous object made of some unknown material. Which of the following quantities would be the same for the whole object and for any piece of that object? Explain your reasoning.

   (1) mass
   (2) volume
   (3) density
   (4) heat capacity
   (5) specific heat

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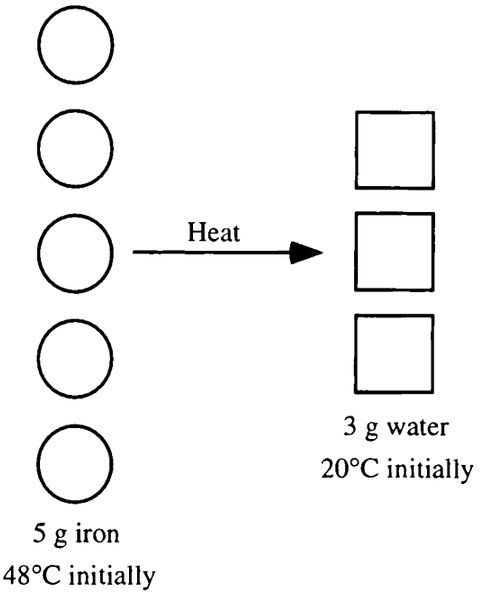
Some properties of matter are specific to a given object, while other properties are characteristic of the material from which the object is made. *Characteristic properties* are those that are the same for any object made of a particular material. (See Section 10 of *Properties of Matter.*) Such properties can be listed in tables and used to help identify the material from which an object is constructed.

Exercise 4.9

Which of the properties listed in the preceding exercise are characteristic properties and which are not? Explain your reasoning.

Exercise 4.10

Suppose 5 g of iron at 48°C are placed in contact with 3 g of water at 20°C. (Use 0.1 cal/g/C° for the specific heat of iron.) The situation is illustrated at right.



A. What happens to the temperature of each object when one calorie of heat is transferred?

B. Determine the final temperature by imagining that heat is transferred from the iron to the water until equilibrium is reached. Record your calculations in a table like the one below.

Heat transferred (cal)	Temp. of iron (°C)	Temp. of water (°C)
0 cal	48	20

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Exercise 4.11

A physics student has two cubes of iron. One cube has three times the volume of the other. The cubes are heated over identical flames for the same amount of time. The cubes are then removed and the temperature of each cube is immediately recorded.

How will the thermometer readings compare?

Exercise 4.12

A physics student has three cubes of metal: a 100 g cube of iron, a 300 g cube of iron, and a 100 g cube of aluminum. The student puts the cubes into a beaker of boiling water. The cubes remain in boiling water until each cube is as hot as it is going to get. The cubes are then removed and the temperature of each cube is immediately recorded.

A. How will the thermometer readings compare?

B. Which cube reached its final temperature most quickly?

C. Discuss how the answer to this exercise differs from the answer to the preceding exercise.

✔ Check your results with a staff member.

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Section 5. Distinguishing between heat capacity and specific heat

Heat capacity and specific heat are not the same. Although both quantities are involved in heat transfer calculations, they are not used in exactly the same way. As you saw in Section 4, heat capacity is a property of a particular object. Specific heat, on the other hand, is a characteristic property; that is, specific heat has the same value for all objects made of a given substance. The difference between heat capacity and specific heat can be seen in their interpretations:

> The *heat capacity* of an object indicates the amount of heat that will change the temperature of the entire object by one degree.
>
> The *specific heat* of a substance indicates the amount of heat that will change the temperature of one gram of the substance by one degree.

Specific heat applies to one gram of a substance, whereas heat capacity applies to a whole object.

Exercise 5.1

In each of the following cases, state whether it is possible to have a pair of objects like the ones described. In other words, tell whether such a pair might exist. If so, give a specific example of two such objects. If not, explain why not. (Use the tables in the Appendix to help you find specific examples for your answers.)

A. One object has the larger mass; the other has the larger heat capacity.

Sample solution for A:

> 10 g of water has a heat capacity of 10 cal/C° while 50 g of iron has a heat capacity of 5 cal/C°. Thus 10 g of water and 50 g of iron are a pair of objects of the required kind.

Find another example for this case.

B. One object has the larger volume but the other has the larger heat capacity.

C. One object has the larger heat capacity but the other has the larger specific heat.

D. One object has the larger specific heat but the other has the larger heat capacity per unit mass.

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Exercise 5.2

Two objects interact thermally. The first has heat capacity K_1 and the second has heat capacity K_2. The first object's temperature decreases by an amount ΔT_1, and the second object's temperature increases by an amount ΔT_2. Write an equation that illustrates the relationship among these four quantities.

✔ Explain your reasoning to a staff member.

Exercise 5.3

"If two objects interact thermally, the object with the larger _____ will have the smaller temperature change." Which of the quantities listed below makes this statement always true for interactions involving temperature changes only?

- volume
- mass
- specific heat
- heat capacity

✔ Explain your reasoning to a staff member.

Exercise 5.4

Make an analogy between heat capacity and density. Describe which quantities correspond to one another. Explain your reasoning.

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Section 6. Proportional reasoning with heat and temperature

In this module, we have studied the relationship between heat transfer, temperature change, and heat capacity. In *Properties of Matter,* we studied the relationship between mass, volume, and density. Even though these are very different concepts, the same kind of reasoning used with mass, volume, and density can also be used with heat transfer. A mathematical analogy can be made between heat capacity and density as follows: the quantity of heat transferred corresponds to mass; temperature change corresponds to volume; and heat capacity corresponds to density. In other words, the heat capacity of an object tells us the number of calories transferred for each Celsius degree temperature change, just as the density of a homogeneous substance tells us the mass in grams of each cubic centimeter of material. Even though density is a characteristic property and heat capacity is not, we can still make an analogy between the two and use the same kind of reasoning.

The solutions to the sample problem below illustrate how the concepts of heat capacity and specific heat can be used to solve problems involving heat transfer.

Sample problem

It takes 120 cal of heat to raise the temperature of 24 g of a substance by 10 C°. If 280 cal of heat is delivered to 40 g of this substance, how much will the temperature change?

Sample solutions

We are asked for the temperature change when 40 g of a substance receive 280 cal of heat. Two methods of solution follow.

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Solution 1

We can find the number of calories each gram receives by dividing the 280 calories equally among all 40 grams. If we find the specific heat, that is, how many calories it takes to change each gram by one Celsius degree, then we can find the temperature change of each gram (and the temperature change of the entire object). We must use the other information given in the problem to find the specific heat.

Find the specific heat:

We know 120 cal increase the temperature of 24 g by 10 C°. The 120 calories are equally shared by all of the grams, so (120/24) = 5 cal change one gram by 10 C°. To change temperature by only 1 C°, each gram must receive only 1/10 as much heat or (5/10) = 0.5 cal. Thus 0.5 cal changes one gram by 1 C°. This is the specific heat.

Find the temperature change of 40 g:

Use the specific heat to see what happens when 40 g receive 280 cal. The heat is equally shared, so each gram gets (280/40) = 7 cal. Each 0.5-calorie package raises one gram by 1 C°, so we must find out how many 0.5-calorie packages are in the 7 cal each gram receives. We get (7/0.5) = 14 packages. Each package changes one gram by 1 C°, so the temperature of each gram increases by 14 C°. Therefore, the temperature of the entire 40 g increases 14 C°.

Solution 2

We know 40 g of the substance receive 280 cal. If we find the heat capacity, that is, how many calories it takes to change the entire object by one Celsius degree, then we can divide 280 by that number to find the temperature change of the entire object.

Find the specific heat:

As above, obtain the specific heat: 0.5 cal/g/C°.

Find the temperature change of 40 g:

To warm 40 g of the substance by 1 C°, each gram must receive 0.5 cal. Thus it takes 40 × 0.5 = 20 cal to warm 40 g by 1 C°. (This is the heat capacity.) We know the object receives 280 cal or (280/20) = 14 of these 20-calorie packages. Each package increases the temperature of the whole object by 1 C°, so the object changes temperature by 14 C°.

Exercise 6.1

Two samples of a liquid are being heated. One sample has a mass of 70 g and the other has a mass of 900 g. If 400 cal of heat produce a 15 C° temperature change in the smaller sample, how much heat is needed for a 10 C° change in the larger sample? Explain. Do not use algebra.

✔ Discuss your reasoning with a staff member.

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Algebra in heat problems

The basic formula for heat transfer can be written using the following reasoning: Suppose we have an object with heat capacity K. This means that an amount of heat K is required to raise the temperature of the object by one Celsius degree. If the object's temperature is raised from T_i to T_f, the total temperature change is $T_f - T_i$. The amount of heat Q that is required to produce the entire temperature change is thus:

$$Q = K(T_f - T_i)$$

If the object described in this equation is cooling off, T_f is less than T_i and Q is negative. Many people like to use the sign of the heat in this way to tell them whether heat is lost or gained. In *Heat and Temperature,* however, we will always treat the heat as a positive quantity and rely on the description of the situation to tell us whether the heat is gained or lost. We will use the symbol ΔT to represent the change in temperature as a positive quantity, whether the temperature increases or decreases. Thus ΔT will be $T_i - T_f$ for an object that is cooling and ΔT will be $T_f - T_i$ for an object that is warming. Our basic formula is thus:

$$Q = K\,\Delta T$$

Sample problem

Two objects interact thermally. The heat capacity of the hot object is 2.5 times as large as the heat capacity of the cool object. If the hot object's temperature falls by 10 C°, by how much does the temperature of the cold object rise?

Sample solution

Let us write an equation for each object:

$$Q_2 = K_2\,\Delta T_2$$

$$Q_1 = K_1\,\Delta T_1$$

If we divide these two equations, we obtain an equation that is always true.

$$\frac{Q_2}{Q_1} = \frac{K_2\,\Delta T_2}{K_1\,\Delta T_1}$$

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Since the two objects are interacting thermally, the heat lost by the hot object and the heat gained by the cold object will be equal. For this problem, the equation reduces to:

$$\frac{\Delta T_2}{\Delta T_1} = \frac{K_1}{K_2}$$

The relation between heat capacity and temperature change in this situation is an inverse proportion. Right away we know that the object with the larger heat capacity will have the smaller temperature change. Solving for the unknown temperature change, we get:

$$\Delta T_2 = \frac{K_1}{K_2}\Delta T_1 = 2.5(10\ \text{C°}) = 25\ \text{C°}$$

Exercise 6.2

Two objects interact thermally. One object has a heat capacity of 75 cal/C°, and its temperature falls by 30 C°. If the second object's temperature rises by 45 C°, what is its heat capacity?

If the objects involved in a problem are homogeneous, we may be given the specific heat and mass of the objects rather than their heat capacities. The heat capacity must then be expressed in terms of the specific heat, c, and the mass, M. (Recall Exercise 4.7.) Our formula then becomes:

$$Q = c\,M\,\Delta T$$

Sample problem

Suppose we do an experiment with 80 g of plastic and we note that a certain amount of heat produces a temperature change of 12 C°. If we apply 1.5 times as much heat to 48 g of the same kind of plastic, what will be its temperature change?

Sample solution

We begin by writing an equation for each object:

$$Q_2 = c_2 M_2 \Delta T_2$$

$$Q_1 = c_1 M_1 \Delta T_1$$

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The proportions among the various variables can be easily seen by dividing one equation by the other and expressing the result in terms of dimensionless ratios (i.e., ratios in which the numerator and denominator have the same units):

$$\frac{Q_2}{Q_1} = \frac{c_2 M_2 \Delta T_2}{c_1 M_1 \Delta T_1}$$

In this problem, we are asked for the temperature change, so we rearrange the equation so that just the temperature changes appear on one side:

$$\frac{\Delta T_2}{\Delta T_1} = \frac{Q_2 c_1 M_1}{Q_1 c_2 M_2}$$

Direct and inverse proportions are easy to recognize in an equation that is composed of dimensionless ratios, like the previous equation. In a direct proportion, the subscripts in the dimensionless ratios are in the same order on both sides of the equation. For an inverse proportion, the subscripts are reversed.

In the equation above, *2* is above *1* in the ratio of the temperature changes. On the other side of the equation, *2* is above *1* in the ratio of the heat transfers, but *1* is above *2* in the ratio of the specific heats and in the ratio of the masses. The equation could thus be described in words like this: the temperature change is directly proportional to the heat transfer and inversely proportional to the specific heat and the mass.

In this problem, both objects are made of the same material, so the specific heats are the same and the ratio of c_1 to c_2 is equal to one. Solving for the unknown temperature change, we have:

$$\Delta T_2 = \frac{Q_2}{Q_1} \frac{M_1}{M_2} \Delta T_1$$

$$\Delta T_2 = \left(\frac{1.5}{1}\right)\left(\frac{80}{48}\right)(12\ \text{C°}) = 30\ \text{C°}$$

Exercise 6.3

If 20 cal of heat is required to raise a certain sample of stone 200 C°, how much heat is required to raise one million times as much stone by 5 C°?

Exercise 6.4

Two pieces of copper are heated. The second receives 0.15 times as much heat as the first and its temperature rises 0.45 times as much. If the mass of the first piece is 35 g, what is the mass of the second piece?

✔ Discuss your reasoning with a staff member.

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Reading proportions from an equation

In many cases, it is often harder to solve a problem algebraically than it is to use a formula as a guide for proportional reasoning. In this case, the equation is used only to determine whether quantities are directly proportional or inversely proportional.

Sample problem

Suppose we have two samples of the same kind of oil. The first, which has a mass of 50 g, warms by 18 C° when it receives a certain amount of heat. If half as much heat is transferred to the second sample, which has a mass of 150 g, how much will its temperature change?

Sample solution

The basic formula for heat transfer is: $Q = c\,M\,\Delta T$

In this problem, we are asked for the temperature change, so we solve for ΔT:

$$\Delta T = \frac{Q}{c\,M}$$

We now use the formula as a guide for proportional reasoning. We can tell whether a proportion is direct or inverse by checking to see whether two quantities increase and decrease together (a direct proportion) or whether one increases while the other decreases (an inverse proportion). From the formula above, we see that the temperature change is directly proportional to the heat transfer and inversely proportional to the mass and specific heat of the sample.

The ratio of the mass of the second sample to the mass of the first is 150/50 or 3. The ratio of the heat received by the second sample to the heat received by the first sample is one-half. Since both samples are the same substance, c is the same for both samples and the ratio is 1.

In this case, the temperature change is *directly* proportional to the heat transferred. Since the heat transferred to the second sample is half that transferred to the first, the effect is to reduce the temperature change by a factor of one-half. However, the temperature change is also *inversely* proportional to the mass. Since the mass of the second sample is three times that of the first, the temperature change is reduced by a factor of one-third. The combined effect is to reduce the temperature change by a factor of (l/2)(1/3) or l/6. The temperature change of the second sample will be:

$$\Delta T = \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)(18\ \text{C}^\circ) = 3\ \text{C}^\circ$$

Exercise 6.5

A piece of aluminum with a mass of 24 g is heated and the resulting rise in temperature noted. If a piece of iron receives 0.6 times as much heat and its temperature rises 4.8 times as much, what is its mass? Explain. Use proportional reasoning to solve this problem.

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Section 7. Phase changes

Melting, freezing, boiling or evaporation, and condensation (the reverse of evaporation) can be described in terms of heat transfer. These processes, in which one substance changes form (from a solid to a liquid or a liquid to a gas, for example), are called *phase transitions.* Ice, water, and water vapor or steam are said to be different *phases* of water. For interactions involving phase changes, the basic description in terms of heat transfer remains the same: heat lost equals heat gained. In the experiments and exercises in this section we examine phase changes in more detail.

Experiment 7.1

Place 20 g of room temperature de-ionized water in a Styrofoam cup and record the temperature of the water. Shake or drain some ice to get rid of excess water and add the ice to the de-ionized water. Use a lot of ice (about one or two times the volume of the water). Stir continuously with a thermometer.

Notice that the temperature falls. When the temperature stops decreasing (at approximately 0°C), *immediately* pour the water into a graduated cylinder, leaving the ice behind. Use a funnel when pouring the water to trap any ice that spills.

Record the volume of water in the graduated cylinder and determine how much ice melted.

Repeat the experiment with 40 g, 60 g, and 80 g of room temperature water. Be sure there is always plenty of ice in the cup.

A. For each experiment, calculate the heat lost by the room temperature water and the heat gained by the ice. Record your results in a table.

B. Make a graph of the heat gained by the ice versus the mass of ice melted.

C. The graph should be approximately a straight line. Draw the best straight line you can and compute the slope.

   What does the fact that the graph is straight indicate?

   How do you interpret the slope of the graph?

D. Discuss how the behavior of ice when heat is transferred to it differs from the behavior of cold water when heat is transferred to it.

✔ Discuss your results with a staff member.

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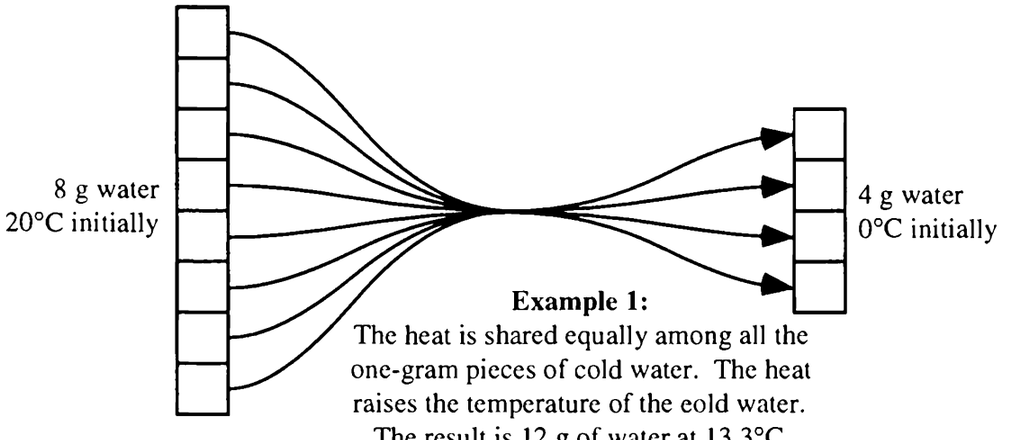
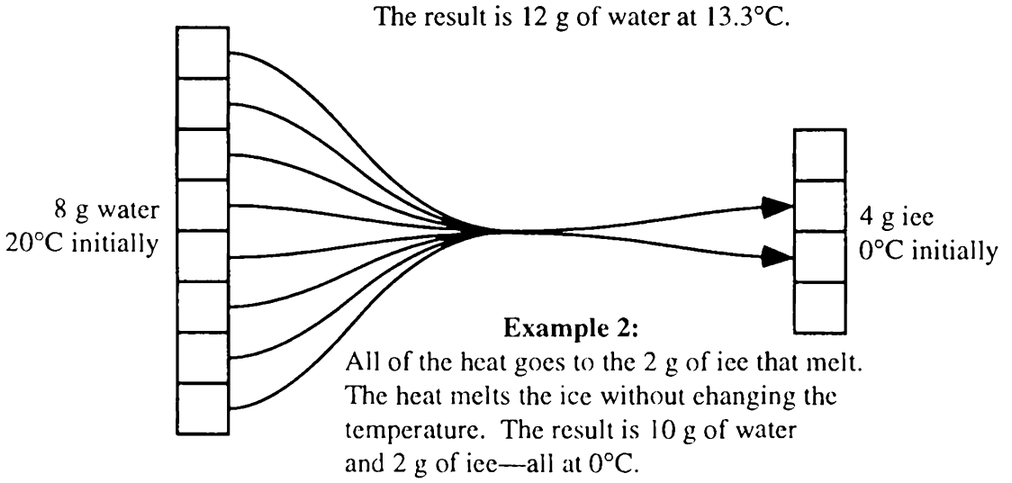
The amount of heat required to melt one gram of ice is called the *latent heat of fusion* of ice. A table of latent heats of fusion for various substances is in the Appendix. Compare your result for the latent heat of fusion of water with the value given in the table.

The result of transferring heat to the ice in the previous experiment was to melt the ice; the ice did not change temperature while it was melting. In our earlier experiments the effect of transferring heat to an object was to increase the temperature of the object. These examples illustrate a difference in the behavior of objects that are and are not undergoing a phase transition. In each case, transfer of heat is thought of as the cause of what happens, but the effect is different. There is another difference as well. In interactions with no phase change, heat received by an object is evenly distributed, that is, shared equally among all of the grams of the object. In the case of melting, all of the heat goes to just the ice that melts. None goes to the unmelted ice.

These differences are illustrated in the two interactions shown below. The first is the



Example 1:
The heat is shared equally among all the one-gram pieces of cold water. The heat raises the temperature of the eold water. The result is 12 g of water at 13.3°C.



Example 2:
All of the heat goes to the 2 g of iee that melt. The heat melts the ice without ehanging the temperature. The result is 10 g of water and 2 g of iee—all at 0°C.

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interaction between 8 g of water at 20°C and 4 g of *water* at 0°C. The second is the interaction between 8 g of water at 20°C and 4 g of *ice* at 0°C.

Exercise 7.2

A 25 g ice cube is put in 200 g of water at 20°C. Determine the final temperature. The steps below are intended as a guide to help you answer this question.

Imagine sending heat from the hot material to the cold material until equilibrium is reached. Since ice and water can only coexist in equilibrium at 0°C, all of the ice must be melted before the temperature of the cold material can be raised above 0°C. At the outset, it is not obvious whether all of the ice will melt.

Keep track of your calculations in a table like the one following.

Heat transferred	Temp. of added water	Mass of ice still unmelted	Mass of water melted from ice	Temp. of melted water
0 cal	20°C	25 g	0 g	0°C
800 cal	16°C	15 g	10 g	

✔ Discuss your reasoning with a staff member.

Experiment 7.3

The purpose of this experiment is to find out how much heat is required to vaporize water. Because high temperatures and large amounts of heat are required for this experiment, it will be necessary to heat the water with a Bunsen burner or a hot plate.

The experiment consists of heating water in a metal can and seeing how much heat is required to boil away some of the water. First, however, we find the amount of heat transferred to the can–water system per unit of time.

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A. Obtain a can and find its mass. Assume the can is mostly iron and calculate its heat capacity. Put 100 g of room temperature water in the can. What is the heat capacity of the water?

B. Measure the initial temperature of the water. Then, turn on the burner and let it heat the can and water for 60 seconds. (If you are using a hot plate, leave the hot plate on for several minutes before starting this part of the experiment.)

After 60 seconds, remove the burner from the can, but *do not turn off the burner or alter the flame.* Gently stir the water in the can with a thermometer and note the highest temperature attained by the can–water system.

Caution: The can is now hot!

Calculate the amount of heat received by the can–water system.

Calculate the amount of heat received each second.

C. Empty the can, cool it with tap water, dry the outside, and put 100 g of room temperature water in it. Place the can on the same support used previously. Measure the initial temperature of the water. Then place the burner, which should still be burning at the same rate, under the can for 300 seconds (5 minutes).

After 300 seconds, remove the flame and shut it off. Pour the contents of the hot can into a graduated cylinder and measure the volume.

What mass of water has been vaporized?

D. Calculate the amount of heat delivered to the can–water system.

Calculate the amount of heat used to raise the temperature of the can and water to 100°C. Explain your reasoning.

Calculate the amount of heat used to vaporize the water. Explain your reasoning.

According to your experiment, how much heat is required to vaporize one gram of water? Explain.

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The amount of heat required to vaporize one gram of water is called the *latent heat of vaporization* of water. A table of latent heats for various substances can be found in the Appendix. Compare your result for the latent heat of vaporization of water with the value listed in the table.

Exercise 7.4

Suppose that 18 g of steam at 100°C interacts with 50 g of ice at 0°C. What is the final temperature? Explain your reasoning. (*Hint:* Consider whether all the steam must condense in order to melt all the ice.)

Exercise 7.5

A total of 22,500 cal of heat is delivered to a sample of ice at 0°C. The result is that half of the sample boils away. Find the initial mass of the ice.

✔ Discuss your reasoning with a staff member.

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Section 8. Heat in everyday life

In this section, we will examine several everyday phenomena from the point of view of heat transfer.

Exercise 8.1

Consider the following situation. A large pot of water is boiling on an electric hot plate. It has been boiling steadily for some time.

A. Describe all of the heat transfers taking place among the following three elements: the water, the bottom of the pot, and the hot plate. Tell what happens to each of these objects when it gives or receives heat. Explain your reasoning.

B. Tell what is happening to the temperature of each of the following: the water, the bottom of the pot, and the hot plate. Explain your reasoning.

C. How does the temperature of the pot bottom compare with the temperature of the water? How does the temperature of the pot bottom compare with the temperature of the hot plate? Explain.

Exercise 8.2

In this exercise, we will compare heating up leftovers in an oven versus in a steamer. (In a steamer, food is placed in the steam above boiling water.)

Both processes involve interaction between the cold food and hot gas (hot air in the oven or steam in the steamer). The hot air in an oven at 200°C (approximately 400°F) is at a much higher temperature than the 100°C steam in a steamer. Explain why it is nevertheless faster to warm up leftovers by steaming them than by putting them in the oven.

Exercise 8.3

Water need not be at a temperature of 100°C in order to vaporize. Room temperature water spilled on a table will eventually vaporize—quickly if the air is dry. This process is called evaporation.

How can we calculate the heat of vaporization of water when it is at a temperature below 100°C? Here is a way of thinking about the process that gives a good approximation: imagine heating the water to 100°, vaporizing it, and then cooling the vapor to 20°C. For this calculation, we will think of the vapor as remaining a gas during the cooling process.

The net heat gained by the water as it undergoes these three processes will be approximately the same as the heat of vaporization of water at 20°C.

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Use the method described above to estimate the heat of vaporization of water at 20°C. Assume the specific heat of water vapor is 0.5 cal/g/°C at all temperatures.

Compare your answer to the accepted value of the heat of vaporization at 20°C, which is 585 cal/g.

Exercise 8.4

On hot days, the temperature of the air may be higher than the temperature of the human body, which is about 37°C. The body still produces heat, however, and must transfer heat away in order to maintain a constant temperature. Since the body cannot transfer heat to the air, which is at a higher temperature, another method of losing heat must exist. Perspiration plays a key role in removing heat from the body in such circumstances.

For the purposes of this problem, think of the process of perspiration as follows: Water inside the body moves through the skin to sit on the outside of the body. The passage through the skin does not involve any transfer of heat so the temperature of the water is not affected significantly by this movement.

In answering the questions below, think of the system consisting of these three objects: the air, the water on the outside of the body, and the body itself (not including the water on the outside of the body).

A. Describe all of the thermal interactions taking place among the three elements in the system.

B. Explain how the body is cooled by perspiration.

C. Where does the heat lost by the body go?

Exercise 8.5

Imagine a room that measures 6 meters by 8 meters by 3 meters.

A. What is the heat capacity of the air in the room? (The density of air under normal conditions is about 0.0012 g/cm3.)

B. Suppose the room contains 100 kg (about 200 lbs) of wood furniture. Calculate the heat capacity of the furniture. (Assume that the specific heat of wood is 0.4 cal/g/C°.)

C. Assume the walls, ceiling, and floor are all 2 cm thick and are made of material with an average heat capacity per unit volume of 0.2 cal/C°/cm3. What is the combined heat capacity of the floor, walls, and ceiling?

D. Express the heat capacities of the air, of the furniture, and of the floor–walls–ceiling as percentages of the total heat capacity.

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Exercise 8.6

Room temperature is a rather sophisticated concept. In normal circumstances, everything in a room is at the same temperature. If a small hot object is then placed on a table in the room, a complex series of interactions is initiated. First the air and the table interact with the object. Then the now slightly warm air and table interact with the other furniture, the walls, the floor, and so on. In short, the small hot object interacts directly or indirectly with the entire room.

A. Use your estimate of the heat capacity of a room from Exercise 8.5 to compute the ratio of the heat capacities of the room and a 250 mL cup of coffee. (Assume that coffee and water have the same specific heat.)

B. If the initial temperature of the room is 20.00°C and that of the coffee is 80.00°C, what will be the final temperature of both?

In describing ordinary experience, we often say that hot or cold objects eventually arrive at "room temperature." Used in this way, "room temperature" means more than just a particular value of the temperature, like 20°C. By using the term "room temperature," we are indirectly stating that the heat capacity of the room is vast compared with the heat capacities of normally encountered hot or cold objects. This must be the case if normal hot or cold objects produce no noticeable change in the temperature of the room.

The concept of room temperature rests on the experience that the temperature of the room appears not to be affected by ordinary hot and cold objects. As illustrated in part B, the room appears not to be affected because its heat capacity is so large that its temperature is changed only slightly by the heat transferred by cups of coffee, ice cubes, and so on.

C. Consider the situation of filling a bathtub full of hot water when the bathroom is a very small room. Does the concept of room temperature apply here? Explain your reasoning.

✔ Discuss your responses to Exercises 8.1–8.6 with a staff member.

Exercise 8.7

Besides the calorie, there are several other units of heat in common use. This problem introduces two of them.

A. The "calorie" used in discussing the energy content of food represents 1000 times as much heat as the calorie we have been using. The food calorie is sometimes written with a capital C (Calorie, Cal). Sometimes it is called a kilocalorie to distinguish it from our calorie, which is based on one gram of water. Usually, however, one must decide from the context which calorie an author means. If the topic is food, the calorie is 1000 of our calories. If the topic is physics, chemistry, or engineering, the calorie is the one we have been using.

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Almost all of the energy content of food used by the body ends up as heat to warm the body. Thus, a 100 Calorie baked potato can be expected eventually to produce 100 Calories (that is, 100,000 calories) of heat to warm the body.

How much would a 200 Cal candy bar cause a 50 kg person's temperature to rise if the person could not cool off? Assume the person is made entirely of water, which is a good approximation.

B. A British thermal unit (Btu) is the amount of heat required to change the temperature of one pound of water one degree Fahrenheit.

How many calories is a Btu? Explain your reasoning.

Experiment 8.8

Look up the heats of vaporization of water and alcohol. From this information, which would you expect to feel cooler on the skin, a drop of water or a drop of alcohol? Explain your reasoning.

Try this experiment. If the results are not consistent with your prediction, try to reconcile them.

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Supplementary problems for *Heat and Temperature*

Problem 1.1

Assume you have an *uncalibrated* thermometer. Write an operational definition for the temperature of an object or substance. Do not assume that you have access to a calibrated thermometer.

Problem 1.2

A glass of ice water has been sitting in front of two students for some time before they begin to discuss it. Consider the following dialog between them.

Student 1: *"The ice is colder than the water is. When I put my hand into the water it feels cold, but when I pick up a piece of ice it feels much colder. So the water must be warmer than the ice is."*

Student 2: *"But we know the water and the ice are at the same temperature. When two objects are in contact for a while, they eventually reach the same temperature."*

Do you agree with student 1, student 2, or neither? Explain your reasoning.

Problem 3.1

Consider the following dialog between two students.

Student 1: *"When hot water and cold water are mixed, some of the temperature of the hot water goes over to the cold water until they meet at an equilibrium temperature."*

Student 2: *"That's wrong. In some cases, the temperature of the cold water might increase by more than the temperature of the hot water decreases. Degrees don't have to come from anywhere."*

Do you agree with student 1, student 2, or neither? Support your answer with data from your experiments.

Problem 3.2

Suppose you have some 20°C water and some boiling water. How much of each would you use to make 4 kg of water at 45°C?

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Problem 3.3

Suppose that 50 g of water at 2°C is mixed with 100 g of water at 20°C (room temperature). The resulting temperature is observed to be 15°C.

A. What would you have predicted for the final temperature?

B. If we blame the discrepancy on heat absorbed from the environment, how much heat must we say was absorbed?

C. What is the ratio of the heat transferred between the samples of water to the heat transferred from outside?

Problem 3.4

Two samples of water were mixed yielding a 28°C mixture. The cold water was initially 20°C. There was 1.5 times as much hot water as cold water. What was the initial temperature of the hot water?

Problem 3.5

A student mixes 40 g of water at 20°C with 70 g of water at 43°C. Imagine sending heat in steps from the hot water to the cold water. Use this method to predict the final temperature to within 0.001°C. This may require working with very small amounts of heat to close in on the equilibrium temperature. To achieve this accuracy, it is necessary to retain four decimal places in all temperatures calculated along the way.

Problem 3.6

Suppose we have two samples of water, A and B. Sample A has a mass of 500 g and sample B has a mass of 300 g. These samples of water are heated over identical flames and then mixed.

Consider the following two experiments:

Experiment 1: Sample A is heated for 5 minutes and sample B is heated for 3 minutes.

Experiment 2: Sample A is heated for 3 minutes and sample B is heated for 5 minutes.

Compared to the final temperature after mixing in Experiment 1, would the final temperature in Experiment 2 be higher, lower, or the same? Explain your reasoning.

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Problem 3.7

We have the following samples of water:

X: 100 g at 20°C Y: 200 g at 26°C Z: 150 g at 40°C

A. Suppose we first mix X and Y. This mixture is then mixed with Z.

(1) What will be the final temperature?

(2) How much heat does X gain (or lose) in all?

(3) How much heat does Y gain (or lose) in all?

(4) How much heat does Z gain (or lose) in all?

(5) How are the three heats calculated above related?

B. Suppose, instead, we first mix Y and Z. This mixture is then mixed with X.

(1) What will be the final temperature?

(2) How much heat does X gain (or lose) in all?

(3) How much heat does Y gain (or lose) in all?

(4) How much heat does Z gain (or lose) in all?

(5) How do your answers to 1–4 compare with your answers in part A?

Problem 4.1

An object initially at 20°C interacts with an object initially at 150°C. The heat capacity of the colder object is 3.5 times as large as the heat capacity of the hotter object. What is the equilibrium temperature? Explain your reasoning.

Problem 4.2

An object at 20°C with heat capacity 100 cal/C° is placed in 40 g of water at 100°C. After equilibrium is reached, the object is removed from the hot water and placed in 200 g of water at 0°C. What is the final temperature of the object? Explain.

Problem 4.3

Two objects are heated over separate flames. One object has three times the heat capacity of the other. The flames are adjusted so that the temperatures of the two objects rise at the same rate.

Now suppose the objects are cooled to room temperature. Then the objects are heated again, but this time with the flames switched. The first object to reach a temperature of 80°C does so in 15 seconds. How much time is required for the second object to reach 80°C? Explain your reasoning.

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Problem 4.4

Suppose that a 400 g copper object (specific heat 0.092 cal/g/C°) receives 900 cal of heat. What is the temperature change? Explain your reasoning. Do not use algebra.

Problem 4.5

What is the mass of a piece of copper (specific heat 0.092 cal/g/C°) that changes temperature by 17 C° upon receiving 2400 cal of heat? Explain. Do not use algebra.

Problem 4.6

A 125 g piece of porcelain is initially at 20°C. Upon receiving 520 cal of heat, this piece of porcelain attains a temperature of 36°C. How much more heat is required to further raise the temperature to 47 C°? Explain your reasoning. Do not use algebra.

Problem 4.7

The concept of heat transfer can be applied to interactions involving more than two objects. The basic rule is still the same—heat lost equals heat gained—but several objects may gain or lose heat.

A. In this problem, there are three objects: a piece of aluminum at 110°C with heat capacity 100 cal/C°, a piece of brass at 42°C with heat capacity 300 cal/C°, and a tank of water at 20°C with heat capacity 800 cal/C°. Suppose the aluminum and brass are put into the tank of water. Find the final temperature by imagining transferring heat among these three substances until equilibrium is reached. Note that we can imagine sending heat from any one of the objects to either or both of the others.

   Keep track of your calculations in a table like the one below. It may be easier to make only one kind of transfer at a time as has been done in the first two example steps shown. (In the table, *A* stands for aluminum, *B* for brass, and *W* for water.)

Heat transferred (calories)						Temperature		
A to B	A to W	B to A	B to W	W to A	W to B	A	B	W
						110°C	42°C	20°C
	800					102°C	42°C	21°C
			2400			102°C	34°C	24°C

B. What is the net heat gained or lost by each object?

C. What relation is there among these three numbers?

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Problem 4.8

It is observed that when a 20 g metal object at 25°C receives 120 cal of heat, its temperature rises to 55°C. How much heat is required to raise one gram of this metal by 1 C° in temperature?

What is the specific heat of this metal? Explain your reasoning. Do not use algebra.

Problem 4.9

One calorie of heat will raise one gram of mercury 30 C°. How much heat is required to raise 100 g of mercury 80 C°? Explain your reasoning. Do not use algebra.

Problem 4.10

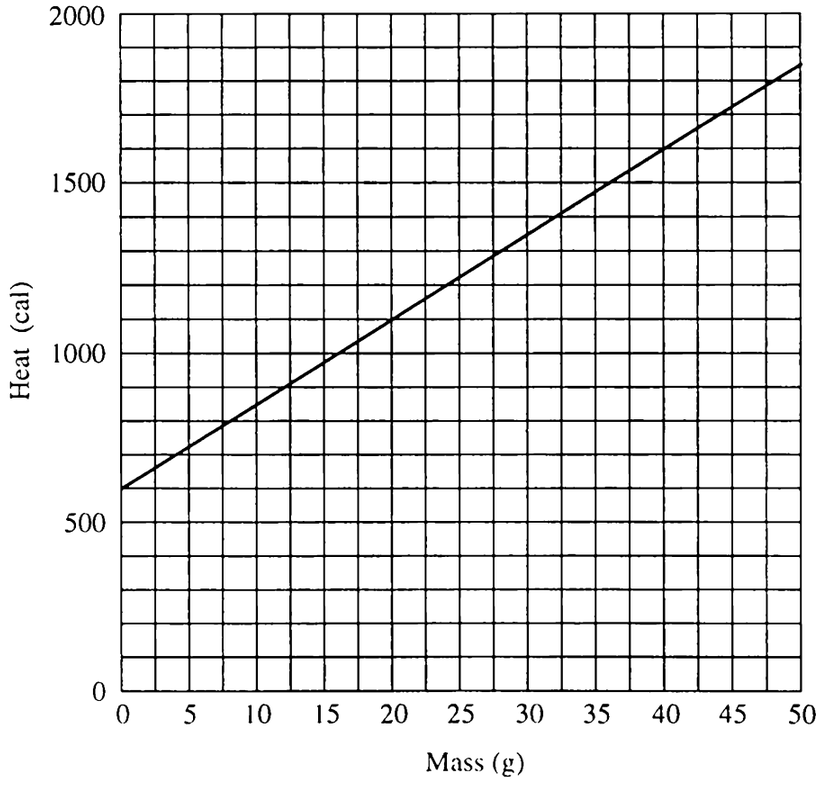
Suppose equal *volumes* of iron and water interact thermally. Which will change temperature more? If they are initially 30 C° apart in temperature, how much will the temperature of each change? Explain your reasoning.

Problem 4.11

Suppose we have a compound object: an iron cowboy riding an aluminum horse. The horse has a mass of 10 g and the cowboy has a mass of 30 g. What is the heat capacity of this compound object? Use the tables at the end of the module. Explain your reasoning.

Problem 4.12

A student heated various amounts of liquid in a pot over a constant flame and recorded how much heat was required to bring the pot–liquid system to the boiling temperature of the liquid, 83°C. The initial temperature of the system was the same each time. The specific heat of the liquid was 0.5 cal/g/C°. The graph shows the total heat delivered to the pot–liquid system versus the mass of liquid in the pot.



A. The graph does not go through the origin. How do you interpret the 600 cal offset?

B. What was the initial temperature?

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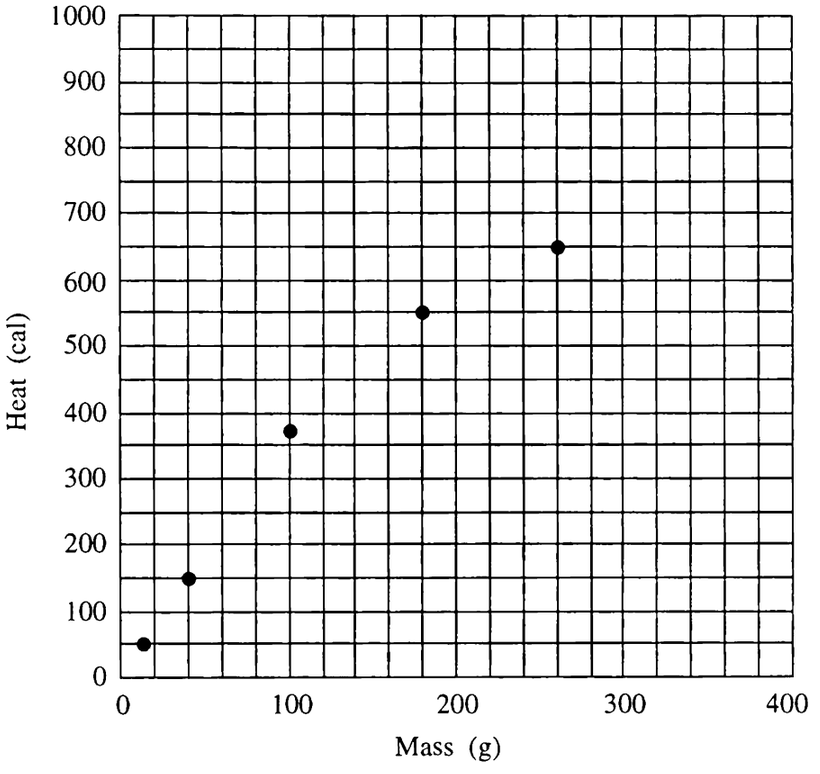
Problem 4.13

A toolsmith is making a metal object. The object is to be put into boiling water in order to cool off the water to 70°C. The object must be able to soak up a lot of heat without being too heavy. The toolsmith has two metals to work with, A and B. First she puts a 15 g piece of metal A into boiling water and measures how much heat this piece absorbs in rising from 20°C to 70°C. Then she uses larger and larger pieces of A, each time dropping the piece into different amounts of boiling water and finding out how much heat the piece absorbs in rising from 20°C to 70°C. Eventually, she runs out of metal A and starts using pieces that are combinations of metal A and metal B. The graph below shows the heat absorbed by the pieces in rising from 20°C to 70°C plotted against the mass of the pieces.



A. What are the specific heats of metals A and B?

B. How much of each substance was added in all?

C. What is the heat capacity of the final combination?

Problem 5.1

It is always important to distinguish between quantities that apply to an object as a whole and those that apply to just one unit of the substance of which the object is made.

Draw the following analogy: The relation between specific heat and heat capacity is analogous to the relation between density and mass.

Explain the analogy in detail, telling what corresponds to mass, to volume, and to density. Be sure to tell how the corresponding parts are similar.

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Problem 6.1

A student did an experiment in which he mixed some water at 70°C with some water at 20°C, but he forgot to measure how much water he used at each temperature. He did determine the mass of the mixture to be 120 g and the final temperature to be 50°C.

Solve algebraically for the initial masses of the hot and cold water. Explain your reasoning.

Problem 6.2

Solve this problem algebraically using the two methods outlined below.

Problem: 100 g of water at 10°C are mixed with 600 g of water at 70°C. What is the final temperature?

First solution: Let T be the final temperature.

A. Is T the answer?

B. Write an expression for the heat lost by the hot water.

C. Write an expression for the heat gained by the cold water.

D. Write an equation and tell why you set the two sides equal.

E. Answer the question.

Second solution: Let Q be the amount of heat transferred in the interaction.

A. Is Q the answer?

B. Write an expression for the final temperature of the hot water.

C. Write an expression for the final temperature of the cold water.

D. Write an equation and tell why you set the two sides equal.

E. Answer the question.

Problem 6.3

Two objects interact thermally. One object has a mass of 200 g and an initial temperature of 70°C. The second object, which is made of a different material, has a mass of 50 g and an initial temperature of 20°C. The final temperature of both is 40°C. What is the ratio of the specific heats of the two materials?

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Problem 6.4

Suppose two blocks of the same metal are heated. The first block receives 490 cal of heat and rises 18 C° in temperature. The second block receives 280 cal of heat and rises 36 C° in temperature. If the mass of the first block is 35 g, what is the mass of the second? Solve using dimensionless ratios.

Problem 6.5

It is observed that 800 cal of heat will raise the temperature of a certain stone by 18 C°. How much heat must be supplied to raise 4.7 times as much stone by 4 C°? Solve using dimensionless ratios.

Problem 6.6

One object has 2.3 times the mass of another. When they interact thermally, the heavier object changes temperature by 14 C°; the lighter one changes by 10 C°. If the lighter object is made of iron, what is the specific heat of the other object? Solve using dimensionless ratios.

Problem 6.7

A 400 g object receives 80% as much heat as a second object of the same substance and changes temperature 1.2 times as much as the second object. What is the mass of the second object? Solve using dimensionless ratios.

Problem 6.8

We have two objects and one of them is clearly iron. The second object, which is made of an unknown metal, has a mass that is one-third as great as the iron object. The second object changes temperature 5.9 times as much as the iron object when the two interact thermally. What do you think the other metal might be? Solve using dimensionless ratios.

Problem 6.9

The specific heat of copper is 0.092 cal/g/C° and the specific heat of silver is 0.056 cal/g/C°. A piece of silver with a mass 0.02 times as much as a piece of copper receives 0.33 times as much heat as the piece of copper. If the copper's temperature change is 20 C°, what is the temperature change of the silver? Solve using dimensionless ratios.

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Problem 6.10

In this problem, the calculations have been done for you. The questions ask about the reasoning involved.

Problem: Suppose 300 g of a substance increases 20 C° in temperature upon receiving 200 cal of heat. A second sample of the same substance increases 6 C° upon receiving 50 cal of heat. What is the mass of the second sample?

Solution:

$$(1)\ \frac{200}{20} = 10$$

$$(2)\ \frac{10}{300} = 0.033$$

$$(3)\ 0.033 \times 6 = 0.2$$

$$(4)\ \frac{50}{0.2} = 250$$

A. How do you interpret the number *10* in step 1? Explain why the division was done in step 1.

B. How do you interpret the number *0.033* in step 2? Explain why the division was done in step 2.

C. How do you interpret the number *0.2* in step 3? Explain why the multiplication was done in step 3.

D. How do you interpret the number *250* in step 4? Explain why the division was done in step 4.

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Problem 6.11

In this problem, the calculations have been done for you. The questions ask about the reasoning involved.

Problem: A 250 g piece of a certain kind of metal receives 1000 cal of heat and increases its temperature from 20°C to 44°C. If 300 cal of heat is delivered to 120 g of this metal, what will be the temperature change?

Solution:

$$(1)\quad 44 - 20 = 24$$

$$(2)\quad \frac{1000}{250} = 4$$

$$(3)\quad \frac{4}{24} = \frac{1}{6}$$

$$(4)\quad \frac{1}{6} \times 120 = 20$$

$$(5)\quad \frac{300}{20} = 15$$

A. How do you interpret the number *24* in step 1? Explain why the subtraction was done in step 1.

B. How do you interpret the number *4* in step 2? Explain why the division was done in step 2.

C. How do you interpret the number *1/6* in step 3? Explain why the division was done in step 3.

D. How do you interpret the number *20* in step 4? Explain why the multiplication was done in step 4.

E. How do you interpret the number *15* in step 5? Explain why the division was done in step 5.

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Problem 6.12

In this problem, the calculations have been done for you. The questions ask about the reasoning involved.

Problem: A certain piece of glass rises 40 C° in temperature upon receiving 1200 cal of heat. If 2.3 times as much glass receives 1800 cal of heat, how much will its temperature change?

Solution:

$$(1)\ \frac{1200}{40} = 30$$

$$(2)\ 2.3 \times 30 = 70$$

$$(3)\ \frac{1800}{70} = 26$$

A. How do you interpret the number *30* in step 1? Explain why the division was done in step 1.

B. How do you interpret the number *70* in step 2? Explain why the multiplication was done in step 2.

C. How do you interpret the number *26* in step 3? Explain why the division was done in step 3.

Problem 6.13

In this problem, the calculations have been done for you. The questions ask about the reasoning involved.

Problem: Two samples of the same liquid are being heated. The mass of the first sample is 70 g. It rises 18 C° in temperature upon receiving 980 cal of heat. If the second sample rises 36 C° upon receiving 560 cal of heat, what is its mass?

Solution:

$$(1)\ \frac{560}{980} = \frac{4}{7}$$

$$(2)\ \frac{36}{18} = 2$$

$$(3)\ \frac{70}{2} \times \frac{4}{7} = 20$$

A. How do you interpret the number *4/7* in step 1? Explain why the division was done in step 1

B. How do you interpret the number *2* in step 2? Explain why the division was done in step 2.

C. How do you interpret the number *20* in step 3? Explain why the multiplication was done in step 3.

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Problem 6.14

In this problem, the calculations have been done for you. The questions ask about the reasoning involved.

Problem: Two pieces of the same metal are being heated. One piece has a mass of 35 g. It is observed that 70 cal of heat raise the temperature of this piece by 15 C°. If the second piece has a mass of 450 g, how much heat is required to raise its temperature 10 C°?

Solution:

$$(1)\ \frac{70}{35} = 2$$

$$(2)\ \frac{2}{15} = 0.13$$

$$(3)\ 0.13 \times 450 = 60$$

$$(4)\ 60 \times 10 = 600$$

A. How do you interpret the number *2* in step 1? Explain why the division was done in step 1.

B. How do you interpret the number *0.13* in step 2? Explain why the division was done in step 2.

C. How do you interpret the number *60* in step 3? Explain why the multiplication was done in step 3.

D. How do you interpret the number *600* in step 4? Explain why the multiplication was done in step 4.

Problem 6.15

In this problem, the calculations have been done for you. The questions ask about the reasoning involved.

Problem: A piece of a certain metal rises 200 C° in temperature upon receiving 20 cal of heat. How much heat is required to raise twice as much metal by 40 C°?

Solution:

$$(1)\ \frac{200}{40} = 5$$

$$(2)\ \frac{20 \times 2}{5} = 8$$

A. How do you interpret the number *5* in step 1? Explain why the division was done in step 1.

B. How do you interpret the number *8* in step 2? Explain why the multiplication and the division were done in step 2.

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Problem 6.16

A total of 400 cal of heat is required to raise the temperature of a particular object from 20°C to a final temperature T. Write an expression for the heat capacity of the object. Explain your reasoning.

Problem 6.17

Write an expression for the heat capacity of 40 cm^3 of a substance with density D and specific heat 0.03 cal/g/C°. Explain your reasoning.

Problem 6.18

An amount of heat Q is necessary to raise a 50 g object of a certain substance by 40 C°. Write an expression for the heat capacity of the object. Explain your reasoning.

Problem 6.19

Two objects interact thermally and reach a final temperature T. The initial temperatures are 20°C and 80°C. Write an expression for the ratio of the heat capacities. Be sure to tell whether your answer is the ratio of the hot object's heat capacity to the cold object's or vice versa. Explain your reasoning.

Problem 6.20

An amount of heat Q is lost by an 80 g object initially at 120°C when it is placed in 200 g of water initially at 20°C. Write an expression for the amount of heat gained by the water. Explain your reasoning.

Problem 6.21

A total of 8000 cal of heat is delivered to a 100 g sample of a particular substance at 50°C. Write an expression for the final temperature in terms of the specific heat, c, of the substance. Explain your reasoning.

Problem 6.22

An engine part is made of two 50 g pieces of different metals. One metal has specific heat 0.06 cal/g/C° and the other has specific heat c. Write an expression for the amount of heat required to raise the temperature of the engine part by 150 C°. Explain your reasoning.

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Problem 6.23

An object with heat capacity K is to be heated in a water bath of 50 g of water. A total of 1300 cal of heat is delivered to the object–bath system. Write an expression for the temperature change of the object. Explain your reasoning.

Problem 6.24

An object is made of brass and iron, and the mass of iron is three times as much as the mass of brass. How much heat is required to raise the temperature of this object by 100 C°? Answer by writing an expression in terms of M, where M is the mass of iron in the object. Explain your reasoning.

Problem 6.25

Suppose an object with heat capacity 600 cal/C° rises 30 C° in temperature upon receiving a certain amount of heat. Some more material is then added to the object and it is found that it now requires an additional amount of heat Q to raise the temperature of the object by 30 C°. Write an expression for the new heat capacity of the object. Explain.

Problem 6.26

An aluminum tank is holding water at 20°C. The combined mass of the tank and water is 5 kg. After 500 g of boiling water is poured into the tank, the final temperature of the system is 30°C. How much water was initially in the tank ? Explain your reasoning.

Problem 6.27

Suppose we want to make an object with a mass of 1000 g and a heat capacity of 100 cal/C°. This object is to be made from two metals with specific heats 0.09 cal/g/C° and 0.16 cal/g/C°. How much of each metal should we use? Explain your reasoning.

Problem 6.28

A pot is used to heat water on an electric hot plate. When the pot is full of water, the pot and the water rise 10 C° in 30 seconds. If the pot is only half full, the pot and the water rise 18 C° in 30 seconds. A full pot holds 1000 g of water. What is the heat capacity of the pot? Explain your reasoning.

Problem 7.1

A 100 g piece of very cold iron was placed in 20 g of water at 20°C. When the interaction was finished, 15 g of water had frozen. What were the initial and final temperatures of the iron? Explain your reasoning.

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Problem 7.2

If 200 g of steam at 100°C interact thermally with 500 g of ice at 0°C, what is the final temperature? Explain your reasoning.

Problem 7.3

If 100 g of steam at 100°C interact thermally with 900 g of ice at 0°C, what is the final temperature? Explain your reasoning.

Problem 7.4

If 20 g of steam at 100°C interact thermally with 140 g of ice at 0°C, what is the final temperature? Explain your reasoning.

Problem 7.5

It takes 10,000 cal of heat to raise 50 g of a certain metal from room temperature to its melting point. Its latent heat of fusion is F. Write an expression for the amount of heat that must be applied to 50 g of this metal to melt half of it starting from room temperature. Explain your reasoning.

Problem 7.6

A certain liquid with latent heat of vaporization L is at its boiling point. Initially the mass of liquid is 500 g. As 7000 cal of heat are delivered, some of the liquid boils off and some remains. Write an expression for how much liquid remains. Explain your reasoning.

Problem 7.7

A certain solid melts at 53°C. The latent heat is 30 cal/g and the specific heat of the liquid is 0.3 cal/g/C°. Suppose that 40 g of this solid substance are at its melting point. Write an expression for how much heat is required to raise it to a final temperature T. Explain your reasoning.

Problem 7.8

A sample of a certain solid at its melting temperature receives 900 cal of heat. The latent heat of melting is 40 cal/g. The final temperature of the sample is 20 C° above the melting point. Write an expression for the specific heat of the liquid in terms of the mass M of the sample. Explain your reasoning.

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Problem 7.9

An 80 g sample of liquid is raised from 20°C to the boiling point at 70°C and then completely vaporized. The total heat required is 3000 cal. Write an expression for the latent heat of vaporization in terms of the heat capacity, K, of the sample of liquid. Explain your reasoning.

Problem 7.10

A certain amount of ice is to be melted by thermal interaction. Object A could melt all of the ice if its initial temperature were 100°C. Object B could melt all of the ice if its initial temperature were 40°C. Suppose A and B were combined into one object. What must the initial temperature of this compound object be for it to melt all the ice? Explain your reasoning.

Problem 7.11

Two metal tanks and their contents interact thermally. One tank is initially at 37°C. This tank's mass is 5 kg and it contains 2 liters of water. The other tank is made of 2.7 kg of the same metal, and it initially contains 800 g of ice at 0°C. After the tanks interact, the equilibrium temperature is 10°C. What is the specific heat of the tank metal? Explain your reasoning.

Problem 7.12

An ice cube at 0°C was placed in 500 g of water at 60°C. The final temperature was 18°C. What was the mass of the ice cube? Explain your reasoning.

Problem 7.13

Here are some data from a hypothetical experiment:

> Suppose that 40 g of ice at –20°C is placed in 100 g of water at 20°C in a plastic foam cup. When equilibrium is reached, there are 120 g of water and 20 g of ice in the cup.

A. Quantitatively discuss these results in terms of heat transfer; that is, keep track of all the heat transferred: where it came from, where it went, what its effects were. Use 80 cal/g for the heat of fusion of water.

B. What property of ice can be determined from these data? Determine it.

Problem 7.14

Suppose we transfer half as many calories to an ice cube as are necessary to entirely melt it. What happens is that half of the 1 g pieces of ice melt while the other half remain unchanged. Thus, heat transfer is not divided equally among all the grams of ice, but rather gram after gram is melted until there is no more heat available.

Contrast this process with the spreading out of heat when a temperature change occurs. How are these processes the same and how are they different?

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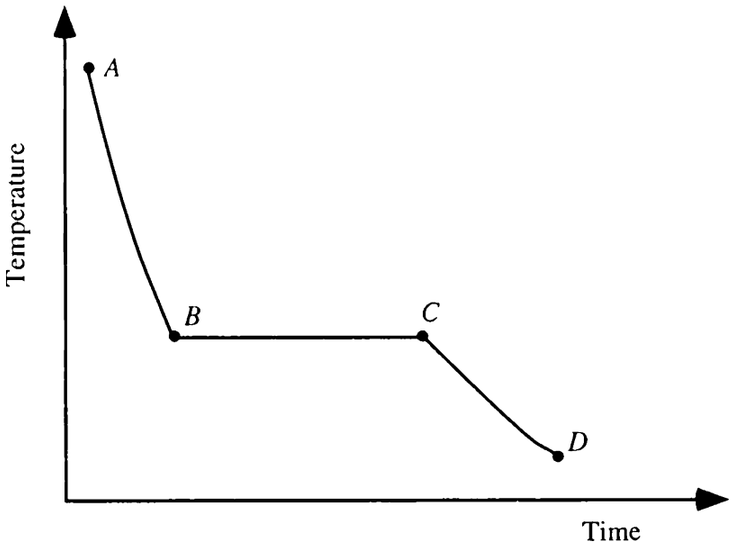
Problem 7.15

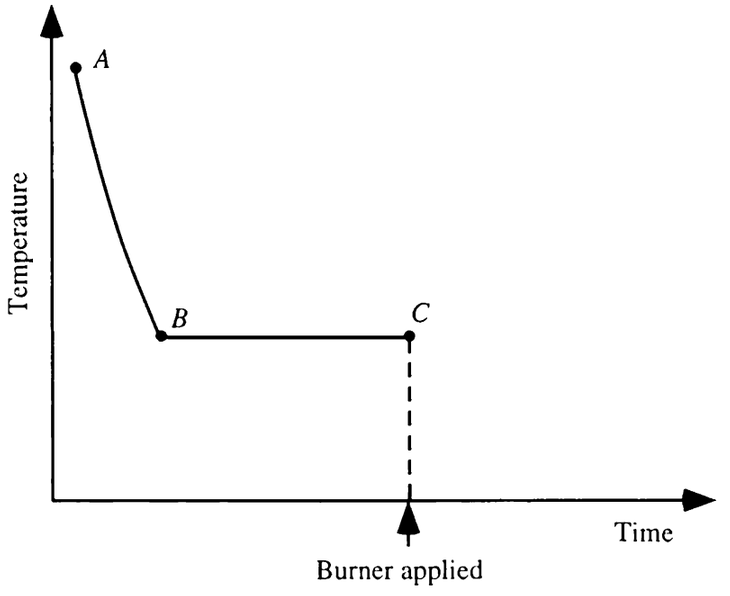
A. A hot liquid was left out in a room to cool. Its temperature was recorded every few seconds. The following graph shows the behavior of the temperature over time.



(1) What is happening to the substance between *A* and *B*? between *B* and *C*? between *C* and *D*?

(2) Discuss the heat transfers taking place in the intervals *A* to *B*, *B* to *C*, and *C* to *D*.

B. Suppose this experiment is redone, but this time a burner is put to the substance at time *C*.



(1) Sketch the graph from point *C* on.

(2) Tell what is happening to the substance and describe the heat transfers taking place for the part of the graph following point *C*.

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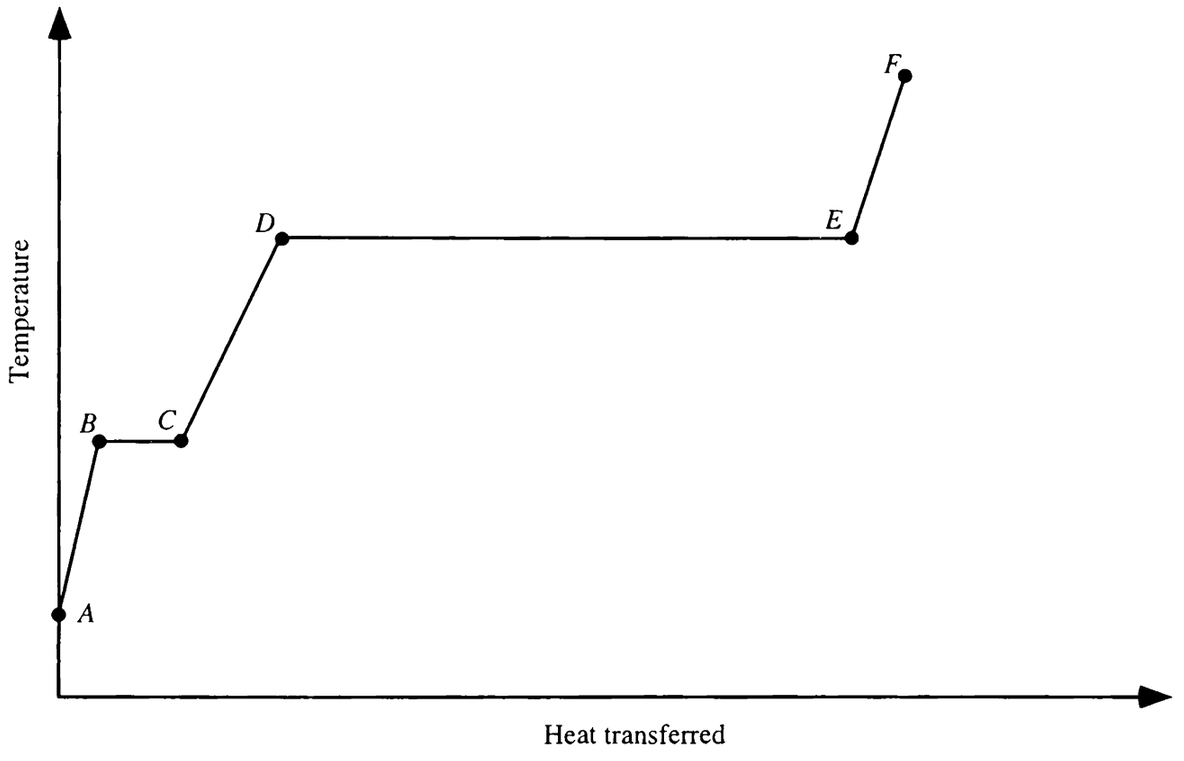
Problem 7.16

Very cold ice is placed in a closed container. Heat is then applied to the container. Below is a graph of the temperature inside the container plotted against the amount of heat delivered to the system.



Describe what is happening to the substance inside the container during each of the following intervals.

A. From *A* to *B*

B. From *B* to *C*

C. From *C* to *D*

D. From *D* to *E*

E. From *E* to *F*

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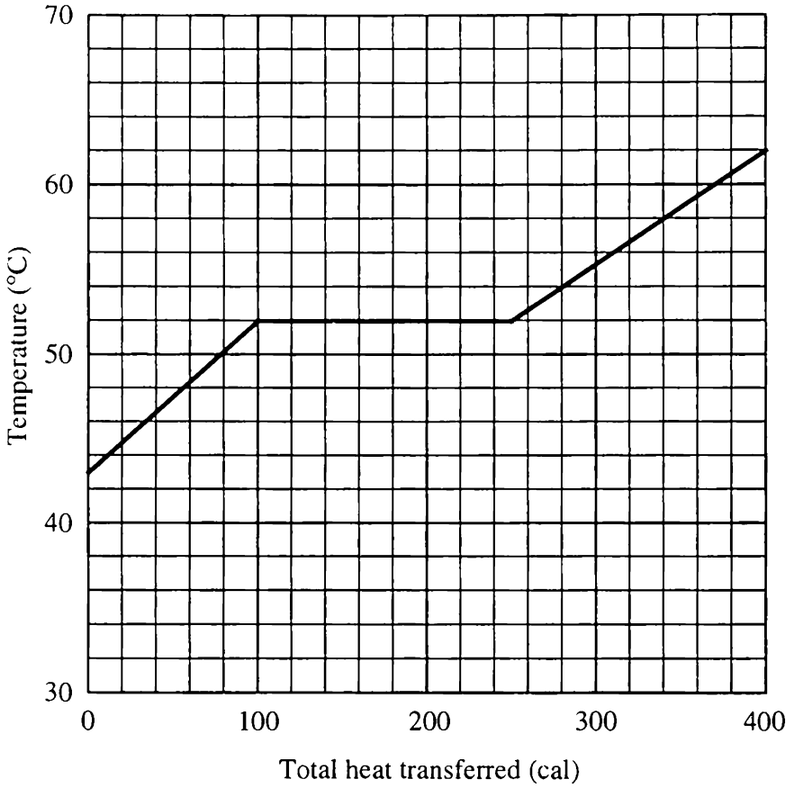
Problem 7.17

A student heated a 50 g sample of a certain substance and recorded the temperature. During the experiment, the substance melted. The graph shows the temperature of the substance plotted against the amount of heat delivered to the substance.



A. Why does the graph not go through the origin?

B. What is the latent heat of fusion of this substance? Tell how you obtain this information from the graph.

C. Tell as much as possible about the specific heat of this substance. Tell how you obtain your information from the graph. Consider both solid and liquid phases.

Problem 8.1

Suppose a house is to be maintained at a constant temperature in the winter.

A. Why is it necessary to continue to heat the house even after it has reached a comfortable temperature?

B. How does the temperature of the walls compare to the temperature inside the house? to the temperature outside?

C. What heat transfers take place involving the walls? What effect do these heat transfers have on the temperature of the walls?

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Problem 8.2

Steam (high temperature water vapor) is extremely dangerous. It can burn severely and quickly. Compare the heat transferred to the skin by one gram of boiling water at 100°C and by one gram of steam at 100°C. Assume the final temperature is 40°C in both cases.

Problem 8.3

In many places, it takes weeks or even months for snow that accumulated during the winter to melt. Why doesn't all of the snow melt as soon as the temperature climbs above 0°C?

Problem 8.4

It is often possible to keep foods quite cold without a refrigerator by wrapping them in a towel soaked in cold water. Why does this procedure work?

Problem 8.5

Warm-blooded animals generally maintain their body temperatures at nearly constant levels. These temperatures are usually higher than the surroundings. For example, humans maintain the insides of their bodies at about 37°C, whereas a normal temperature for the surroundings is 20°C.

A. What can you say about the temperature of skin compared to the inside of a body? What can you say about the temperature of skin compared to the room?

B. How can the body maintain a constant temperature if heat is constantly transferred out of it?

Problem 8.6

Which of the following properties of water make it useful in fighting fires? Explain your reasoning in each case.

A. It does not burn.

B. Its temperature is low compared to burning material.

C. It has a large thermal conductivity.

D. It vaporizes and leaves, taking the heat along with it.

E. It has a large heat of vaporization.

Problem 8.7

A. A spherical ball of ice is briefly submerged in hot water, during which time 10% of the ice melts. Which 10% of the ice would melt? Explain your reasoning.

B. Why does crushed ice cool water faster than does a single cube of the same mass of ice? Explain your reasoning.

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PHYSICS BY INQUIRY

Light and Color

In *Light and Color,* we investigate the formation of shadows and images and experiment with colored light and colored paints. We develop a mental model that enables us to account for our observations and to make predictions for situations that we have not yet observed.

Part A: Light and shadows

We begin our investigation of light by making observations using flashlights, masks, and screens. We use the ideas that we develop in this context to account for more complicated phenomena, such as the formation of images and shadows from extended sources.

Section 1. Introduction to light

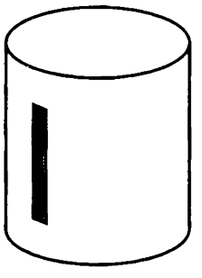
Experiment 1.1

This experiment should be performed in a darkened room.

You will need a small bulb, a socket, two batteries, connecting wires, and some black construction paper. A staff member can show you how to connect the bulb to two batteries in series so that the bulb shines brightly.

A. Look at the lighted bulb. Note that you can see it from many places.

Make a cover for the bulb, as shown at right, by rolling a piece of construction paper into a tube just large enough to fit over the socket. Cut a narrow slit in the paper (about 3 mm wide), then place the tube over the bulb. Place another piece of paper over the top of the tube.



Can you still see the bulb from many different places? If not, where must you place your eye to see the bulb?

Rotate the cover. Now where must you place your eye to see the bulb?

Draw a top view diagram (a picture of what someone would see when looking down from above) to show where you need to place your eye in order to see the bulb. Do this for at least two different positions of the slit.

Describe the arrangement of bulb, slit, and your eye that is necessary for you to be able to see the lighted bulb.

B. Obtain a chalkboard eraser that has chalk dust on it. Have your partner hold the eraser near the slit in the tube and strike the eraser several times with a ruler. What do you observe?

C. People often speak of a *beam* of light. Why do you think people use this word to describe light?

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Experiment 1.2

This experiment should be performed in a darkened room.

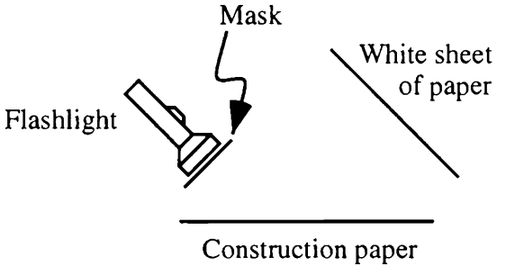
You will need several sheets of colored construction paper, a white sheet of paper, a flashlight, and a mask for the flashlight. Make the mask by cutting a small slit (approximately 25 mm long and 5 mm wide) in an index card or thick piece of paper. Tape the mask to the front of the flashlight.

A. Shine the flashlight through the mask onto various flat surfaces, including colored pieces of construction paper.

Describe what you observe as you vary the angle between the flashlight and the flat surfaces.

Compare what happens when you shine the flashlight on the following types of surfaces: dark-colored surfaces, light-colored surfaces, rough surfaces, and smooth surfaces. Try other types of surfaces as well. Describe the similarities and differences in the results when the different surfaces are used.

B. Hold a piece of white paper above a sheet of white construction paper as shown. Aim the flashlight at the construction paper, not at the white sheet of paper.



How does the appearance of the white paper that you are holding change when the flashlight is turned on and off? How might you account for your observations?

Leave the flashlight on and replace the white construction paper by a piece of red construction paper. Does this change make a difference in the appearance of the sheet of white paper you are holding?

Try other colors of construction paper. How does the color of the construction paper affect what you see on the white sheet of paper?

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Experiment 1.3

A. Look at an unlighted bulb. Notice that even though the bulb is not lighted, you can still see it.

Suppose you took the bulb into a very dark closet. Would you still be able to see it? Explain.

B. Place the cover that you used in Experiment 1.1 over the unlighted bulb.

Where do you have to place your eye to see the bulb?

Use a flashlight with a mask to produce a beam of light.

How do you have to aim the flashlight to make the bulb inside the cover more easily visible?

C. When you are not aiming a flashlight at the bulb, does light from anywhere else strike the bulb? Explain.

✔ Discuss this experiment and Experiment 1.2 with a staff member.

Exercise 1.4

A. Consider the following statement by a student:

"At night it is dark, but during the day it is bright because the sun lights up objects. When objects are lit up I can see them."

Do you agree with the statement by this student? Would you change the statement in any way to make it more correct?

B. Summarize the conditions necessary for you to be able to see an object.

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The observations that you have made thus far are consistent with the following way of thinking about light: An observer sees an object when light travels from that object to the observer's eye. In some cases, the object generates light (e.g., a lighted bulb, the sun, etc.). In other cases, the object (e.g., an unlighted bulb, a piece of paper, etc.) cannot be seen unless it is illuminated by light from another source. Light from the source reaches the object and then travels from the object to the observer's eye. We say that light from the source is *reflected* from the object.

Exercise 1.5

A. Discuss how the experiments that you have performed thus far are consistent with the way of thinking about the motion of light outlined above. Give specific examples.

B. Examine your responses to Exercise 1.4. Do you still agree with your answers to that exercise? If not, modify your answers to make them more complete.

C. In Experiments 1.1 and 1.3 you were able to see the bulb both when it was lighted and when it was not. Describe, both in words and in a diagram, the path that light follows in each of these two cases. On your diagram, use an arrow to indicate the direction that light moves along the path.

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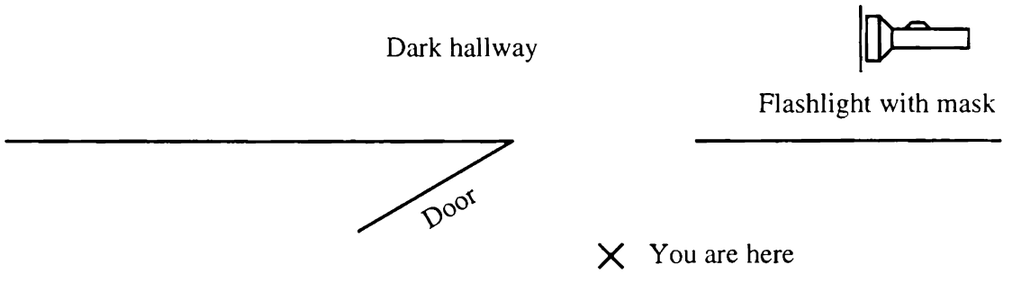
Experiment 1.6

Imagine that your partner is in a dark hallway, and you are by a doorway that enters into the hall. You are facing the hallway and no other lights are on. Your partner aims a flashlight with a mask at the far end of the hall.

Would you be able to tell whether the flashlight is on or off? If so, how could you tell? What would you see? If not, explain why not.



Discuss your ideas with your partner, then carry out the experiment, if possible. If your prediction was incorrect, try to resolve any inconsistencies.

✔ Discuss this experiment and the preceding exercise with a staff member.

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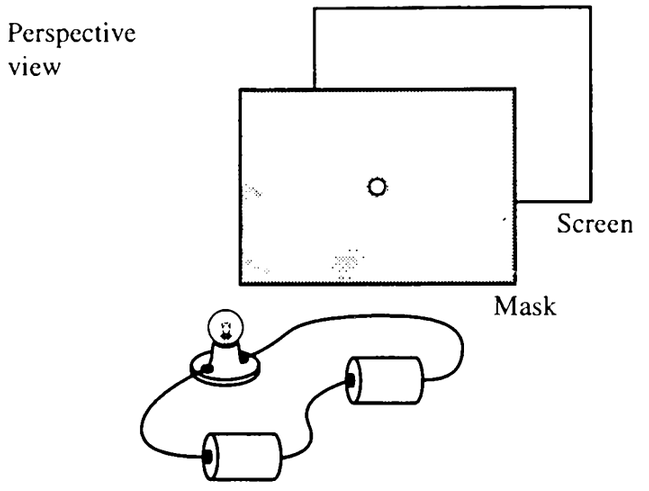
Section 2. Light sources, masks, and screens

Experiment 2.1

This experiment should be performed in a darkened room.

A small bulb, a screen made of white paper, and a mask are arranged as shown. The mask is a sheet of paper with a hole in it that is about 3 mm wide.



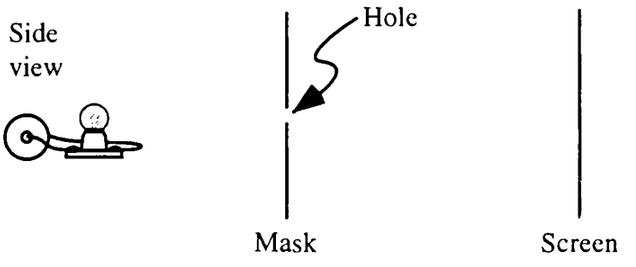
A. Predict what you will see on the screen when the bulb is lighted. Explain.

   Set up the equipment and check your prediction.

B. Predict how what you see on the screen would change if the bulb were moved upward, downward, or side to side. Explain.

   Check your predictions.

   How does the direction that the bright spot moves compare to the direction that the bulb moves?

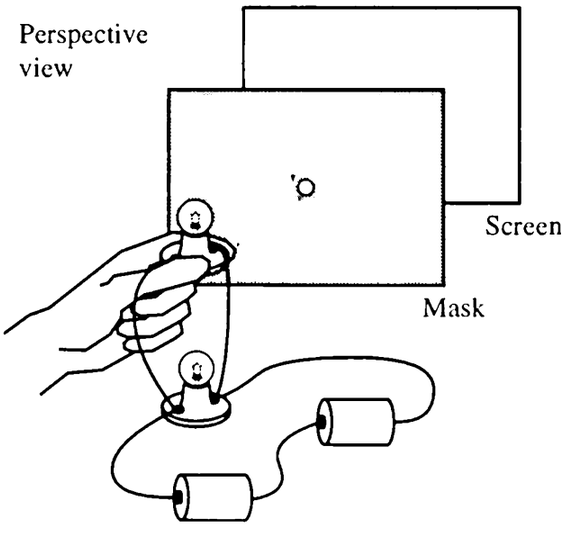


Draw a diagram to account for what you observe.

C. Predict how placing a second bulb above the first would affect what you see on the screen.

   Predict how moving the top bulb upward slightly would affect what you see on the screen.

   Check your predictions.



D. What do your observations suggest about the path taken by light from the bulb to the screen?

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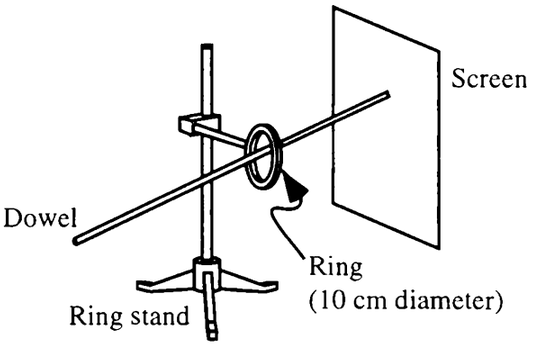
Experiment 2.2

Set up the apparatus shown at right. You will need a long dowel, a ring stand, and a ring that is about 10 cm in diameter.



Demonstrate how you can use this apparatus to help you account for some of the observations that you made in the preceding experiment. The ring represents the hole in the mask.

What did you use the dowel to represent? What property of the dowel makes it a suitable choice for this role?

✔ Discuss this experiment and Experiment 2.1 with a staff member.

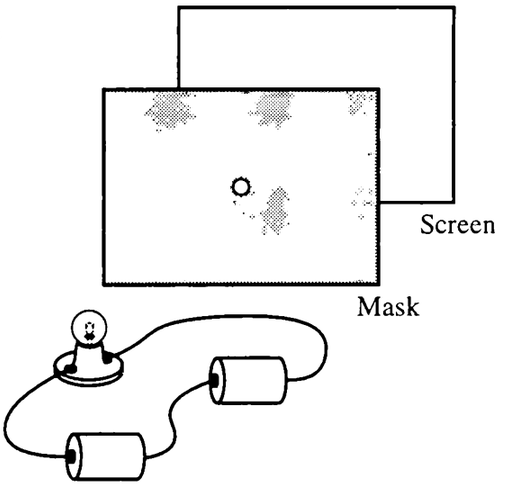
Experiment 2.3

This experiment should be performed in a darkened room.

In this experiment, you will be asked to make several predictions. If you find that your predictions are incorrect, try to resolve the inconsistencies in your explanation before continuing to the next prediction.

A mask with a circular hole is placed between a small lighted bulb and a screen as shown.

Perspective view



Predict how each of the following changes would affect what you see on the screen. Explain your reasoning and include sketches that support your predictions.

A. The bulb is moved *farther* from the mask.

   Check your prediction.

B. The mask is replaced by a mask with a *triangular* hole.

   Check your prediction.

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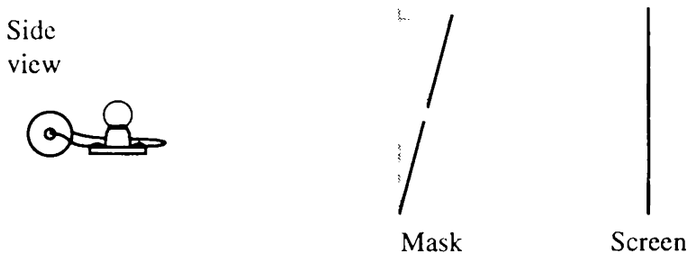
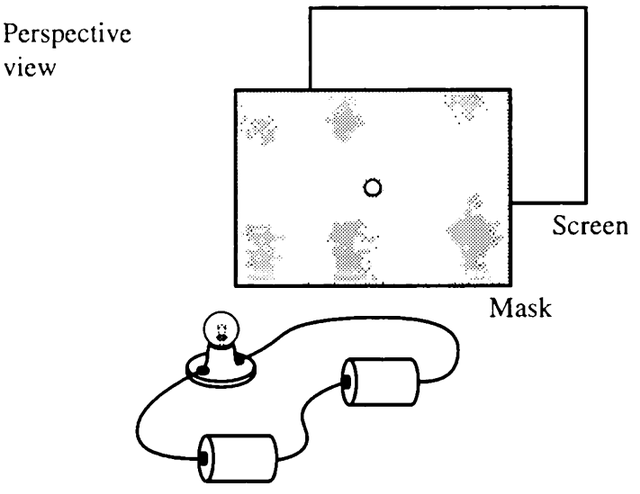
C. The mask is tilted toward the screen as shown in the side view diagram below.



Check your prediction.

Experiment 2.4

A small bulb is placed 10 cm from a mask with a hole that is 4 cm in diameter. The mask is 15 cm from a screen. The bulb is centered on the hole in the mask.



A. On a piece of graph paper, draw a careful life-size diagram to illustrate this physical situation.

Which of the following types of diagrams do you think will be most useful in helping you to determine the size of the bright region of the screen: a perspective view, a top view, or a side view? Explain.

Since the filament of the bulb is small, you can draw it as a dot. A source of light that is represented as a dot on a diagram is often called a *point source* of light. In this module, treat small bulbs as point sources.

From your diagram, measure the height of the bright region on the screen.

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B. Your diagram should contain two triangles: The vertices (corners) of the larger triangle are located at the bulb and at the top and bottom of the bright region on the screen. The vertices of the smaller triangle are located at the bulb and at the top and bottom of the hole in the mask. Outline both triangles on your sketch.

Which features of these triangles are the same and which features are different?

(1) Measure the length of one side of the larger triangle and the length of the corresponding side of the smaller triangle. Calculate the following ratio of the two lengths:

$$\frac{\text{length of side on larger triangle}}{\text{length of corresponding side on smaller triangle}}$$

Make the same calculation as above for each of the two remaining sides of the triangles.

Are the three ratios essentially the same or are they different?

(2) Compare the angles in the two triangles.

How do the corresponding angles of the triangles compare?

What is the ratio of corresponding angles in the two triangles?

C. Triangles such as the two that you have drawn, where one is a scaled version of the other, are called *similar triangles.*

Suppose that you were given two triangles. Describe a method by which you could determine whether or not the triangles are similar.

It is not always possible to make a full-size diagram of a physical setup. For example, if a bulb and screen are very far apart, it may be necessary to make a sketch that is smaller than life-size. A diagram in which each distance is shown reduced or enlarged by the same multiplicative factor is called a *scale diagram.*

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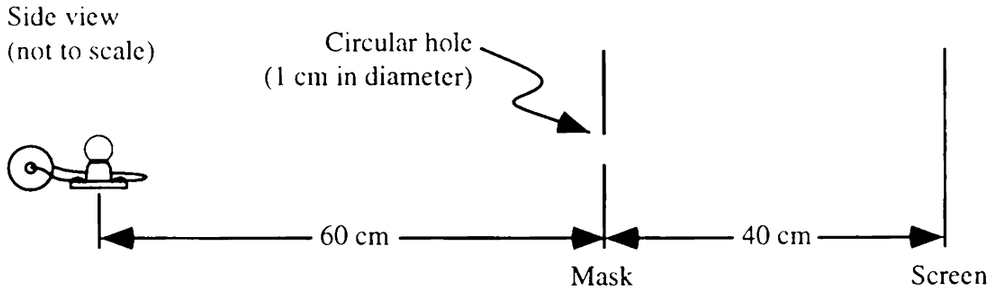
Experiment 2.5

A bulb is placed in front of a mask and screen as shown below.



A. Use the properties of similar triangles to write an equation that you can use to determine the height, h, of the bright region on the screen.

Calculate h. What are the units of h?

Is the height of the bright region on the screen greater than, less than, or equal to the diameter of the hole in the mask? Explain how you can tell from your equation and how you can tell from the diagram.

Set up the apparatus and check your prediction for the height of the bright region on the screen.

B. Write an equation for the height, h, of the bright region on the screen in terms of the diameter, d, of the hole in the mask.

Check your equation by substituting 1 cm for the diameter of the hole. You should obtain the same answer as in part A.

Suppose that the diameter of the hole in the mask were doubled.

Would the height of the bright region on the screen also double? Explain how can you tell from your equation.

C. Compare the following ratios without making any measurements:

$$\frac{\text{height of bright region on the screen}}{\text{distance from bulb to screen}} \quad \text{and} \quad \frac{\text{height of hole in mask}}{\text{distance from bulb to mask}}$$

D. Suppose the following changes are made to the mask above. In each case, make a rough sketch, and decide whether or not you can use the idea of similar triangles to find the size of the bright region on the screen.

(1) The mask is raised so that the bulb is not even with the center of the hole in the mask.

(2) The top of the mask is tilted toward the screen (as in part C of Experiment 2.3).

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Exercise 2.6

A mask with a small circular hole is placed between a bulb and a screen. The hole is 5 cm in diameter. The mask is 45 cm from the bulb and 60 cm from the screen.

Determine the size of the lit area on the screen. Explain your reasoning.

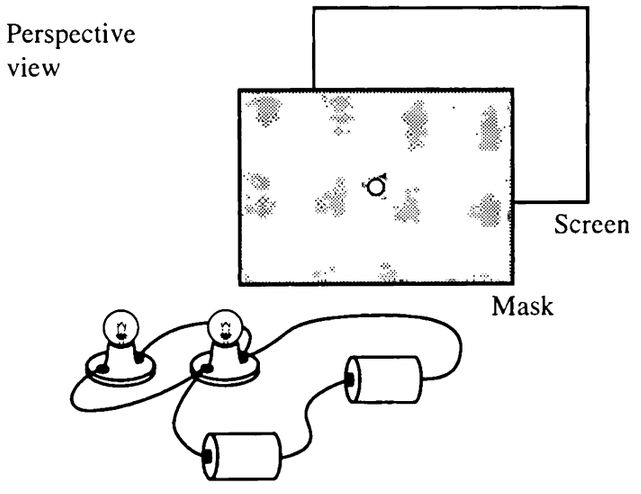
✔ Discuss this experiment and Experiment 2.5 with a staff member.

Experiment 2.5 and Exercise 2.6 illustrate how an understanding of the properties of similar triangles can allow us to calculate quantities without having to measure them in real life or from a sketch. Even so, it is often still useful to make a sketch of the physical situation. To be useful, however, the sketch must be accurate enough to show which triangles, if any, are similar.

Experiment 2.7

This experiment should be performed in a darkened room.

A mask is placed between two small bulbs and a screen as shown.



A. Predict what you would see on the screen. Explain your reasoning.

   Set up the apparatus, and check your prediction.

B. Predict what you would see on the screen if you were to add a third bulb in line with the other two bulbs.

   Check your prediction.

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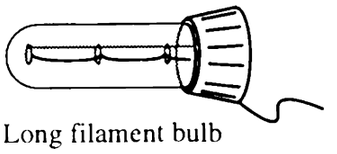
C. Predict what you would see on the screen if you were to replace the three small bulbs by a long filament bulb. Explain your reasoning.



Long filament bulb

Obtain a bulb with a long filament and check your prediction.

Describe how your observation is consistent with the idea that we can consider a long filament bulb as a string of closely spaced small bulbs.

D. Predict what you would see on the screen when two long filament bulbs are arranged to form a T-shaped light source. Use a diagram to help explain your prediction.

Check your prediction. Draw a side view diagram and a top view diagram to account for your observation.

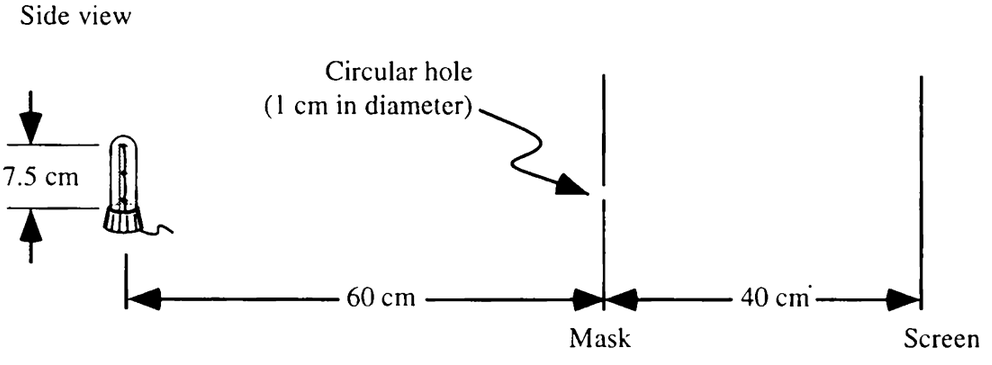
E. Predict what you would see on the screen when two long filament bulbs are arranged to form an L-shaped light source. Use a diagram to help explain your prediction.

Check your prediction. Draw a side view diagram and a top view diagram to account for your observation.

✔ Discuss your results with a staff member.

Experiment 2.8

Consider the situation pictured below. Except for the hole in the mask, the diagram has been drawn to scale.



A. Sketch a diagram illustrating the approximate size of the lit area on the screen. Explain your reasoning.

B. If the diameter of the hole in the mask were halved, would the height of the lit area on the screen also become half as tall? Explain your reasoning both in words and with a sketch.

Check your prediction. If your prediction was incorrect, resolve the inconsistency.

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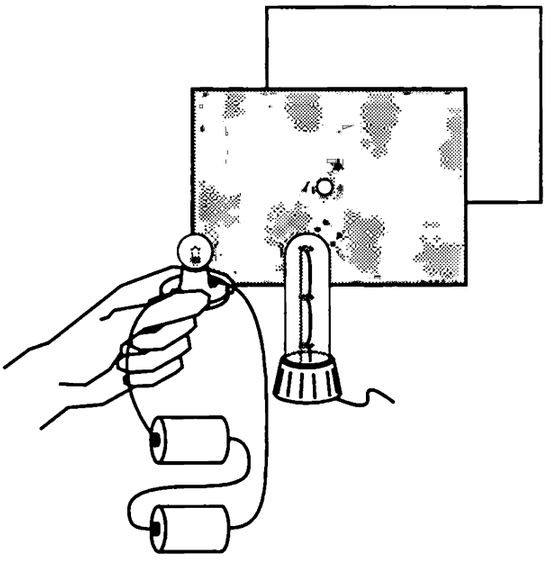
C. If the hole were very small, say the size of a pinhole, would the lit region on the screen be taller than, shorter than, or the same height as the filament? Draw a diagram that supports your prediction.

Determine the exact height of the lit region on the screen in this case. (*Hint:* Use the idea of similar triangles developed in Experiment 2.5 and Exercise 2.6.)

Check your prediction.

D. Sketch what you think you would see on the screen when both a small bulb and a long filament bulb are held in front of the mask as shown below. The bulbs are the same distance from the mask and the bulb is held level with the top of the long filament bulb.



If the diameter of the hole in the mask were halved, would the *height* of each bright region on the screen also halve? Explain your reasoning.

If the diameter of the hole in the mask were halved, would the *width* of each bright region on the screen also halve? Explain your reasoning.

Check your predictions. If any of your predictions were incorrect, resolve the inconsistency.

✔ Discuss your results with a staff member.

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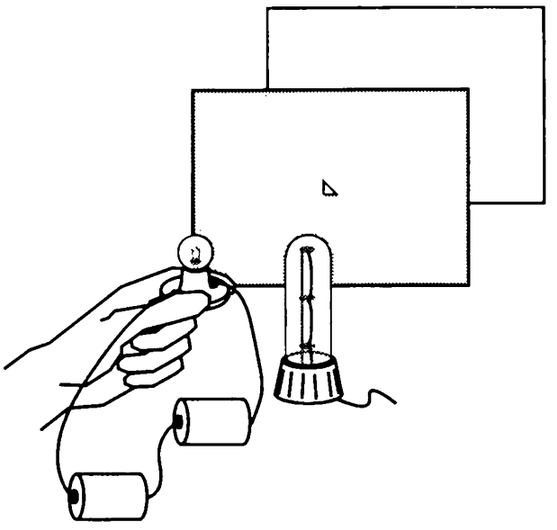
Experiment 2.9

This experiment should be performed in a darkened room.

In this experiment, you will be making several predictions. If you find that your predictions are incorrect, try to find the error in your explanation before continuing to the next prediction.

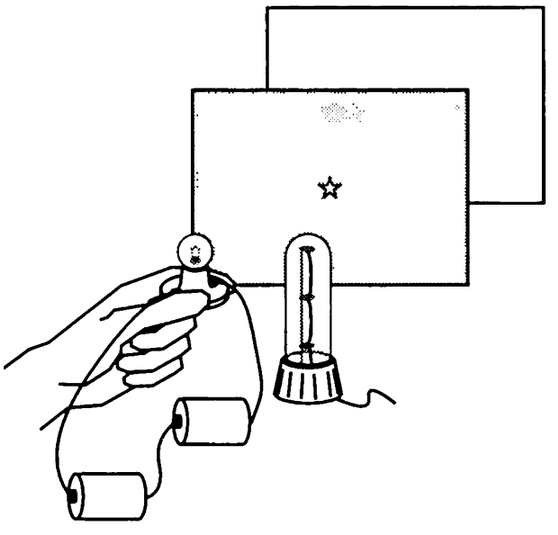
A. Predict what you would see on the screen when a small bulb is held near the top of a long filament bulb, and both are held in front of a mask with a *triangular* hole as shown at right. Explain your reasoning.

Check your prediction.



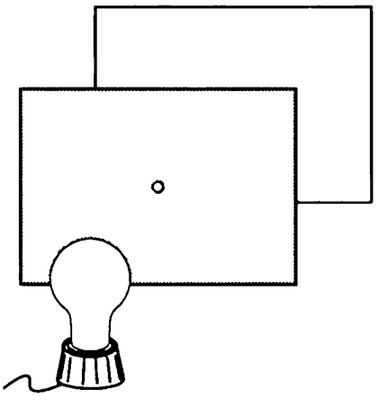
B. Predict what you would see on the screen if instead the hole in the mask were star-shaped. Explain your reasoning.

Check your prediction.



C. Predict what you would see on the screen if an ordinary frosted bulb were placed in front of a mask with a circular hole. Explain.

Check your prediction.



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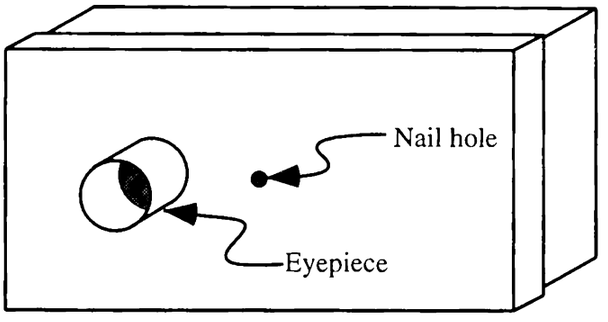
Section 3. Pinhole cameras

The ideas developed in the preceding sections can be used as the basis for explaining many phenomena involving light. In this section, we apply these ideas to help us understand a simple type of camera.

Experiment 3.1

For this experiment you will need a box about the size of a shoe box, a knife, some tape, a pair of scissors, and a piece of cardboard tubing about 3 cm long.



A. On one of the largest sides of the box, cut a hole for an eyepiece. The hole should be off-center as shown. Make the hole small enough that the cardboard tube fits snugly inside. Secure the tube in the hole with about one-half centimeter of the tube extending into the box.

   Inside the box, on the wall opposite the hole, tape a piece of plain white paper. The paper should cover the entire side of the box.

   The box should not have any holes other than the one that you just cut; it must be completely dark inside when the eyepiece is covered. To check the box for holes, hold the eyepiece to one of your eyes and cup your hands around that eye to block light from the sides. Then close your other eye and peer into the box. Wait a few moments for your eye to adjust to the darkness. If the inside of the box is lit, find the holes and cover them with black tape.

   On the same side of the box as the eyepiece, use a nail to punch a hole about 4 cm from the eyepiece. Make the edges of the nail hole as smooth as possible.

B. Stand with your back to a window or another bright spot in the room. Look into the box through the eyepiece. You may need to cup your hands around your eye to block the light from the sides. Describe what you see in the box.

   Note: Wait a few moments to allow your eyes to adjust to the darkness. If you do not see anything, move the box slightly from side to side or ask your partner to help block the light on the sides of your eye. If you still do not see anything, ask the staff for help.

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C. Move toward and away from the object that you are viewing with the box.

How does changing your distance from what you are viewing affect what you see inside your box?

D. Make two more holes close to the first nail hole: one larger than the original hole; the other, smaller.

Use your fingers to cover two of the three holes and view an object with the box. Then repeat twice more, each time leaving a different hole uncovered.

How does the size of the uncovered nail hole affect what you see inside the box?

The device you constructed in the preceding experiment is called a *pinhole camera.* The "picture" you observed on the side of the box opposite the hole is called an *image.*

In the following experiments, we examine how an image is formed in a pinhole camera.

Experiment 3.2

This experiment should be performed in a darkened room.

A. Use tape to cover all but one of the holes in your pinhole camera. View a small lighted bulb through your pinhole camera.

Describe what you observe.

B. Predict how the bright spot in your pinhole camera would change if the bulb were moved upward, downward, or side to side. Check your predictions.

How does the direction that the light spot moves compare to the direction that the bulb moves?

Have your partner move the bulb while you are viewing it in your pinhole camera. Try to infer the direction in which your partner is moving the bulb.

Draw diagram(s) to account for your observations.

C. Predict how the image in your pinhole camera would change if the light bulb were moved toward or away from the pinhole camera. Check your predictions.

Draw diagram(s) to account for your observations.

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Experiment 3.3

This experiment should be performed in a darkened room.

A. Set up two small bulbs side by side on a table. Predict what you will see when you look at the bulbs through your pinhole camera, then check your prediction.

B. Replace the two small bulbs by a long-filament bulb. Predict what you will see when you look at the long-filament bulb through your pinhole camera. Check your prediction.

   Imagine that the long filament bulb is oriented vertically. Predict what you would see in the box if your partner were to cover the top half of the long filament bulb. Check your prediction.

C. For this part of the experiment, you will need an overhead projector to project a large lighted area in the shape of the letter *L* on the wall.

   Sketch what you think you will see inside your pinhole camera when you view the letter *L* through your pinhole camera. Explain your reasoning.

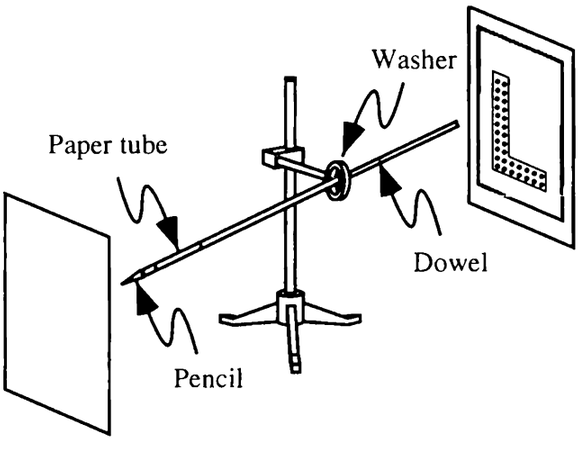
   Check your prediction.

   How does the image in your camera differ from what is projected on the wall? Use the ideas that you have developed thus far to account for any differences.

✔ Explain your reasoning to a staff member.

Experiment 3.4

The equipment you used in Experiment 2.2 can help you to visualize how a pinhole camera works. Set up a ring stand, a washer and a dowel as shown. The diameter of the dowel should be slightly less than that of the washer.



Fasten a pencil to the dowel by wrapping paper around both the dowel and the pencil and then taping the paper to the dowel. The pencil should slide freely in and out of the paper tube.

Draw a large letter *L* on a sheet of paper. Tape the *L* and a blank sheet of paper to pieces of cardboard. Support each piece of cardboard vertically in the locations shown.

We can imagine that the *L* is made up of many small dots, spaced very close to one another. Draw some dots on your letter *L* as shown.

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A. Hold the end of the dowel without the pencil in the middle of one of the dots in the *L*. While holding that end in place, mark the blank piece of paper with the pencil. Make the mark as large as the hole in the washer allows.

Predict the shape you will obtain after making a mark on the blank paper for each of the dots on the pattern. Sketch your prediction. Check your answer by repeating the procedure above for each of the dots that make up the *L*.

How does the orientation of the diagram that you generated compare to the orientation of the original diagram?

B. Describe how the activity you performed in part A of this experiment can help you to account for the observations you made while using the pinhole camera in part C of the preceding experiment.

C. Predict how the "image" that you generated in part A would change if the paper containing the original pattern were moved closer to the ring stand and washer.

What would be the analogous change for someone looking at an image in a pinhole camera?

Check your prediction using a pinhole camera and the projection of a transparency of the letter *L*.

D. Predict how the diagram that you generated in part A would change if the size of the washer hole were enlarged.

What would be the analogous change for someone looking at an image in a pinhole camera?

Check your prediction by replacing the washer by one with a larger hole and repeating the procedure of part A.

✔ Discuss this experiment with a staff member.

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Experiment 3.5

This experiment should be performed in a darkened room.

Obtain transparencies of various letters and words from your instructor. Use an overhead projector to project the transparencies on a wall or screen.

A. Project one of the transparencies and predict what you would see in the camera when looking at the projection. Explain your reasoning. Draw a sketch of your prediction, then check your answer.

B. Look in your pinhole camera and have your partner project a different transparency for you. Try to determine what is being projected on the wall only on the basis of what you see in your camera.

C. View one of the projected letters through your pinhole camera. Change the size of the hole in the camera and describe how the image changes. Explain this effect using the ideas about light that you have developed thus far. (See part D of Experiment 3.4.)

D. Look at a building or other large object in your camera. Explain the formation of the image using the ideas about light that you have developed thus far.

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Section 4. Shadows

Experiment 4.1

This experiment should be performed in a darkened room.

A. Place a nail on its head so it stands upright on your table. (A small piece of clay will hold the nail in place.) Use a small lighted bulb to cast a shadow of the nail on a sheet of white paper held vertically behind the nail. Experiment with the equipment as you try to answer the following questions:

   (1) How can you make the shadow of the nail larger? How can you make the shadow smaller? What factors affect the size of the shadow?

   (2) Does the shadow "fill up" space in the way that a light beam does, or is it a flat thing? Would it make sense to speak of a "shadow beam" as we speak of a light beam?

   (3) Can an object have more than one shadow? If so, describe the circumstances under which it can have more than one. If not, explain why it cannot.

B. Place the sheet of paper underneath the nail and hold the bulb above the table so that the shadow is on the paper.

   What factors affect the shape and size of the shadow now?

Experiment 4.2

This experiment should be performed in a darkened room.

A. Imagine that a small light bulb is placed 8 cm from a nail and a vertical screen is placed 15 cm beyond the nail. Let h represent the height of the nail. Draw a scale diagram of this arrangement and predict the length of the shadow of the nail.

   Set up the equipment and check your prediction.

B. Suppose the bulb is held 20 cm above the tabletop and 15 cm horizontally from the nail. Predict the length of the shadow that is cast on the table by the nail.

   Set up the equipment and check your prediction.

✔ Check your results with a staff member.

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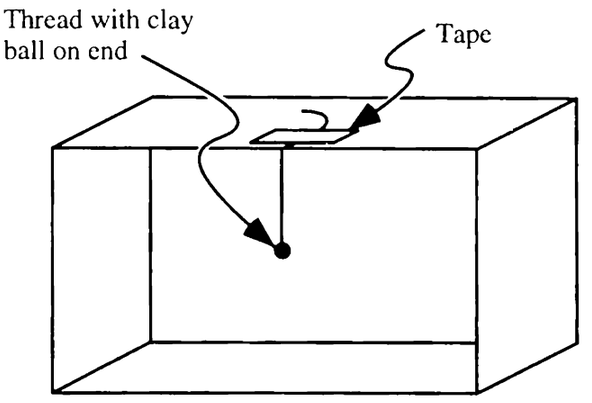
Experiment 4.3

In this experiment, you will make a shadow box. You will need clay, thread, tape, white paper, and a box with a removable lid (or flaps that can be folded out of the way or removed). If you are working near a window, turn your box so that the open side of the box faces away from the window.



Tape white paper to the back of the box to serve as a screen. Suspend a small ball of clay, about 1 cm in diameter, from thread as shown below.

This experiment should be performed in a darkened room.

A. Hold a small lighted bulb in front of the clay ball.

   Describe what you observe. Draw a diagram to help you account for your observation.

B. For each of the following situations, first predict how the shadow will change, then check your prediction. Use diagrams and the ideas developed thus far to explain your predictions and observations.

   (1) The clay ball is made larger.

   (2) The clay ball is made smaller.

   (3) The clay is formed into a triangular shape.

C. Sketch what you expect to see on the screen at the back of the box if two small lighted bulbs are held side-by-side in front of the ball.

   How do you expect what you see on the screen to change as the bulbs are moved closer together? Explain.

   Check your predictions and try to resolve any inconsistencies.

D. Discuss the following student's statement.

   "A shadow is formed when no light reaches a certain portion of a screen or wall."

✔ Discuss your results with a staff member.

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Experiment 4.4

Perform this experiment in a darkened room. You will need a long filament bulb and a shadow box with a clay ball about 1 cm in diameter.

A. *Predict* what you would see in the box if the long filament bulb were held vertically in front of the clay ball. Explain your reasoning.

   Check your prediction. Try to resolve any inconsistencies between your prediction and observation.

B. *Predict* how the shadow would change if the long filament bulb were brought closer to the clay ball. How would the shadow change if the bulb were moved farther from the clay ball?

   Check your predictions. Draw a diagram to help explain your observations.

C. Use your hand or a piece of dark paper to cover the top half of the long filament bulb.

   How does the shadow change? How can you account for your observations?

D. In Section 2, we treated the long filament bulb as though it consisted of a line of point sources. Explain how that idea can be used to explain your observations in this experiment.

Experiment 4.5

Perform this experiment in a darkened room. In this experiment, you will be making several predictions. If you find that a prediction is incorrect, try to resolve any inconsistencies before continuing to the next prediction.

A. *Predict* what you will see in your shadow box when two long filament bulbs are used to make a T-shaped light source. Use diagrams to explain your prediction.

   Check your prediction.

B. *Predict* what you will see in your shadow box when two long filament bulbs are used to make an L-shaped light source.

   Check your prediction.

C. *Predict* what you will see in your shadow box when the clay is made into a triangular shape (the light source is still L-shaped).

   Check your prediction. If your prediction is incorrect, try to resolve the inconsistency.

✔ Discuss your results with a staff member.

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Part B: Pigments and colored light

In Part B of this module, we conduct experiments with colored paints and colored light. We determine how to mix paints of different colors to obtain a particular color of paint and how to combine light of different colors to obtain a particular color of light. On the basis of our observations, we construct a model that helps us to predict the color an object will be when viewed under light of different colors.

Section 5. Pigments

Experiment 5.1

Obtain a large sheet of paper, a paintbrush, a container of water, and the following colors of paint: blue, cyan (turquoise), green, magenta (purplish-pink), red, and yellow. Before you begin the experiment, make sure that you can associate each color with its name.

A. Explore what happens when you mix two or more colors together. Record your results. Experiment with mixing the colors in various proportions. Continue to mix paints until you know the color that will result when you mix any two colors. (You may find it useful to make a table to record your results.)

B. Can you make a mixture of two of your original colors that is the same color as one of the other colors that you were given?

   Is it possible to reproduce each of your original colors by mixing two of the other colors? Make a list of those that you can reproduce and another list of those that you cannot.

C. How can you make gray or black from the colors you have been given?

   Can you make gray or black by mixing only two colors of paint? If so, how?

D. What is the smallest number of different paint colors that you would need to make all of the other colors that you have been given? Which colors could you use?

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Since the colors cyan, magenta, and yellow can be mixed to form a very wide range of colors, they are often called the *primary colors of pigment* or paint. When two primary colors are mixed in certain proportions, the resulting colors are called the *secondary colors* of pigment. In the following experiment we explore the properties of the primary and secondary colors. (*Note:* Artists often use a different set of colors as their primary colors. The difference between the two choices is discussed later in this section.)

Experiment 5.2

A. Fill in the blanks below to indicate the secondary colors that result when each pair of primary colors are mixed (in the correct proportions):

cyan + magenta = _____

cyan + yellow = _____

yellow + magenta = _____

B. What happens if you mix a primary color with a secondary color? Do you get a primary color, a secondary color, or a new color?

What happens if you mix a secondary color with a secondary color? Do you get a primary color, a secondary color, or a new color?

C. How can you make dark or muddy colors from the paints that you have been given?

How can you make light or bright colors from the paints that you have been given?

D. Which secondary color has no magenta in it? What color do you obtain if you mix that color with magenta?

The color you mixed with magenta is called the *complementary color* to magenta or the *complement* of magenta.

What is the complementary color to cyan? What color do you obtain if you mix cyan with its complementary color?

What is the complementary color to yellow? What color do you obtain if you mix yellow with its complement?

E. Artists usually call red, blue, and yellow the primary colors of paint. Discuss why cyan, magenta and yellow may be a better choice.

✔ Discuss your conclusions with a staff member.

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Early paints were made from natural materials. Shades of red, for example, are easy to make from iron oxides. However, not all colors occur readily in nature. Artists have chosen a set of primary colors that are readily available, yet can be mixed to form a wide range of colors. Many people regard these colors as a useful and acceptable set of primary colors. Yet not all colors can be made from red, blue, and yellow (e.g., magenta and cyan). For the purpose of this course, we will use magenta, cyan, and yellow as the primary colors for paints.

Experiment 5.3

Obtain several water soluble transparency markers of various colors (including green, black, and brown), some white paper towels, and a beaker with a few centimeters of water in it.

A. Cut one of the paper towels into strips that are 5–10 cm wide. Use the green marker to make a band of color across the width of one strip. The band should be about 1 cm wide and about 5 cm from the bottom of the strip. In pencil, write the name of the color at the top of the strip.

   Place the paper strip into the beaker so that the bottom end is under the water, but the colored band is above the water surface. Hang the top of the strip over the top of the beaker. Leave the strip in the water for several minutes and describe what you observe.

B. *Predict* what you would see if you were to repeat the experiment in part A for each of the different colored markers.

   You may want to begin the next experiment while waiting for the results of this experiment. However, remember to check your paper towels occasionally.

   Discuss your observations with your partner.

C. Do the results of parts A and B support the choice of magenta, cyan, and yellow as primary colors? Explain.

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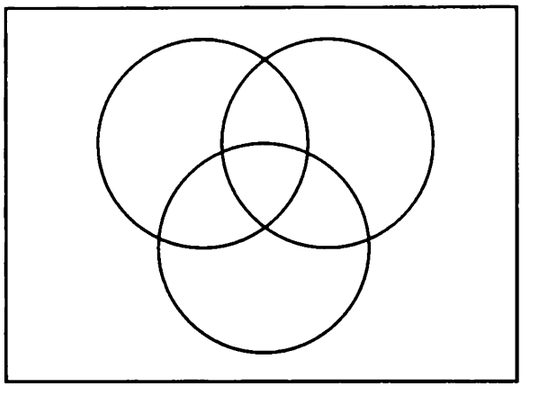
Exercise 5.4

Your observations of mixing paints can be summarized in the following diagram.

Imagine that you were to paint three overlapping circles each in a different primary color of paint. (See figure below.)



A. In each of the different regions of the diagram, write the name of the color of that region. You may wish to use colored pens or crayons to help you remember what each of the colors looks like.

B. Where on the diagram are the secondary colors?

   Where on the diagram are the complements of the primary colors?

   What color do you get when you mix two complementary colors of paint? Do you get the same color for all three pairs of complementary colors? Can these mixtures be represented by a unique color and area on the diagram above?

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Section 6. Colored light

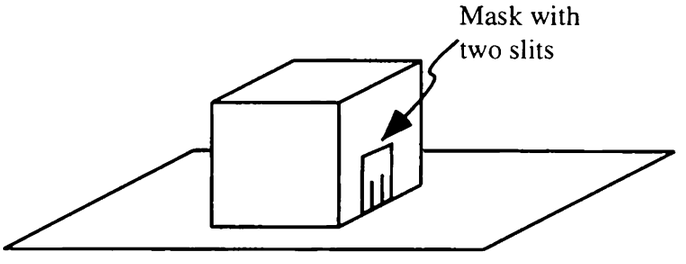
In Section 5 we examined how colored pigments combine to make other colors. In this section and the next we develop a way of thinking about light that allows us to account for our observations in terms of the light that reaches our eyes. We begin by making observations of colored beams of light.

Experiment 6.1

This experiment should be performed in a darkened room.

You can make a colored beam of light by placing a colored acetate (a thin sheet of plastic) into a beam of light. For this experiment, you will need the following acetates: blue, cyan, green, magenta, red, and yellow.

Obtain a mirror, a large piece of white paper, and a "light box." The light box should have a bright light bulb inside and an opening on the side over which masks can be placed. For this experiment, you will need masks that have two and three slits in them. (See a staff member for more detailed instructions on making a light box.)



A. Place the light box on the large sheet of paper. Use a mask with two slits. The two beams from the mask should illuminate the paper. Place a colored acetate in the path of one of the beams. Describe your observations.

   How does the appearance of a lit region of the paper close to the light box compare to a lit region far from the light box? Describe the similarities and the differences.

B. Place an acetate of a different color from the first in front of the second beam. Use a mirror to make the two beams cross. Describe the color you see on the paper where the beams meet.

   How can you mix "a little" red light with "a lot of" green light? Describe your method and the resulting color mix. What happens if you mix "a little" green light with "a lot of" red light?

   Describe how you can get a range of colors using just the two colored beams.

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C. Repeat part B using all possible combinations of your acetates. Record your observations. As you work, try to answer the following questions:

Can you make two colored beams cross to form a color that is the same as one of the six acetates that you were given? (Use a mask with three slits so that you can compare the color that you produce with the color that you are trying to match.)

Are there any colors of light that seem to be primary in the same sense that certain colors were primary with paint? Explain.

What are the primary colors for light?

Are the *primary colors of light* the same as the primary colors of paint?

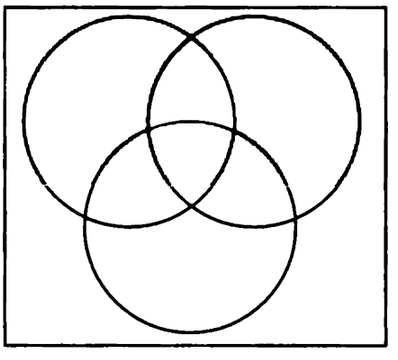
What are the *secondary colors of light?* Are the secondary colors of light the same as the secondary colors of paint?

What color do you obtain if you mix all the primary colors of light?

What color do you obtain when you combine all the secondary colors of light?

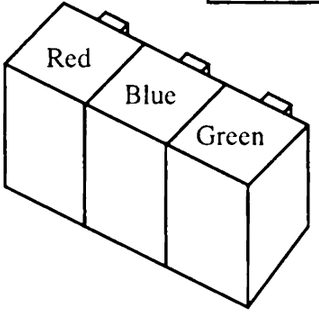
✔ Discuss the results of this experiment with a staff member.

Exercise 6.2



Three light sources project light onto a screen as shown. Each source projects a different circular region of color.

A. Label each part of the screen with the appropriate color. (Use colored pens or crayons to help you remember what the colors look like.)



B. The color that results when two primary colors of light are mixed is called the *complement* of the remaining primary color.

Identify the *complementary* color for each primary color of light.

What color do you obtain when you mix two complementary colors of light? Do you obtain the same result no matter which pair you mix? Can this be represented by a unique color and area on the diagram?

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Exercise 6.3

Describe some of the differences between mixing colors of light and mixing colors of paint. (*Hint:* Compare the results of Exercise 5.4 and Exercise 6.2.)

Experiment 6.4

This experiment should be performed in a darkened room.

Obtain a prism or a diffraction grating from a staff member. Place the prism or grating into a beam from your light box. Vary the orientation until you see a broad rainbow of colors. The arrangement of colors is called the *spectrum* of colors.

A. Does each of the primary colors of light appear in the spectrum?

   Does each of the secondary colors of light appear in the spectrum?

B. One at a time, place each of the acetates between the screen and the prism (or diffraction grating). Start with acetates that correspond to primary colors of light, then try the others. Record your observations.

   Describe how what you see on the screen *changes* when an acetate is placed into the beam. Does it matter whether the acetate is placed in front of the prism (or diffraction grating) or behind it?

C. What seems to be the function of the acetate pieces: do they seem to "add" color to light or do they "take away" color from light? Explain your reasoning.

   What seems to be the function of the prism or the diffraction grating: does it separate light into different colors, or does it add colors to the light? Explain. How can you distinguish between these possibilities using a diffraction grating and colored acetates?

✔ Explain your reasoning to a staff member.

Experiment 6.5

This experiment should be performed in a darkened room.

Put your light box on a large sheet of white paper. For this experiment, use a mask with only a single slit.

A. Place a green acetate into the beam from your light box to produce a beam of green light.

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(1) Predict what you would see on the screen if you were to place a red acetate into the beam of green light. Explain.

Predict what you would see on the screen if you were to place a green acetate into a beam of red light. Explain.

Perform the experiments and check your predictions. Try to account for any discrepancies between what you predicted and what you observe.

(2) Predict what you would see on the screen if you were to place a magenta acetate into the beam of green light. Explain.

Predict what you would see on the screen if you were to place a green acetate into a beam of magenta light. Explain.

Perform the experiments and check your predictions. Try to account for any discrepancies between what you predicted and what you observe.

B. Repeat the experiment describe in part A with acetates of various colors. Record your results. In each case, make a prediction before performing the experiment. Try to account for your observations in terms of the role of the acetate pieces and your knowledge of how colored lights combine to form new colors.

When you have finished, explain how you can predict the color that will result when two acetates are placed in a beam one after the other.

C. In this experiment and in Experiment 6.1, you used pairs of acetates to produce colored light. The rules for predicting the color that results is different in the two methods. Compare the rules for the two methods and describe how they are related.

✔ Explain your reasoning to a staff member.

Experiment 6.6

This experiment should be performed in a darkened room.

Use two small bulbs to produce two shadows of a clay ball suspended in a shadow box (see Experiment 4.3).

Predict what you would see on the screen if a red acetate were placed over one bulb and a green acetate were placed over the other bulb.

What would you see if the two shadows were distinct (i.e., do not overlap)? Explain.

What would you see if the two shadows were to "overlap?" Explain.

Check your predictions. If your predictions were incorrect, try to resolve any inconsistencies.

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Section 7. Pigments under colored lights

Experiment 7.1

Review Experiment 1.2 before starting this experiment.

Cut six large squares from a large sheet of black paper. Paint each square a different color using the paints from Experiment 5.1.

Obtain red, blue, and green acetate sheets, a light box, and a mask with a single slit.

A. Place a green acetate into the beam from your light box. Arrange the three squares corresponding to the *primary* colors of *light* so that the beam falls on all three at the same time. Describe your observations.

Which colors of pigment reflect green light most strongly?

Repeat the experiment with red and blue acetates. Record your observations.

When a particular color of light is only weakly reflected from a particular color of pigment, what do you suppose has happened to the light falling on that pigment? Explain your reasoning.

Make a table that shows how strongly each of the primary colors of light is reflected by pigments corresponding to primary colors of light.

What pattern do you observe in your data?

B. Place a green acetate into the beam from your light box. Arrange the three squares corresponding to the *secondary* colors of *light* so that the beam falls on all three at the same time. Describe your observations?

Which colors of pigment reflect green light most strongly?

Repeat the experiment with red and blue acetates. Record your observations.

Make a table that shows how strongly each of the primary colors of light is reflected by pigments corresponding to secondary colors of light.

What pattern do you observe in your data in the second table?

C. Summarize your findings in terms of a simple rule for predicting the color you will see when a primary color of light falls on a given pigment.

Use your rule as a basis for predicting the color you would see in the following cases. Explain your reasoning, then check your answers.

- yellow pigment is illuminated by cyan light
- magenta paint is illuminated with green light

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Exercise 7.2

A. A student mixes cyan and yellow paint together and illuminates the mixture with the beam from a flashlight (without an acetate in the beam). The room is otherwise dark.

   Use the rule you devised in part C to predict the color you would see. Is your prediction consistent with your observations in Experiment 5.1?

   Check your prediction.

B. Use your model for how paints appear under colored lights to explain why paints tend to become darker as more pigments are added.

C. Suppose that a black piece of paper and a white piece of paper are placed under green light.

   Predict the apparent color of each sheet of paper.

   Check your answer by shining a beam of green light simultaneously across both black and white pieces of paper. Describe your observations.

   Repeat the experiment for red and blue light.

D. The light from an ordinary bulb is sometimes called *white light.*

   Explain why this term is appropriate. Base your answer on your observations of how white paper appears under various colored lights.

E. Earlier in this module, you developed the idea that an observer can see an object because light goes from the object to the observer's eyes. Modify this statement to account for why a blue object appears blue or a yellow object appears yellow.

✔ Discuss this exercise and the results of Experiment 7.1 with a staff member.

Exercise 7.3

How could you write a message with crayons so that the message is "hidden" and will appear only when illuminated by light containing a particular color?

How could you write a message that would appear as *two* different messages when illuminated by two different colored light beams?

How could you write a hidden message that would appear as *three* different messages when illuminated by three different colored light beams?

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Experiment 7.4

For this experiment you will need a magnifying glass and color pictures in a magazine or book. Use the magnifying glass to look closely at several pictures. Describe what you observe.

Color pictures are often printed using a technique called a *four-color process,* in which many small colored dots are used to form a picture. Find a picture that was printed using the four-color process.

What four colors were used to make your picture? Explain how a picture that contains only four colors can appear to have many different colors.

You have seen that the way in which colored pigments combine to form new colors is different from the way in which colored lights combine to form new colors.

Do the colored dots combine to form new colors in the way that pigments combine or in the way that colored lights combine?

Which color or colors of dots should there be in a yellow region?

What color or colors of dots should there be in a blue region?

Experiment 7.5

For this experiment you will need your light box, colored paper, and acetates that correspond to the primary colors of light.

Shine light beams of the three primary colors on sheets of paper of different colors. Compare the results of this experiment with the results obtained in Experiment 7.1.

Discuss your observations with your partner. Discuss how you could use your observations to predict how to mix paints to match the colors of the paper. Explain your reasoning.

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Experiment 7.6

Read this experiment and then tell a staff member that you are ready to see the demonstration.

In this experiment you will view colored paper strips under three different colors of light. On the basis of what you observe, you are to predict the color of the strips when viewed under ordinary (white) light.

A. Go to a darkened room in which overhead projectors are set up with red, blue, and green acetates. In the room, there should be paper strips numbered from one through nine. Place each strip in turn under one of the three colored lights. Record the color that you observe for each strip.

   Repeat the procedure above for the other two colors of light. Record your observations.

B. On the basis of your observations, predict the color of each strip when viewed under room lights. After you have made a prediction for each strip, take the colored strips into a lit room and check your answers.

   Which of your original observations under the colored light were closest to the colors you now observe for the strips?

   If some of your predictions were incorrect, try to resolve the inconsistencies.

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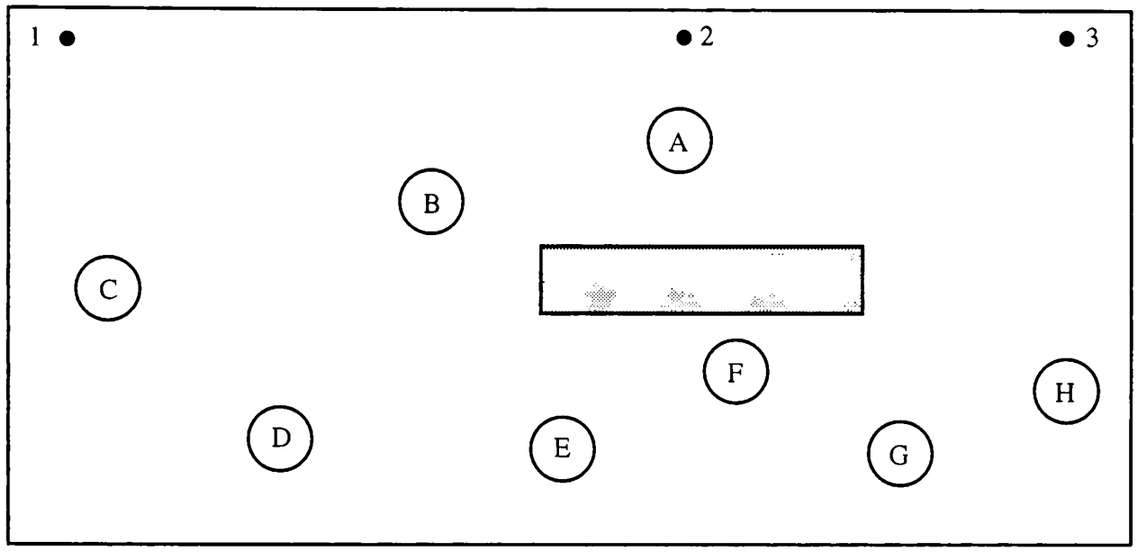
Supplementary problems for *Light and Color*

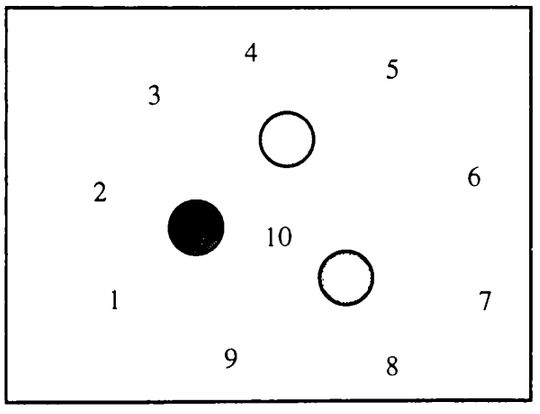
Problem 1.1

The diagram below is a top view diagram of a room containing eight pillars (A–H), three observers (1, 2, and 3) and a wall. Are all of the pillars visible to each observer? For each observer, draw a diagram that shows what the observer would see. Label the objects in your diagram. Explain your reasoning using the ideas developed in Section 1.



Problem 1.2

A student was standing in a room containing three pillars. The diagram below left is a top view diagram of the room. The student made a *rough* sketch of her view of the pillars from where she was standing. Her sketch is below right. On which numbered spot was she standing to see this view of the pillars? Explain how you can tell.



Top view diagram of room

Sketch

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Problem 1.3

Imagine you are in a totally darkened room, with no windows or cracks to let in light. Describe what you would be able to see in the room.

Would you be able to see your hand if you held it at arm's length away from you? Would you be able to see it if you held it right up in front of your face?

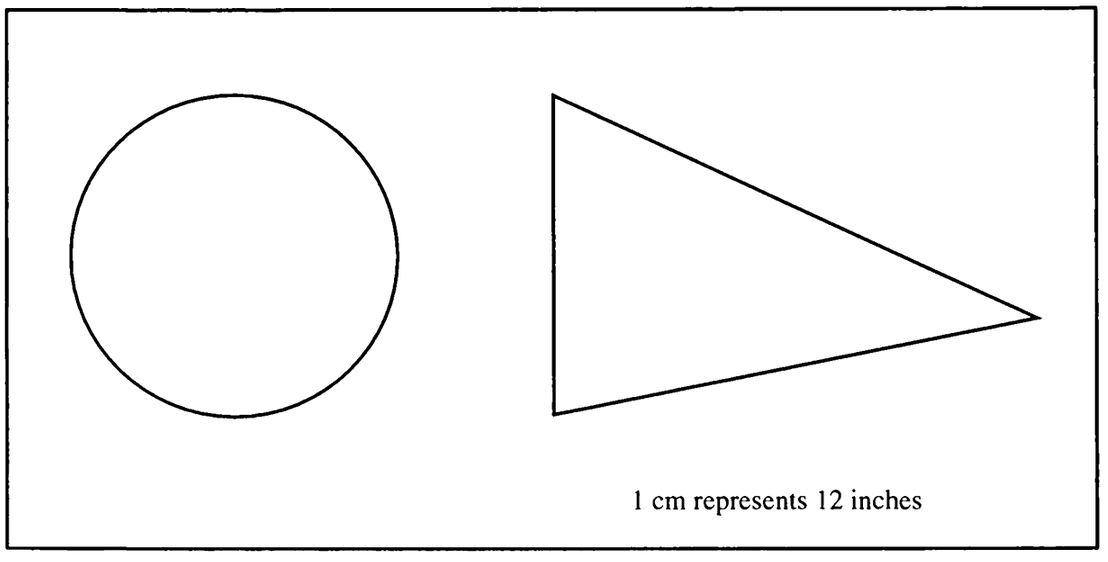
Explain your reasoning.

Problem 2.1

Draw a top view diagram of your classroom.

Problem 2.2

The scale diagram below is drawn so that 1 cm represents 12 inches.



A. What is the diameter of the actual circle?

B. What is the length of each side of the actual triangle?

C. What is the area of the actual triangle?

D. What are the measures of the angles inside the actual triangle?

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Problem 2.3

A. Draw both a top view diagram and a side view diagram of an upright soda can on a table.

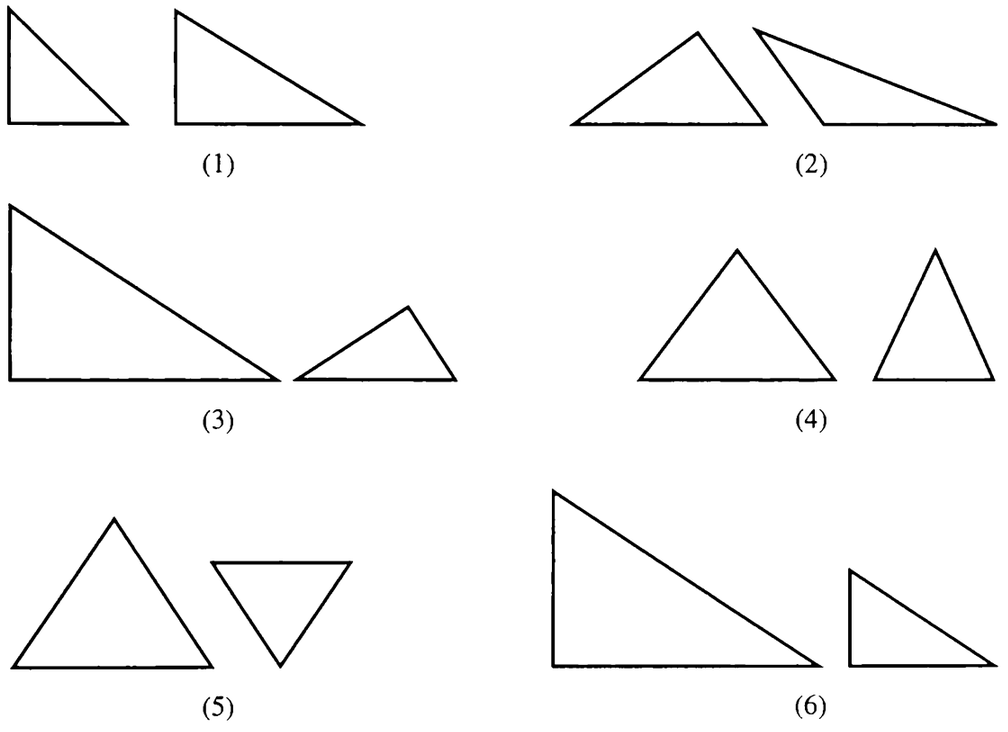
B. Draw a top view diagram of the apparatus used in part A of Experiment 2.7.

Problem 2.4

A. What is the relationship between corresponding angles in similar triangles?

What is the relationship between corresponding sides in similar triangles?

B. Which of the following pairs of triangles are similar triangles? Explain your reasoning. You may use a ruler or protractor to help you answer these questions.



C. Was it necessary in part B to have both a ruler *and* a protractor? Would either one have been sufficient? Explain your reasoning.

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Problem 2.5

There are two poles on a level street that are 25 m apart. One pole is 10 m tall; the other, 15 m tall. (Assume your eyes are 2 m above ground level.)

A. Where on the street, if anywhere, could you stand so that:

(1) the tops of the poles appear level to you.

(2) the shorter pole completely blocks your view of the taller pole.

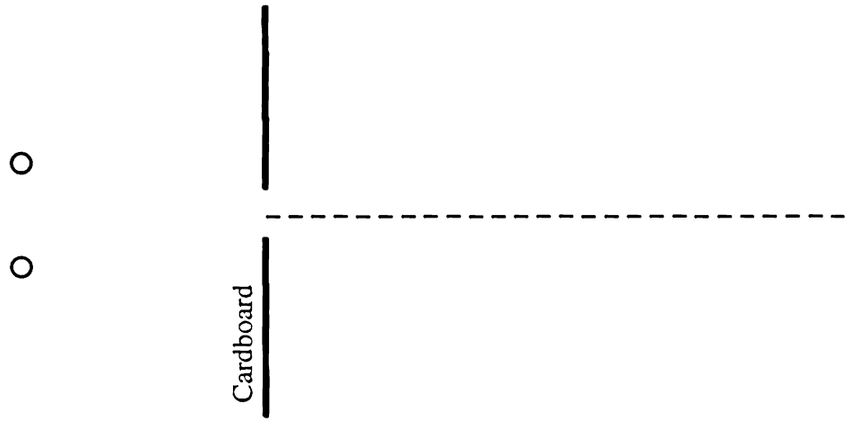
(3) the taller pole completely blocks your view of the shorter pole.

Draw a side view scale diagram that represents this situation, and use your scale diagram to answer these questions. Choose your scale so that your diagram and answers fill a piece of paper.

B. Using your scale diagram, determine how far your eyes are from the top of the taller pole if your feet are 15 m from the bottom of the pole.

Problem 2.6

Two small lighted bulbs and a piece of cardboard with a circular hole in it are arranged along a line as shown in the cross-sectional top view scale diagram below.



A. Where along the dashed line could a screen be placed so that the bright areas formed on the screen by the bulbs are distinct (i.e., do not overlap at all)? If the bright areas on the screen are distinct for a range of screen locations, indicate the range clearly on the diagram. Explain and draw a diagram to justify your answer.

B. Where along the dashed line could the screen be placed so that the bright areas formed on the screen by the bulbs overlap (at least partially)? If the bright areas on the screen overlap for a range of screen locations, indicate the range clearly on the diagram. Draw a diagram to justify your answer.

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Problem 2.7

A mask with a circular hole is placed between a small lighted bulb and a screen. How would each of the following changes affect the size of the bright region on the screen? In each case, explain your reasoning, and draw sketches to support your answer.

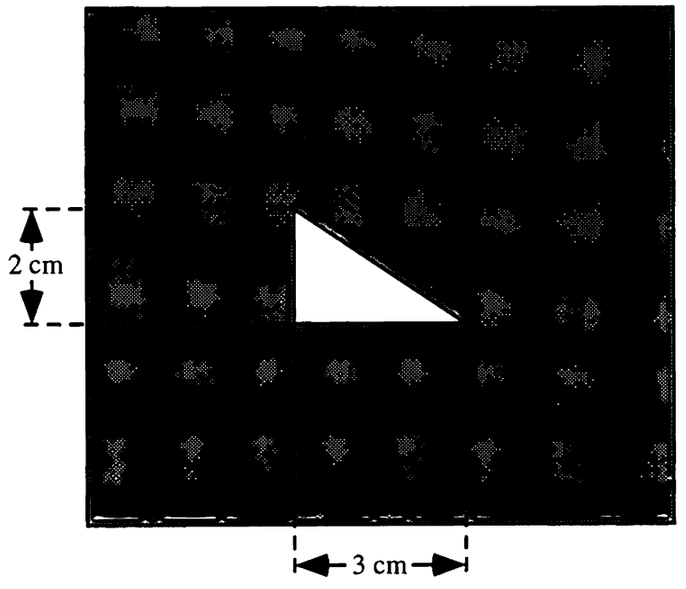
A. The screen is moved closer to the mask (without moving the bulb or mask).

B. The mask is moved closer to the screen (without moving the screen or bulb).

C. The bulb is moved farther from the mask and screen (without moving the mask or screen).

Problem 2.8

A small bulb is placed in front of the mask shown below. The bulb is level with the bottom of the hole in the mask. When viewed from the top, the bulb is aligned with the left side of the hole in the mask. A screen is placed 20 cm behind the mask. The bright region on the screen is 15 cm tall.



A. Draw a picture of the bright region that you would see on the screen.

B. Determine the width of the bright region at the widest part. Explain your reasoning in each step.

C. How far is the bulb from the mask? Explain your reasoning.

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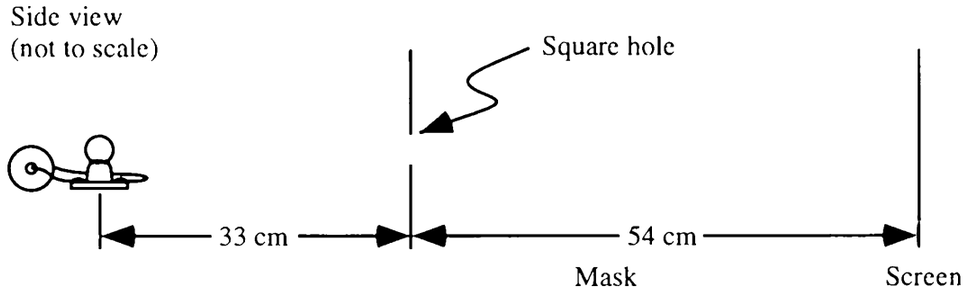
Problem 2.9

A mask with a square hole is placed between a screen and a very small light bulb as shown in the cross-sectional side view below. The bright area on the screen is 9.8 cm tall. Determine the height of the square hole in the mask. Show your work.

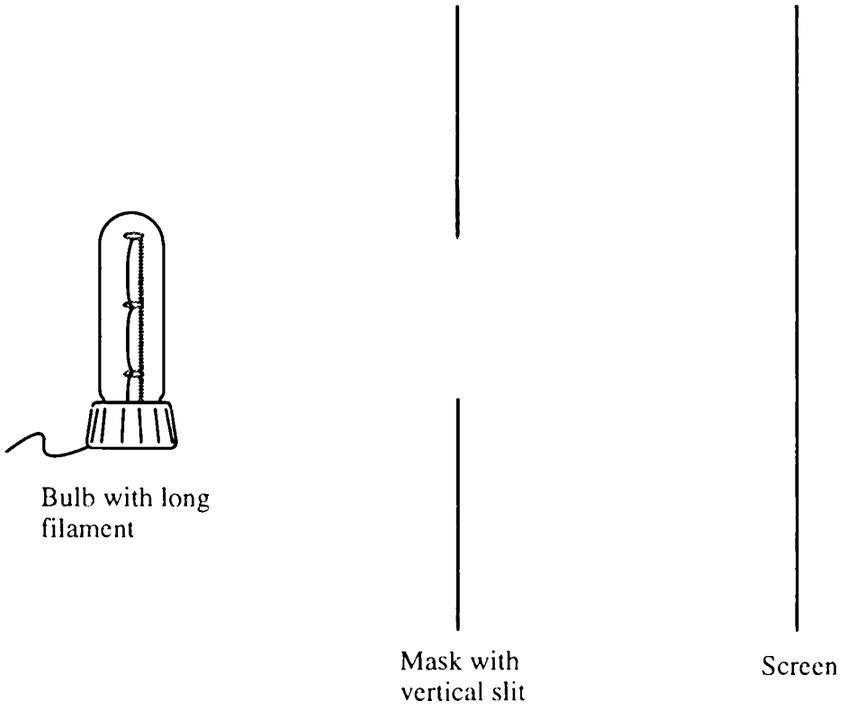


Problem 2.10

A light bulb with a long filament is held vertically in front of a vertical slit in a piece of cardboard. A cross-sectional side view is shown in the scale diagram below. (The length of the slit is the same as the length of the filament, and the filament and screen are equidistant from the piece of cardboard.)

A. Describe the shape of the bright area on the screen on the other side of the slit.

B. Will the bright area be taller than, shorter than, or the same height as the filament? Explain your reasoning.



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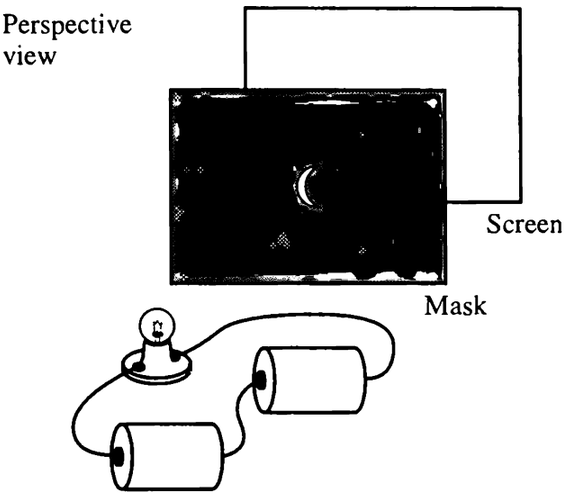
Problem 2.11

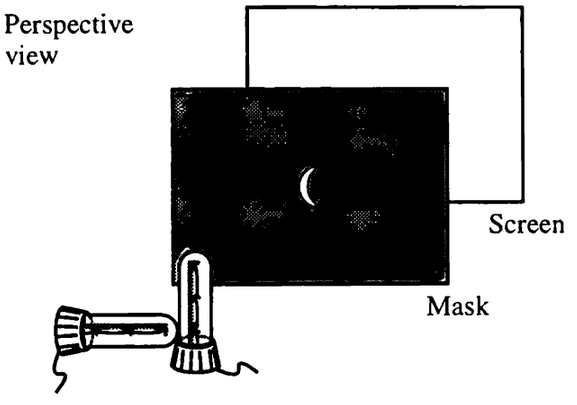
A. A mask that contains a small crescent-shaped hole is placed between a small bulb and a screen as shown at right.

Sketch what you would see on the screen when the bulb is turned on. Explain.



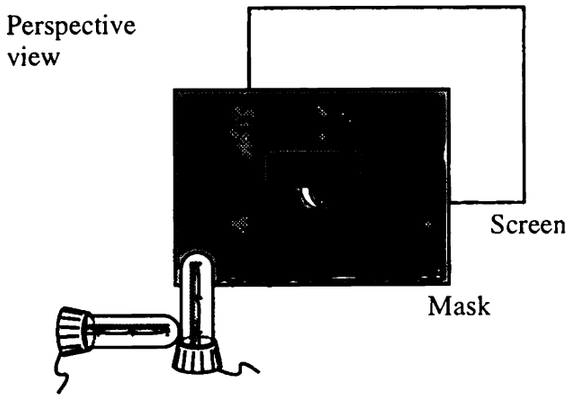
B. The small bulb is replaced by two long filament light bulbs that form a J-shaped light source.

Sketch what you would see on the screen when the bulbs are turned on. Explain your reasoning in words.



C. Suppose that the *top half* of the hole were covered as shown in the diagram.

Sketch what you would see on the screen, and explain your reasoning.



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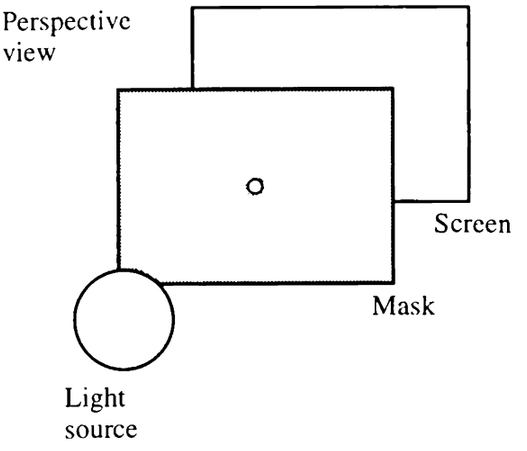
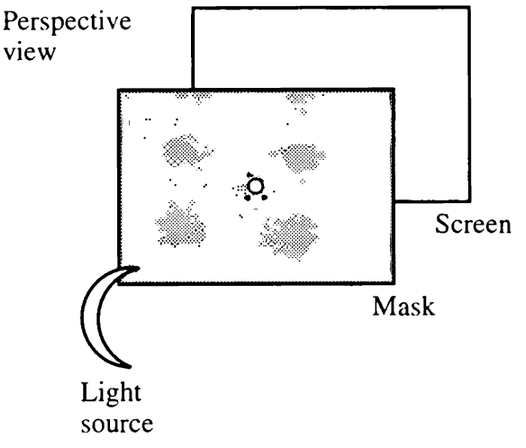
Problem 2.12

A student on vacation in the tropics is preparing for an eclipse of the sun. Instead of looking directly at the sun, she is going to use a mask with a small hole in it and a piece of paper for a screen.

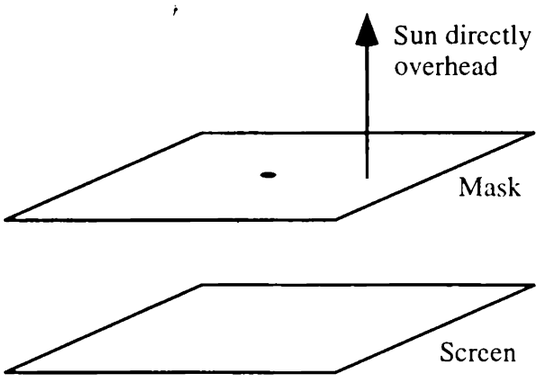


A. She wants to know what to expect before the eclipse occurs, so she sets up a round light source, the mask and screen as shown at right.

   Sketch what she would see on the screen when the light source is turned on. Assume the hole is very small. Explain your reasoning.



B. The student then blocks part of the light to form a crescent (one of the shapes of the sun during an eclipse). Sketch what she would see on the screen in this case. Explain.



C. The student sets up the mask and screen as shown at right. The sun is directly overhead and is not eclipsed. She finds that the width of the lit portion on the screen is 1.0 cm and the distance from the screen to the mask is 1.0 m.

   Calculate the ratio of the sun's diameter to the distance from the mask to the sun. Include a diagram in your answer. Show your work.

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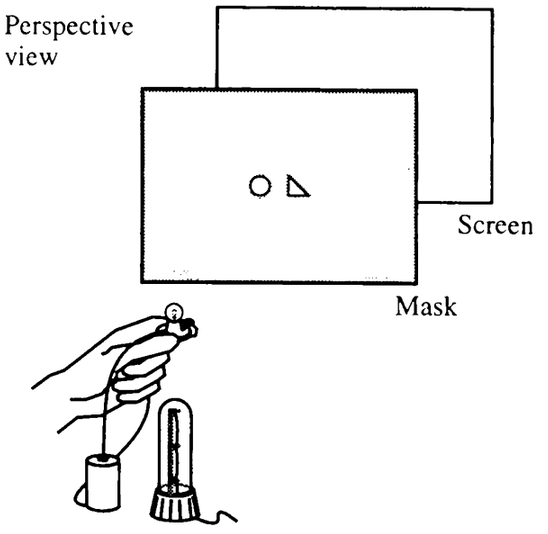
Problem 2.13

A small light bulb and a long filament bulb are arranged as shown in the perspective diagram at right. A mask with a small circular hole and a small triangular hole is placed between a screen and the bulbs, as shown.

Sketch what you would see on the screen when both bulbs are lit. Explain your reasoning.

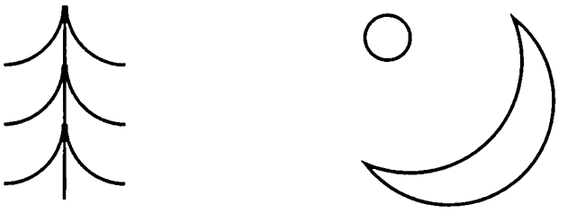


Problem 3.1

You are using your pinhole camera to view pictures that your partner has projected on the wall.

The diagrams at right show two of the pictures that you see in your pinhole camera. (The camera itself is not shown.)

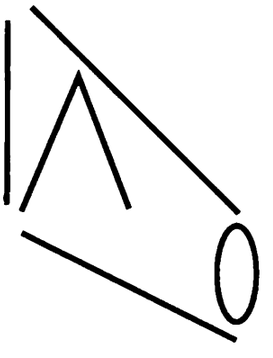
For each diagram, draw a picture of what you would see projected on the wall if you were to turn around.



Problem 3.2

Suppose that you are using a pinhole camera to view a picture being projected on the wall.

You look in the pinhole camera and the design to the right is what you see inside the camera. (The camera itself is not shown.)



A. Describe what feature of the picture on the wall behind you is over your left shoulder. What is over your right shoulder? What is below your left shoulder? What is below your right shoulder?

B. Draw a picture of what you will see when you turn around.

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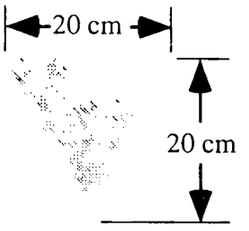
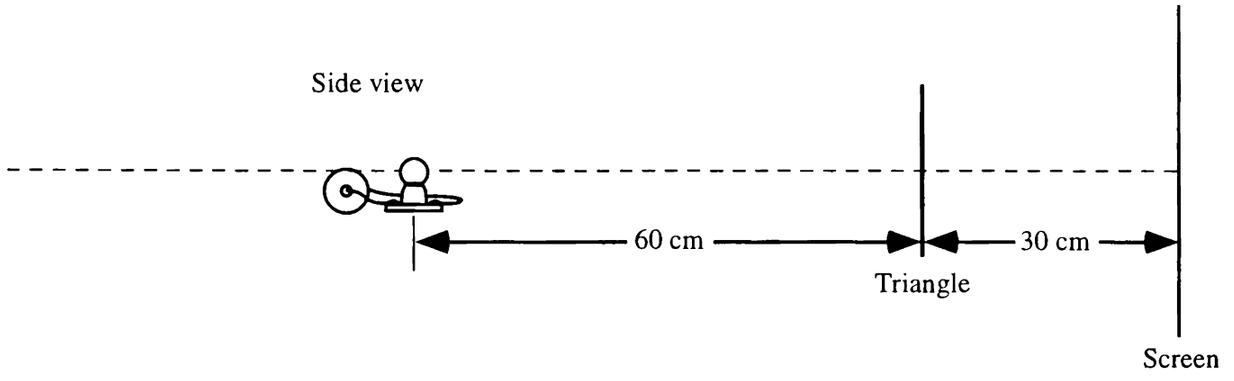
Problem 4.1

A piece of cardboard has been cut out in the shape of a triangle. The dimensions of the triangle are shown at right.



A. Predict the size and shape of the shadow that would be formed on the screen if a small lighted bulb, the cardboard triangle and a screen are arranged along a line as shown. (A line through the bulb and the center of the triangle is perpendicular to the screen.) Explain your reasoning.



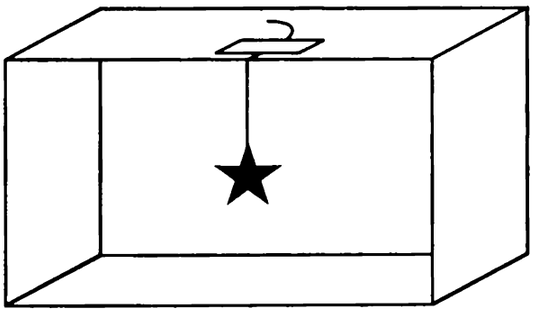
B. Is it possible to place the bulb in another location along the dashed line so that the shadow of the triangle is twice as tall as in part A? If so, where? If not, why not?

C. Is it possible to place the bulb in another location along the dashed line so that the shadow is half as tall as in part A? If so, where? If not, why not?

D. Suppose that the bulb were placed along the line very far away from the triangle and the screen. What would be the approximate shape and size of the shadow? Explain. Describe the light reaching the screen in this case.

Problem 4.2

A small star-shaped cutout is suspended in a shadow box as shown at right.



A. Sketch the shadow that would be cast by a small bulb held in front of the cutout. Explain the reasoning that you used to determine the shape of the shadow.

B. Sketch the shadow that would be cast by a T-shaped light source held in front of the cutout. Explain the reasoning that you used to determine the shape of the shadow.

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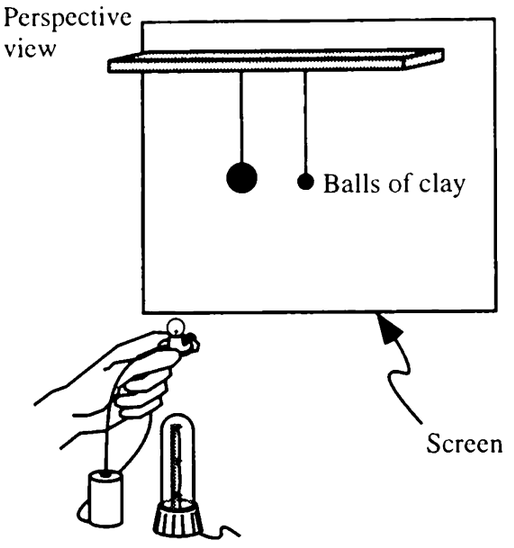
Problem 4.3

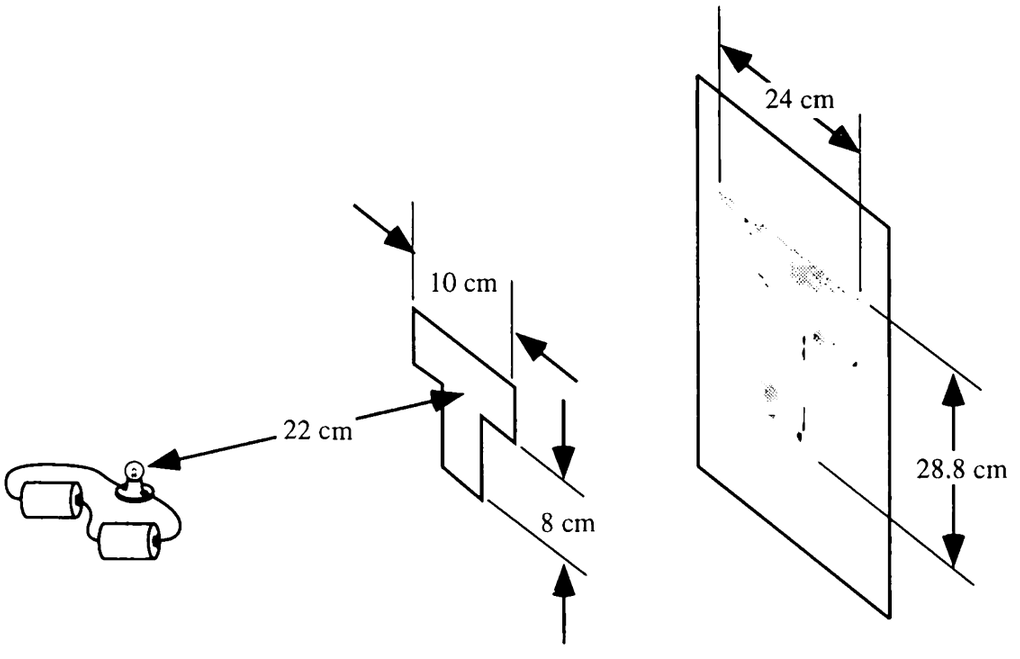
A small light bulb and a long filament bulb are arranged as shown in the perspective diagram at right. Two circular clay balls, *one with twice the diameter of the other,* are placed between the bulbs and a screen, as shown in the diagram at right.

Sketch what you would see on the screen when the bulb is turned on. Explain your reasoning in words.



Problem 4.4

A cardboard letter *T* is placed 22 cm away from a small light source. The cross piece of the *T* is 10 cm wide and the length of the stem is 8.0 cm. A shadow is cast on a screen that is perpendicular to the line joining the light source with the *T*. The width of the cross piece on the shadow is 24 cm and the total height of the shadow (including stem and cross piece) is 28.8 cm.



A. What is the vertical height of the cross piece on the cardboard letter *T*? Explain the reasoning for each step in your solution.

B. How far away from the light source would the screen have to be so that the total height of the shadow would be 1 meter? Again, explain your reasoning.

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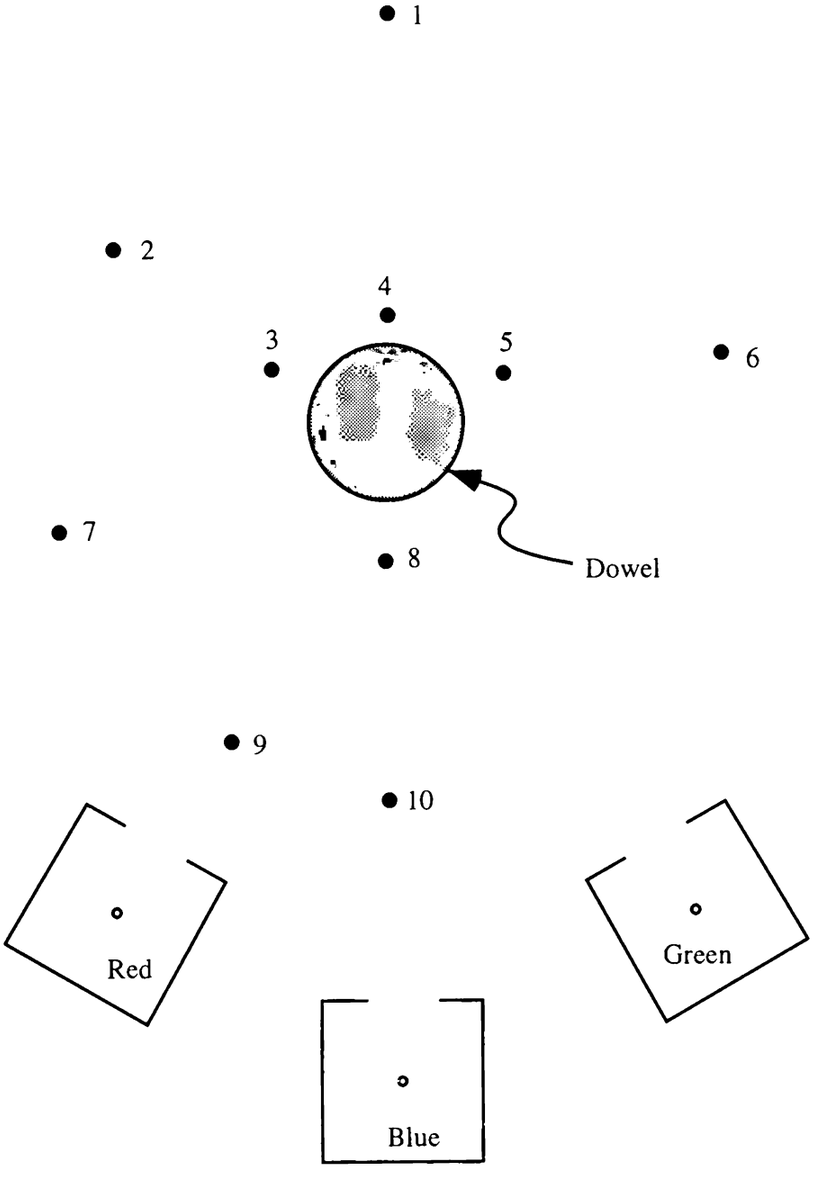
Problem 6.1

Three light boxes are placed on a piece of white paper on top of a table. One of the light boxes produces a red beam, another blue, and a third green as shown. The beams are aimed toward a dowel. The beam of light from each light box falls on both the dowel and on the white paper on the table. (You may treat the light bulbs as point sources of light.)

At each of the labeled points, what color would you see on the paper? Explain your reasoning.



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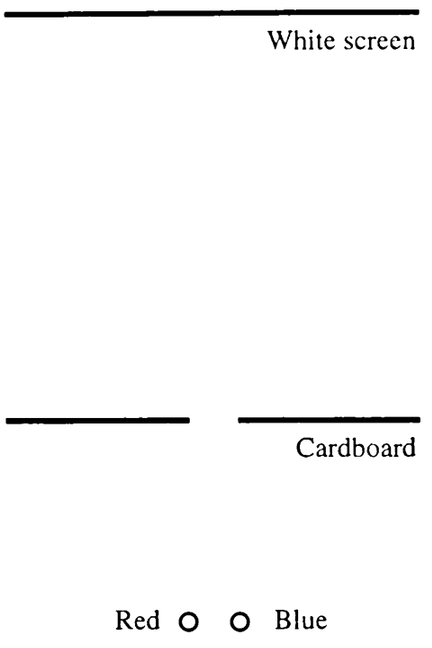
Problem 6.2

A. Two small lighted bulbs, a piece of cardboard with a circular hole in it, and a screen are arranged as shown in the cross-sectional top view diagram at right. There is a red acetate over the bulb on the left, and a blue acetate over the bulb on the right, as indicated.



What would you see on the screen? Sketch your prediction, including what color each part of the screen would appear. Explain your reasoning.

B. Suppose that the cardboard screen were replaced by a small ball of clay the same diameter as the hole in the cardboard. What would you see on the screen in this case? Sketch your prediction including what color each part of the screen would appear. Explain.

Problem 6.3

Black has been referred to as "the presence of all colors" as well as "the absence of all colors." The same has been said of white. For both black and white, describe a situation in which each of these "names" is proper. Explain your reasoning.

Problem 7.1

A student writes the word *red* on a white sheet of paper using a red crayon. The student then writes *cyan* next to this using a cyan crayon.

A. Predict what the student would see if a red acetate were placed on top of the writing. Your prediction should include what word or words are visible, the color(s) that any writing that the student can see would appear and the color of the background. Explain your reasoning.

B. Predict what the student would see if a cyan acetate were placed on top of the writing. Explain your reasoning.

Problem 7.2

The title of a book is printed in red ink on a white background. What would you see if you looked at the cover of the book through a red acetate? a green acetate? Explain your reasoning.

What if the red title were on a black background? How, if at all, would this change what you saw when you looked at the cover through each of the acetates? Explain your reasoning.

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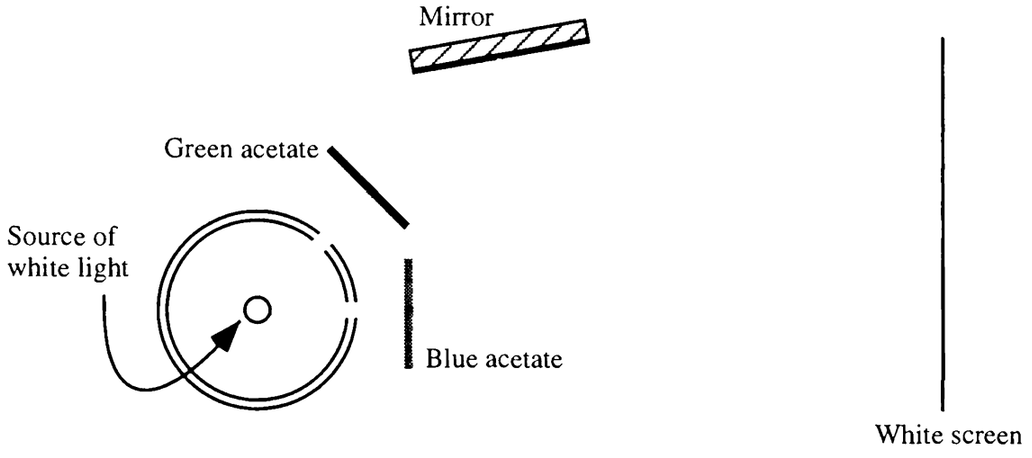
Problem 7.3

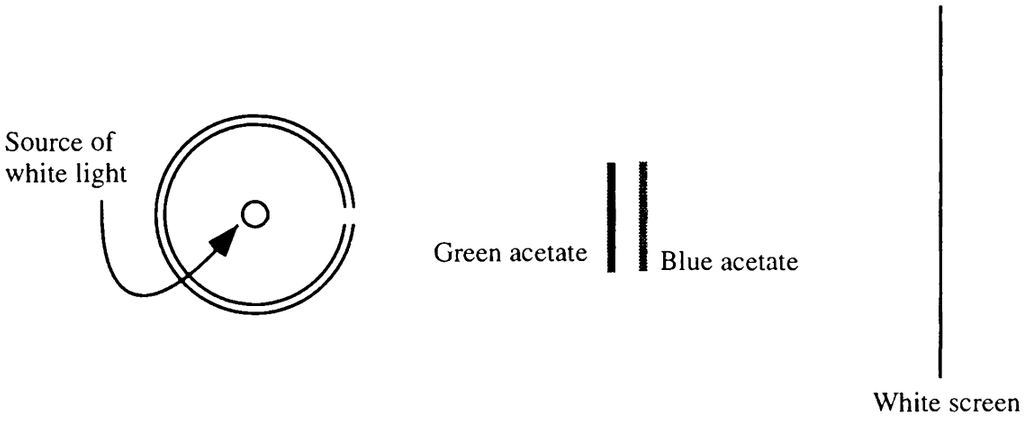
Two slits are placed in front of a source of white light as shown in the diagram below. A green acetate is placed in front of one of the slits and a blue acetate is placed in front of the other. A mirror is used to aim one of the beams so that both beams strike a white screen at the same point. (You may assume that the acetates are "perfect acetates.")



A. What color will you see on the white screen where the two beams are aimed? Explain.

B. If the white screen were replaced with a piece of yellow construction paper, what color would you see where the two beams of light are aimed? Explain.

C. Now suppose that a single slit is placed in front of the light source as shown below and that a green acetate is placed in front of the slit, followed by a blue acetate. What color will the white screen appear now? Does the order of the acetates matter? Explain.



D. If the green acetate in part C is replaced with a cyan acetate so that there is now a cyan acetate followed by a blue acetate, what color will you see on the white screen? Explain.

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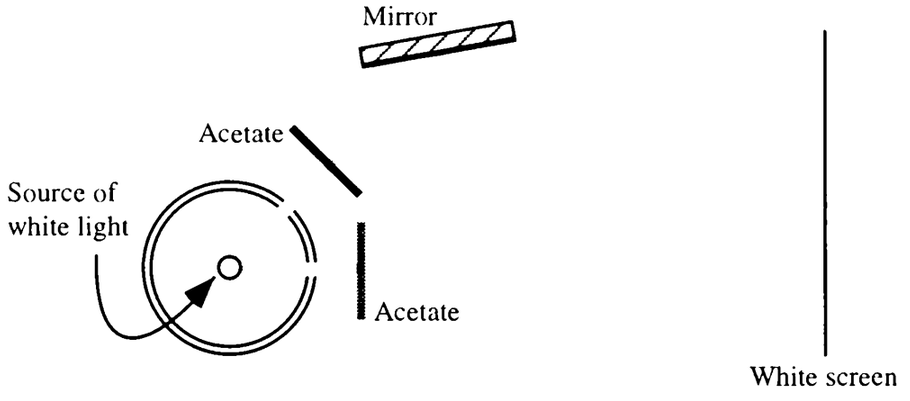
Problem 7.4

The diagram below shows a white light source with two slits. There is one colored acetate in front of each slit. A mirror is used to deflect one of the beams so that both beams strike the white screen at the same point.



A. If one of the acetates is yellow and the other is cyan, what color will you see on the white screen? Explain.

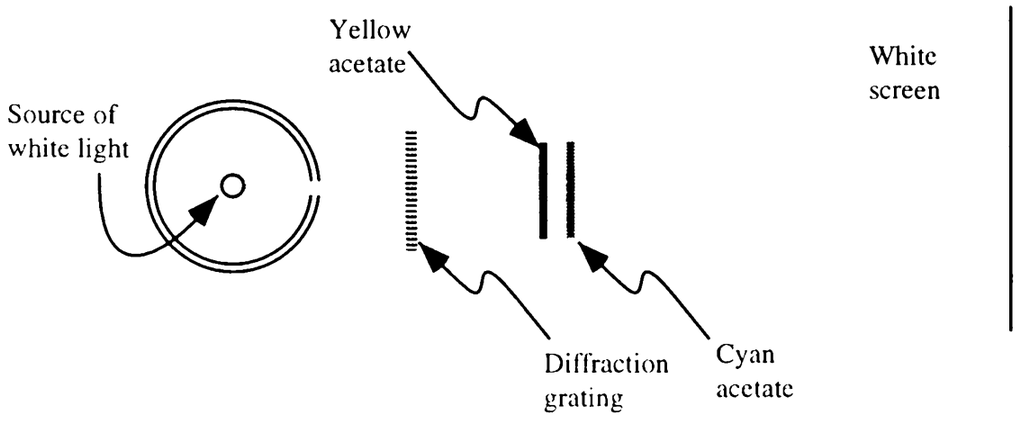
B. If one of the acetates is green and the other is cyan, what color would you see on the white screen? Explain.

C. The acetates are replaced by two other acetates, and the color observed on the screen is magenta. What are the colors of the two acetates in this case? Explain. If there is more than one possibility, explain all of them.

D. Now suppose that acetates are changed again so that a red color is seen on the white screen. What would be the colors of the acetates used this time? Explain. If there is more than one possibility, explain all of them.

Problem 7.5

Suppose that a diffraction grating is placed in front of a slit to produce a band of colors. What will you see on a white screen if a yellow acetate and a cyan acetate are inserted as shown below? Explain.



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PHYSICS BY INQUIRY

Magnets

In *Magnets,* we investigate a particular type of physical interaction. From our observations, we develop a simple model that enables us to account for the behavior of magnets and magnetic materials.

Part A: Behavior of magnets

In Part A, we develop an operational definition for a magnet. We examine the different parts of a magnet, compare the strengths of different magnets, and explore the effects of breaking and combining magnets.

Section 1. Magnetic interactions

Experiment 1.1

Obtain two magnets and a set of objects made of various materials.

A. Bring the two magnets near each other and examine the interaction between them. Record your observations.

Do the magnets need to be in contact to interact? If not, how does the distance between the magnets affect the interaction?

How do the different parts of the magnets interact?

B. Bring various objects from your set near one another and near a magnet from part A. Describe what you observe and record the results in your notebook.

How do the different parts of a magnet interact with the various objects?

C. Arrange the objects into at least three different classes based on their interactions with the two magnets and with each other. Test other objects as well (for example, coins, pencils, paper, paper clips).

How is one class distinguished from another?

For each class, describe how its members interact with:

- other members of the same class
- members of each of the other classes

Do all the metallic objects interact in the same way?

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D. In your notebook make a table like the one below to record your observations from parts A–C. Use a word or phrase to describe the interactions.

Table of interactions

	Class 1	Class 2	Class 3
Class 1			
Class 2			
Class 3			

One of the classes in the preceding experiment consists of objects that interact with one another and with other objects in the same way as the two magnets you were given. We call the objects in this class *permanent magnets,* or simply *magnets.* The interactions you studied in Experiment 1.1 are called *magnetic interactions.*

Exercise 1.2

Formulate an operational definition for a *magnet.* (See Section 5 of *Properties of Matter* for what is meant by an operational definition.) Your operational definition should specify a series of steps that you or someone else could take to determine whether a specific object is a magnet. Do not assume you have access to an object already known to be a magnet.

✔ Check your work with a staff member.

Exercise 1.3

A few of the objects that you worked with in Experiment 1.1 seem to have some, but not all, of the properties of a permanent magnet. Describe how you could distinguish such an object from a magnet. State explicitly what operations you would need to perform. Specify what additional materials, if any, you would need.

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The objects in your set that interact with magnets are said to be *ferromagnetic.* The prefix *ferro* means *iron* in Latin. Many objects that interact magnetically are made of iron. However, a few other materials such as nickel interact in the same way as iron and are also called ferromagnetic.

Experiment 1.4

Obtain a magnet and a ferromagnetic object that is not a magnet. Both should be about the same size and mass. Close your eyes and hold them end-to-end near each other.

Can you tell which is the magnet solely on the basis of the pull that you sense between them?

We have used the term "interaction" to describe the magnetic effects between two objects. Discuss why the use of that term is appropriate.

Exercise 1.5

Suppose you were given two iron bars that are identical except that one is a permanent magnet and the other is not. Without using any other materials, how could you tell which of the two is the magnet? (*Hint:* Recall Experiment 1.1 in which you examined the magnetic interaction between the different parts of various objects.)

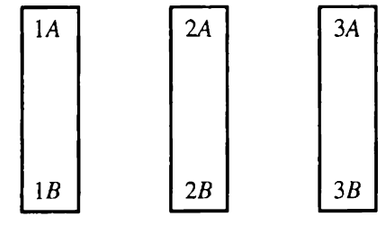
✔ Discuss your method with a staff member.

Exercise 1.6

A student is studying the behavior of 3 objects. The objects are labeled 1, 2, and 3 and the ends of each object are labeled *A* and *B* as shown.

The student finds that:

- the end 1*A* attracts the end 2*A*
- the end 1*A* attracts the end 3*A*
- object 2 and object 3 do not interact



Classify the three objects using the same classes as you used in Experiment 1.1. Explain your reasoning. If you cannot classify some of the objects on the basis of the information given above, explain why not and describe experiments you could perform to help you classify the objects.

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Section 2. The parts of a magnet

Experiment 2.1

For this experiment you will need three bar magnets and a box that has five test spots on it. The test spots are marked A through E.

Place a piece of tape on each end of the three magnets and label the ends I through VI. Place end I next to each of the five test spots, one after the other. Record the interactions in a table like the one below. Repeat this procedure for each of the remaining magnet ends.

	Magnet one		Magnet two		Magnet three	
Test spot	End I	End II	End III	End IV	End V	End VI
A						
B						
C						
D						
E						

A. Are there any magnet ends that can be considered identical? Explain.

Group the magnet ends I–VI into different categories according to their interactions with the five test spots.

How many different categories can you identify?

B. Mark the ends of the magnets with small colored dots. Use the same color for all the ends that interact identically with all five test spots.

How many colors did you require?

C. Is it conceivable that there may be another type of magnet end that is different from the types that you identified above?

If so, what might its entry in the data table look like?

Does your inability to find another type of magnet end definitely indicate that there is no other type?

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Experiment 2.2

For this experiment you will need to use the magnets whose ends you marked with colored dots in the preceding experiment. We will call the ends marked with the same color "like" ends. Those marked with different colors we will call "unlike" ends.

A. What happens when:

- two "like" ends interact?
- two "unlike" ends interact?

B. In Experiment 1.1 you categorized objects into three classes based on their magnetic properties. Use these same classes to classify each of the lettered spots on your test box from Experiment 2.1. Place an appropriately colored dot on each spot that acts like the end of a magnet.

✔ Check your work in this experiment and in Experiment 2.1 with a staff member.

Experiment 2.3

A. Investigate the different parts of one of the bar magnets used in Experiment 2.1. Make rough comparisons of how each part interacts with paper clips or other small ferromagnetic objects.

Draw a sketch of the magnet and identify the different parts. On the diagram, indicate the relative strength of each part of the magnet.

B. Obtain a second bar magnet and test how each of its colored ends interacts with the different parts of the bar magnet in part A. Record your observations.

Do all parts of a magnet interact magnetically with equal strength?

The term *magnetic pole* is used to identify a portion of a magnet that has magnetic properties like the ends of the magnets that you used in the preceding experiments. For a bar magnet, the poles are usually located at the ends of the magnet. However, magnets come in various shapes and the locations of the poles may not be obvious.

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Experiment 2.4

A. Obtain several magnets of various shapes from your instructor. Try to identify the locations of all the poles on each of the magnets.

Are the poles of a bar magnet always located at the ends of the bar?

Outline a series of steps whereby you can (1) determine where the poles are located on a magnet, and (2) identify the type ("color") of each pole. These steps will constitute an operational definition for the term *magnetic pole.*

B. Obtain two flat refrigerator magnets. Investigate the interactions between them.

What do you notice that seems unusual about the magnets?

Can you identify any magnetic poles?

Exercise 2.5

A. Consider the magnets you have worked with thus far in this section.

(1) Do any of the magnets have just *one* type of pole? If so, describe the pole type and draw a sketch of the magnet that shows the pole location.

(2) Do any of the magnets have *two*, and only two, different types of poles? If so, describe the pole types and draw a sketch of the magnet that shows the pole locations.

(3) Do any of the magnets have *more than two* different types of poles? If so, describe the pole types and draw a sketch of the magnet that shows the pole locations.

(4) Do any of the magnets have more than one pole of the same type? If so, describe the pole type(s).

B. From the results of part A:

(1) What can you infer about the number of poles of *different* types in a single magnet?

(2) What can you infer about the number of poles of the *same* type in a single magnet?

✔ Check your work with a staff member.

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Section 3. The earth as a magnet

Experiment 3.1

Note: Perform this experiment together with your classmates. If others have already begun to hang magnets from strings (see part A below), you can use their magnets. However, you must first determine how their color-coding scheme for poles relates to yours.

A. Obtain between six and eight cylindrical or bar magnets. Use one of your color-coded magnets from Experiment 2.1 to color code the poles of the remaining magnets.

Suspend all the magnets from strings in various parts of the room so that they hang parallel to the floor. While you are waiting for the magnets to stop swinging, draw a map of the room that shows the location of each magnet.

When the magnets have stopped swinging, record the orientation of each magnet and its poles on your map. Indicate explicitly the "color" of each pole on your map.

Is there any pattern to the alignment of the magnetic poles? If so, describe the pattern.

Carefully consider the environment of each magnet. List any objects with which the magnet might be interacting.

B. Tape a magnet onto the table and suspend another, smaller, magnet from a string above it. Hold the suspended magnet with your hand so that each pole of the suspended magnet is above the "like" pole of the bottom magnet.

Release the suspended magnet and describe what happens.

C. The results of the experiments above suggest a model or mental picture that can help to account for the behavior of the suspended magnets in part A: We can think of the earth as acting like a large magnet.

On the basis of this model, explain how the situations in parts A and B are similar. (*Hint:* Imagine that the magnet on the table in part B were very large and the suspended magnet were very small.)

D. On the basis of your model, what color would you assign to the magnetic pole of the earth that lies toward:

- the arctic region of the earth?
- the antarctic region of the earth?

Explain your reasoning.

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E. Look around the room for an object made of ferromagnetic material. Suspend a magnet near this object.

Does the alignment of the magnet fit the pattern you discovered in part A for the other suspended magnets? Explain how you can account for any differences.

In part A were there any magnets that did not fit the general pattern you discovered? If so, how might you account for the behavior of those magnets?

✔ Check your work with a staff member.

Exercise 3.2

A. The poles of a magnet are usually labeled with the terms *north pole* and *south pole.* The standard convention is to label the end of a magnet that points toward the Arctic as a "north pole," and the end that points toward the Antarctic as a "south pole."

On the basis of this convention, is the pole of the earth that lies in the Arctic (which we call the *north magnetic pole* of the earth) a north or a south pole?

Is the pole of the earth that lies in the Antarctic (the *south magnetic pole* of the earth) a north or a south pole?

B. Mark the color-coded ends of your magnets with either an "N" or an "S" so that they are consistent with the standard convention.

✔ Check your work with a staff member.

Experiment 3.3

Obtain several small compasses and place them far apart on a sheet of cardboard.

A. Place the set of compasses on an empty cardboard box or on a stack of books.

Examine the compasses and in your notebook sketch what you observe.

How are the results of this experiment similar to that of the hanging magnet activity in Experiment 3.1?

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B. Repeat part A two more times, each time making one of the following changes:

- placing the cardboard and set of compasses directly on the table
- placing an iron object that is not a magnet in the center of the sheet of cardboard and arranging the compasses near the object

In each case, record your observations in your notebook.

C. Hold a single compass in your hand and examine the orientation of the needle.

Which end of the compass points toward the north?

Predict what would happen to the compass needle if you were to bring the north end of a magnet near the compass. Explain your reasoning.

Record your prediction, then check your answer.

D. What do your experiments imply about the needle of a compass? Explain.

Experiment 3.4

During the course of a day, make a shadow plot for the sun. (See Section 1 of *Astronomy by Sight* for directions on making a shadow plot.)

Find the direction of the shortest shadow on your plot. That direction defines a line that we call the "true" or "geographic" north–south line.

Is the direction toward the "north magnetic pole" the same as that of "true" north for your city? If not, find the angle between them.

The deviation of the compass needle from true north is called the *magnetic declination.*

Experiment 3.5

Obtain a globe of the earth and locate the north and south magnetic poles.

Is the declination you measured consistent with what you might expect based on the locations of your city and the marked magnetic poles on the globe? Explain.

How might you expect the magnetic declination to change if you were to move eastward or westward across the earth? Explain.

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Section 4. Comparing the strengths of magnets

Exercise 4.1

Chris and Joe live in different cities. Each has a cylindrical magnet that is 12 cm in length and 1.0 cm in diameter. They often argue about whose magnet is stronger. Describe a procedure that they may follow to determine who has the stronger magnet.

✔ Check your work with a staff member.

Experiment 4.2

A. Obtain several magnets of various sizes and shapes. Use a compass needle to identify the north and south poles of each magnet. Label these "N" and "S."

B. In the first column of a table like the one below, sketch each of your magnets. Show the dimensions of each magnet on your drawing as well as the locations of the poles.

   Use the method you described in Exercise 4.1 to determine the "strength" of the poles of each magnet. Record your results.

Sketch of magnet	"Strength" of north pole	"Strength" of south pole

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C. Base your answers to the following questions on the results of the experiment in part B. Perform additional experiments if necessary.

Is a larger magnet necessarily stronger (or weaker) than a smaller magnet?

Do the north and south poles of a magnet seem to always have essentially identical strengths? If not, what factors (e.g., shape, size, etc.) appear to affect the relative strengths of the poles? Explain how you could tell from your observations.

✔ Check your reasoning with a staff member.

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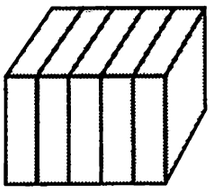
Section 5. Breaking and stacking magnets

Experiment 5.1

Obtain a set of small magnets that are assembled into a magnetic stack as shown at right.



A. Explore the magnetic properties of the stack. Identify the similarities and differences between the behavior of the stack and that of a bar magnet.

   Does the stack behave as though it were a single large magnet? If so, locate the north and south poles and indicate the locations on a diagram. If not, how does the behavior of the stack differ from that of a single magnet of the same size and shape?

B. Examine the interaction between paper clips and the center of the stack (midway between the ends). Describe your results.

Experiment 5.2

In this experiment we will examine what happens when a magnet is broken into smaller pieces. However, rather than actually break a magnet we will use the magnet stack from Experiment 5.1.

A. What justification can you give for regarding the magnetic stack as a large unbroken magnet?

B. Break the stack of magnets in half. Examine both ends of each half of the stack. Does each half have a north and a south pole? If so, where are the poles located?

   Put the two halves back together as they were in Experiment 5.1.

C. Break the stack by removing a single magnet from one end. Identify and label the poles of the magnet you have removed and any poles of the remaining stack. Draw a diagram to show the poles of each part and how the parts had originally fit together.

   Examine the interaction between a paper clip and the center of the stack. Does the center of the new stack behave in the same way as the center of the original magnet stack?

D. Repeat part C, removing one more piece each time until the original stack has been completely disassembled.

E. The original stack consists of several individual magnets, each with its own north and south poles.

   Do the poles of the individual magnets appear to act like poles when part of the stack? Explain.

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Experiment 5.3

Use the small magnets from the preceding experiments for this experiment.

A. Measure the strength of each pole of a single magnet. (Use the procedure from Experiment 4.2.) Record your results in the first line of a table like the one below.

B. Select a second small magnet and record the strength of the poles.

Add the second magnet to the first to make a small stack. Find the strength of the poles of the resulting stack and record your results.

C. Continue to add one small magnet at a time to the stack. At each stage, determine the strength of the added magnet and the strength of the resulting stack. Record your results.

	Strength of added magnet		Strength of stack	
Number of pieces in stack	North pole	South pole	North pole	South pole

D. On a sheet of graph paper, make a plot of the strength of the poles of the stack versus the number of magnets in the stack. Plot the strength of both poles on the same graph. Label your graph and the axes.

Describe how the strength of the stack changes as magnets are added.

Is the strength of the stack at each stage equal to the sum of the strengths of the pieces that form the stack? Explain how you can tell from the graph.

Does the strength of the stack increase without bound (i.e., could you make the stack as strong as you like just by adding magnets?) or does the strength seem to approach a limiting value? Explain.

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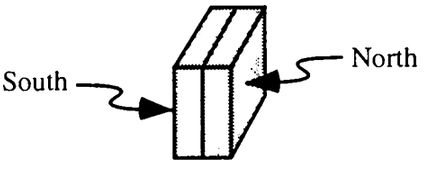
E. Based on your results in this experiment, would you predict that you can greatly increase the strength of a bar magnet by placing it end to end with another bar magnet as shown below? Explain.

Obtain two bar magnets and check your answer.

✔ Discuss this experiment with a staff member.

Experiment 5.4

A. Arrange two small magnets as shown at right.

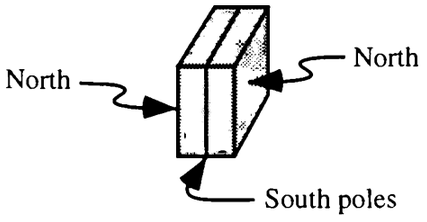


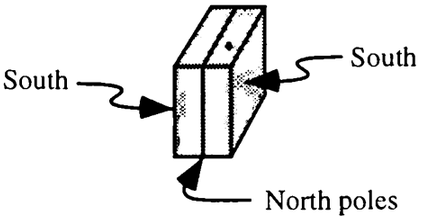
Is this arrangement stable? If not, use a rubber band to keep the magnets together.

Use the procedure from Experiment 4.2 to determine the strength of each end of this small magnet stack.

Record your results in a table like the one in part B.

B. Repeat part A for each of the arrangements of the magnets shown below. Record your observations.





Name of magnet	Stength of one end	Strength of other end
SNSN	_____	_____
NSSN	_____	_____
SNNS	_____	_____

C. In this experiment, you have stacked magnets with the like ends together and with the unlike ends together.

Which of these two methods yields a stronger stack?

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In the last few experiments in this section we have examined two ways of stacking magnets end-to-end and compared the strengths of the resulting magnet stacks. In the following experiment, we consider other ways of stacking magnets.

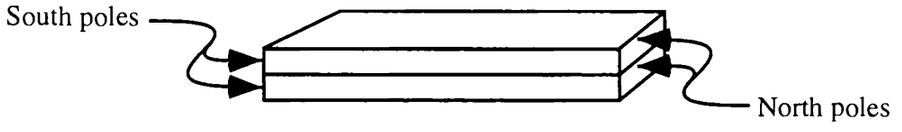
Experiment 5.5

Obtain two bar magnets. Identify and label the north and south poles if they are not already labeled.

A. Use the procedure you employed in Experiment 4.2 and 5.3 to determine the strengths of the poles of the two magnets. Record your results in the first two lines of a table like the one below part C.

B. Arrange the magnets from part A as shown below.

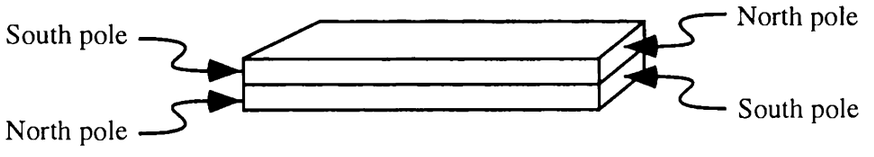
Is this arrangement stable? If not, use a rubber band to keep the magnets together.



Determine the strength of the north and south poles of this magnet combination using the same procedure as above. Record your results.

C. Now arrange the magnets as shown below.

Is this arrangement stable? If not, use a rubber band to keep the magnets together.



Determine the strength of both ends of this magnet combination using the same procedure as above. Record your results.

Name of magnet	Stength of one end	Strength of other end
One	_____	_____
Two	_____	_____
SS-NN	_____	_____
NS-SN	_____	_____

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Exercise 5.6

Suppose that you were given 40 small identical magnets.

Show how you could construct two larger magnets (20 magnets each) of the same size (each two magnets wide and five magnets long) that have different strengths. Base your answer on your observations in the experiments in this section.

✔ Check your work with a staff member.

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Part B: Magnetic materials

In Part A of this module, we investigated magnetic interactions and the behavior of magnets as they were stacked and broken. In Part B, we continue to explore interactions between magnets. On the basis of our observations, we construct a model (a mental picture) that allows us to account for the observed behavior of magnets and magnetic materials.

Section 6. Magnetic fields

Experiment 6.1

Obtain a small compass and a bar magnet, and identify the north and the south pole of each. Place the magnet at the center of a large sheet of paper far from any magnetic or ferromagnetic materials.

A. Use the compass to explore the region surrounding the magnet. Each time that you place the compass on the paper, note the direction in which the north pole of the needle points and mark the direction on the paper. Try various locations both close to and far from the magnet.

(1) Does the compass needle always point directly toward one pole of the magnet or the other?

(2) Mark on the paper the region(s) in which the compass appears to interact strongly with the magnet. Explain how you can tell that the interaction is strong.

Mark on the paper the region(s) in which the compass appears to interact only weakly with the magnet. Explain how you can tell that the interaction is weak.

(3) In what direction does the compass needle point when the compass is held far from the magnet? Note this direction on the paper.

What explanation can you give for the compass needle pointing in that direction?

In which region(s) around the magnet do you think that the direction of the compass needle most nearly reflects the effect of the magnet alone? Explain.

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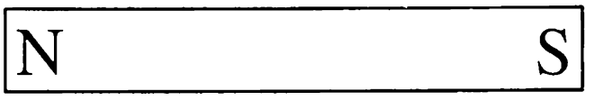
B. In your notebook, sketch a magnet like the one below and show how a compass needle will orient itself at various locations around the magnet. Represent the compass needle in the following way: S⟶N .



Experiment 6.2

A. Place a bar magnet in a plastic bag and lay the magnet flat on a large tray or piece of paper. Cover the magnet with a sturdy piece of paper (part of a manila folder works well) and carefully sprinkle iron filings over the surface of the paper. Be careful to keep the iron filings from coming in direct contact with the magnet.

Gently tap the paper several times and describe what you observe.

In which region(s) do the iron filings appear to interact:

- strongly with the magnet?
- weakly with the magnet?

Explain how you can tell in each case.

B. In your notebook, sketch the bar magnet and the pattern of iron filings.

Note: If you would like to make a permanent record of your iron filing pattern, spray the pattern with an aerosol glue and cover it with a clear plastic sheet. Let the pattern sit for several hours before moving it.

Exercise 6.3

Compare the pattern you obtained by placing the compass around the magnet in Experiment 6.1 with the pattern of iron filings that you obtained in Experiment 6.2.

Describe any similarities and differences between the patterns.

Are the regions in which the magnet appears to interact strongly with the compass the same as those where the magnet interacts strongly with filings?

Are the regions in which the magnet appears to interact weakly with the compass the same as those where the magnet interacts weakly with the filings?

✔ Discuss your answer with a staff member.

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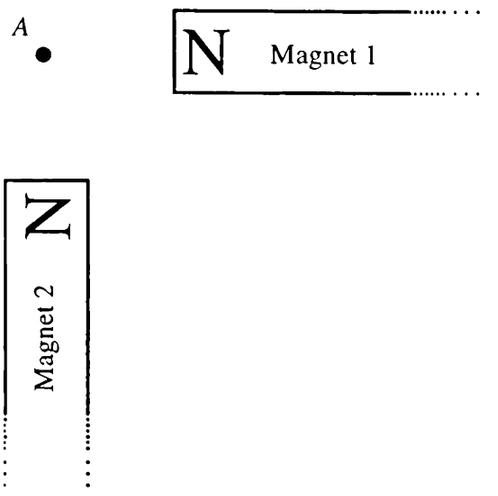
Experiment 6.4

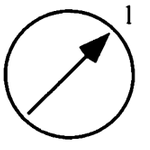
Tape a sheet of paper to a flat surface far from any magnetic or ferromagnetic materials. Find two bar magnets of approximately equal strength. Label them magnet 1 and magnet 2. Place the magnets on the paper at 90° to each other as shown.



A. Trace the position of each magnet on the paper and label the poles. Mark the point that lies along the center-line of both magnets and label it as point *A*. Place a compass at point *A*.

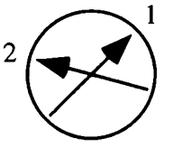
   Remove magnet 2 and hold magnet 1 in place so that it does not move.

   Draw a circle around point *A* to represent a compass. Sketch an arrow in the circle to show the direction in which the north pole of the needle points when it is at point *A*. Label the arrow with a "1" as shown. (*Note:* The direction of the arrow at right is not intended to be the direction that you should measure.)



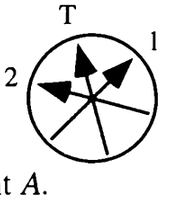
B. Remove magnet 1 and hold magnet 2 in place so that it does not move.

   Draw a second arrow to show the direction in which the north pole of the compass points at point *A*. Label the arrow with a "2."



C. Now place both magnets on top of their outlines on the paper. Hold the magnets so they cannot move.

   Find the direction in which the compass needle points when it is placed at point *A*. Indicate the direction by drawing an arrow and labeling the arrow with a "T" (for "total").



D. Repeat steps A through C with fresh paper three more times. Each time place magnet 1 a little farther to the right of point *A*. Keep the angle between the magnets at 90°.

E. Examine the circles that you drew in parts A–D and the three arrows within each circle.

   (1) How are the directions of the three arrows within each circle related to one another?

   (2) How can you tell which magnet is interacting more strongly with the compass at each location? Explain your reasoning.

   (3) At the start of the experiment you were told to obtain two magnets of approximately equal strength. Do you still believe that the magnets have equal strength? Explain how you can tell from the arrows that you drew in parts A–D.

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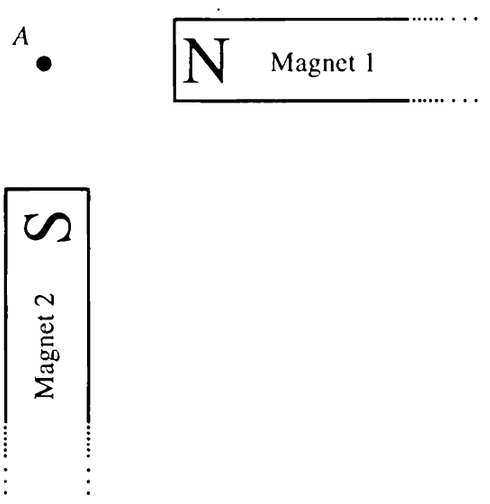
Experiment 6.5

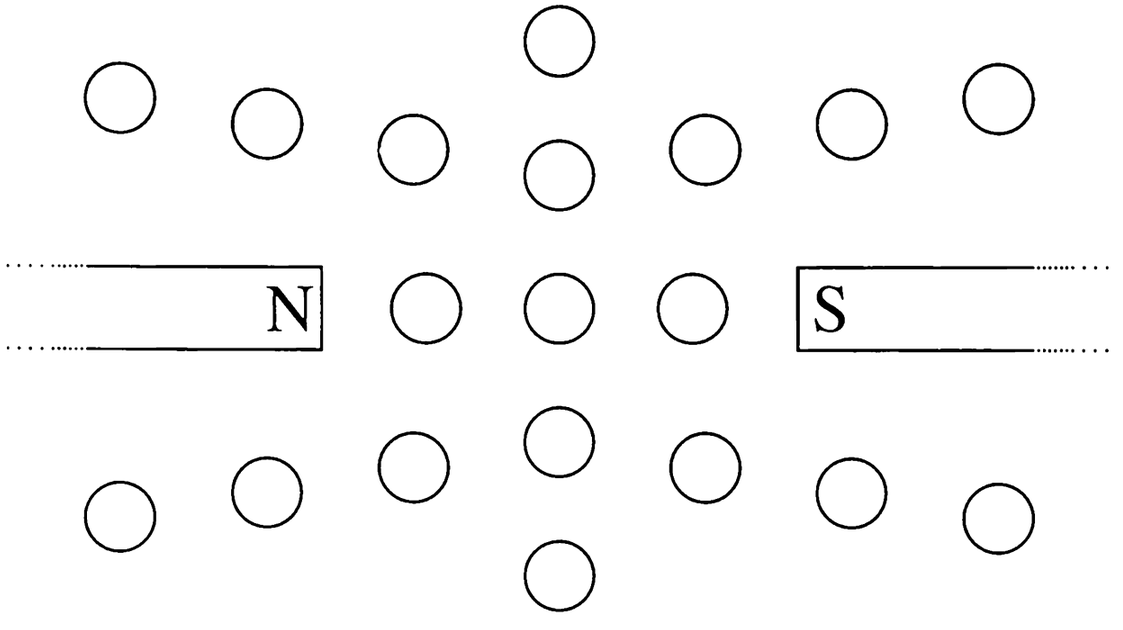
Imagine that magnet 2 from the previous experiment is reversed end-for-end as shown. If a compass were placed at point *A*, in which direction would the north end of the needle point? Explain your reasoning. (Assume that the magnets have equal strength.)



Set up the magnets and check your answer.

Experiment 6.6

Tape a large sheet of paper to a flat surface far away from any magnetic or ferromagnetic materials. Place two magnets on the paper as shown below. The magnets should be far enough apart so that compasses can be placed between them. Trace the position of each magnet on the paper and label the poles.



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A. Remove the right-hand magnet and tape the left-hand magnet to the paper so that it does not move.

Place compasses at the locations near the magnets indicated by the circles on the previous diagram. Draw an arrow in each circle to show the direction in which the north pole of each needle points. Label the arrows with an "L" for "left" (left-hand magnet).

B. Remove the left-hand magnet and tape the right-hand magnet to the paper on top of its outline. Repeat part A. Label each new arrow with an "R" for "right."

C. Now tape both magnets on top of their outlines on the paper. Repeat part A. Label each new arrow with a "T" for "total."

D. For each location where you placed a compass in parts A–C, identify which magnet is interacting more strongly with the compass. Are there any locations where the two magnets have equal effect on the compass? Explain how you can tell.

Are your results above consistent with your results from Experiment 6.1 and 6.2? (In those experiments you identified regions around a magnet in which the magnet interacts strongly and regions in which the magnet interacts weakly.)

✔ Discuss this experiment with a staff member.

Experiment 6.7

Use the method from Experiment 6.2 to produce an iron filing pattern for the configuration of two magnets from Experiment 6.6.

How does the pattern you obtain compare to the pattern of compass needles in the previous experiment? Discuss any similarities and differences.

In your notebook, sketch the magnets and the pattern of iron filings. Indicate those regions where the magnetic interaction seems to be the greatest.

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Experiment 6.8

A. For the following arrangement of two magnets, sketch in your notebook your *prediction* for:

- the pattern that you would obtain if you were to place compasses in the region between and around the magnets.
- the pattern that you would obtain if you were to sprinkle iron filings over the magnets.

Explain your reasoning in each case.

B. Obtain two magnets and check your answer.

✔ Discuss your results with a staff member.

The pattern of iron filings and the pattern of compass needles near a magnet help us to visualize how a magnet interacts with other objects. As you have seen in Experiment 6.7 above, these patterns for a single magnet can also be used to predict magnetic effects when more than one magnet is present. In this case, the effect of the individual magnets does not change, but the effect from each must be taken into account. The resulting iron filing or compass needle pattern reflects the additive effects of all the magnets.

We say that the pattern of iron filings or compass needles illustrates the *magnetic field* of a magnet or group of magnets. The magnetic field serves as a guide to how an object made of a magnetic material will behave when placed near a magnet.

The ability of magnets to act without physical contact is a surprising effect. Many people have pondered whether "action at a distance" is possible without a physical agent or a medium to conduct the effect. Even though our experiments do not answer this question, the magnetic field is a useful concept that helps us to predict magnetic interactions.

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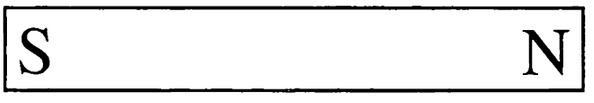
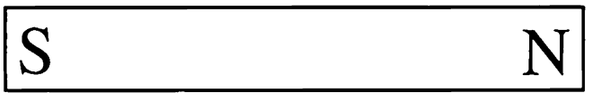
Experiment 6.9

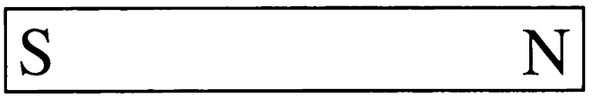
A. For each of the configurations of magnets shown below, predict the pattern of compasses needles (or iron filings) around the magnets. Sketch the pattern in your notebook and indicate the regions in which you think the magnetic field would be the strongest.

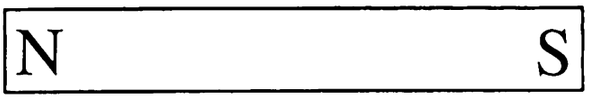
(1)





(2)





(3)

B. Use the methods employed in Experiments 6.1 or 6.2 to check your predictions in part A. If your predictions differ from your observations, discuss how you can resolve the conflict.

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Experiment 6.10

Obtain several magnets that are not bar magnets. Make an iron filing pattern for each magnet.

Make sketches of the patterns that you obtain. Indicate the region(s) where the magnetic field strength is the greatest.

Exercise 6.11

What observations have you made in this section that suggest a method for determining the *direction* of a magnetic field at a particular point?

Use your observations as a basis for writing an operational definition for the direction of a magnetic field at a point.

✔ Discuss this exercise with a staff member.

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Section 7. Magnetic field mapping

Experiment 7.1

Place a large piece of paper on a flat surface far from any magnetic or ferromagnetic materials. Tape a bar magnet to the center of the paper and label the north and south poles. Sketch the outline of the magnet on the paper. On the paper, indicate the direction of the magnetic north pole of the earth.

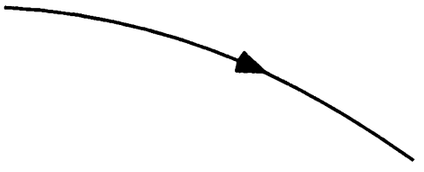
A. Take a small compass and determine which end of the needle is north. Place the compass on the edge of the paper far from the magnet and slide the compass over so that it touches the end of the magnet.

(1) Make two dots on the paper: one at the end of the compass needle next to the magnet, the second at the other end of the needle.

(2) Move the compass so that the end of the needle that was next to the magnet is now directly over the second dot that you drew above. Make a dot on the paper at the other end of the needle.

(3) Continue to move the compass as in part 2 above until the compass comes back to the magnet or leaves the edge of the paper.

(4) Draw a line (not necessarily straight) through the dots. On the line, place an arrowhead to indicate the direction in which the north pole of the compass needle points. (See the example below.)



B. Repeat part A, starting with the compass touching a different part of the magnet. Do this at least 10 more times.

C. Describe the pattern of lines that you drew in parts A and B.

How does the pattern compare to the pattern of compass needles and to the pattern of iron filings around a magnet? (See Section 6.)

D. Imagine that you were to remove the magnet from the paper and to repeat parts A and B *without* the magnet. What would be the pattern of the lines that you drew in this case? Explain.

Remove the magnet and check your answer.

Is the pattern that you obtain in parts A and B due only to the bar magnet? If not, where do you think the pattern is most nearly due to the bar magnet alone? Explain your reasoning.

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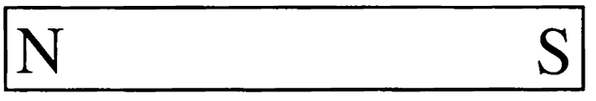
E. Tape the magnet to a fresh piece of paper. Repeat parts A and B with the magnet rotated by 90° with respect to the magnet in part A. Be sure to indicate magnetic north on the paper. (Alternatively, you may compare your diagram with those drawn by your classmates.)

   How can you account for any differences between the pattern of lines on the two diagrams?

   At what distances and locations does the effect of the magnet on the compass needle become negligible? What observations enable you to answer this question?

F. Sketch in your notebook the pattern of lines that you would expect for your bar magnet if no other magnetic fields (including the magnetic field of the earth) were present.



✔ Check your reasoning with a staff member.

Continuous curves drawn using the procedures outlined in Experiment 7.1 are called *magnetic field lines.* The magnetic field is usually represented by field lines rather than by a pattern of iron filings or a pattern of compass needles such as you used in Section 6.

Exercise 7.2

A. In part F of the preceding experiment, you may have drawn field lines that do not return to the magnet. If you were able to extend your map out farther, where do you think these lines might eventually end up?

   What does your answer suggest as a possible rule for drawing the field lines due to a single magnet?

B. In Exercise 6.11 you gave an operational definition for the direction of a magnetic field at a point. Use this definition and your observations in Experiment 7.1 to describe how to use a field line map to determine the direction of the magnetic field at a point.

C. Do you think it is reasonable for a field line map to have field lines that intersect? Explain your reasoning.

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D. Choose two *adjacent* field lines in the pattern that you drew in Experiment 7.1. Compare the strength of the magnetic field at a point where those two lines are close together to the strength at a point where those two lines are farther apart. Explain how you made your comparison.

What feature of the field line pattern do you think you can use to determine how the strength of the magnetic field varies along a *single field line?* Explain.

✔ Check your reasoning with a staff member.

Exercise 7.3

In Experiments 6.6 and 6.9, you sketched the pattern of compass needles around four different arrangements of magnets. In your notebook, sketch the magnetic field lines for each arrangement.

Check to be sure that your field line patterns are consistent with your answers to the questions in the preceding exercise.

Exercise 7.4

In Section 6 and in Experiment 7.1, you represented the magnetic fields of various magnets and combinations of magnets on pieces of paper lying in a horizontal plane. Do you think the magnetic fields only exist in the horizontal plane of the paper or can they be found above and below the piece of paper (i.e., in three dimensions)?

What evidence do you have to support your answer?

Examine a compass needle. Are there any limitations to what a compass can indicate about the direction of the earth's magnetic field?

Experiment 7.5

Obtain a dip needle from your instructor. A dip needle is a compass that can rotate in either a vertical or a horizontal plane.

A. Cover a flat surface with a sheet of paper. Place the dip needle apparatus on the surface so that the needle pivots in a *horizontal* plane. Be sure the surface is far from any magnets or ferromagnetic materials.

In what direction does the north pole of the needle point? How does this direction compare to that of the suspended magnets used in Experiment 3.1?

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B. Draw a line on the paper that indicates magnetic north and south for the earth. Stand the dip needle apparatus upright so that the needle pivots in a *vertical* plane. Align the vertical plane of the needle along the line you have drawn.

Measure the angle that the north pole of the dip needle makes with the vertical and record the angle.

Is the north pole of the dip needle pointing in a direction above or below horizontal?

C. Give an accurate description of the direction of the earth's magnetic field at your location. Explain your reasoning.

✔ Discuss your results with a staff member.

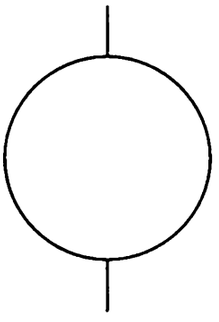
Exercise 7.6

A. Based on what we have observed as the characteristics of the earth's magnetic field, we can think of the earth as one large bar magnet. Based on this model and your results for part C, how do you think the dip needle would orient itself if one were using it:

- at the pole of the earth in the Arctic?
- at the pole of the earth in the Antarctic?
- halfway between the magnetic poles of the earth?

How would the behavior of the dip needle differ in the northern and the southern hemispheres?

B. At right is a sketch of the earth. The axis shows the line along which the *magnetic* poles of the earth lie.



Make an enlargement of this sketch on a separate piece of paper. Label the magnetic poles of the earth and indicate your latitude.

Sketch the magnetic field lines for the earth on your diagram. Be sure the diagram is consistent with your observation of the dip needle in Experiment 7.5 and your predictions from part A.

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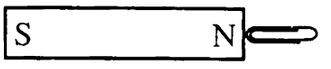
Section 8. Making magnets

Experiment 8.1

A. Obtain a strong magnet, a small compass, and several paper clips.

Place a paper clip on the end of the magnet as shown. After several seconds remove the paper clip and bring one end of the paper clip near the compass.



Describe what you observe. Can you tell whether the paper clip is a magnet from this observation alone?

Rotate the paper clip while holding it very close to the compass. Describe what you observe.

What does your observation imply about the paper clip?

B. Predict how a paper clip that has not been near a magnet will interact with a compass. Explain your reasoning.

Obtain a paper clip and check your prediction. If the paper clip acts like a magnet, can you find a paper clip that does not behave like a magnet?

We say that the paper clip in part A of the previous experiment has been *magnetized.* In the following experiment we explore this effect in more detail.

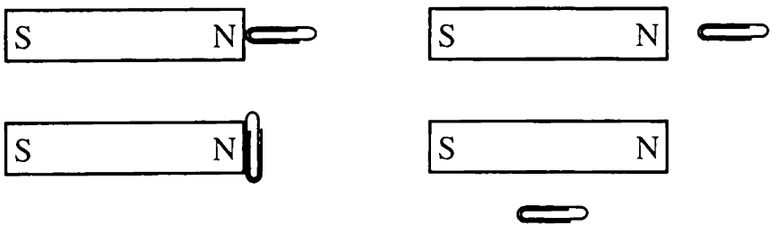
Experiment 8.2

A. Perform experiments to answer the following questions. If other questions occur to you, try to answer them as well. In each case, start with a paper clip that is not magnetized.

(1) Which end of the paper clip in Experiment 8.1 was north and which end was south?

(2) Is direct contact between a magnet and a paper clip necessary in order for the paper clip to become magnetized?

(3) Would a paper clip become magnetized if held as follows:



In each case that the paper clip becomes magnetized, which part of the paper clip is the north pole and which part is the south pole?

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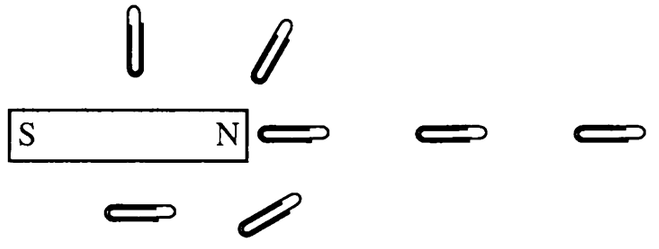
B. Tape a strong magnet to a piece of paper that is far from other magnetic and ferromagnetic materials. Obtain several paper clips that are not magnetized.

Tape a paper clip to the paper so that it is near the magnet but does not touch it. Sketch a diagram in your notebook that shows the location and orientation of the paper clip relative to the magnet.

(1) Use a compass needle to check the behavior of the paper clip after one minute has elapsed. If it has become magnetized, indicate the polarity (that is, the north and south ends of the paper clip) on your diagram.

Place the paper clip back near the magnet and recheck its magnetic behavior again after several more minutes.

(2) Repeat part 1 for several different locations near the magnet. Also try various orientations of the paper clip. Each time use a new paper clip that is not magnetized. The figure below illustrates several locations and orientations that you may find interesting.



C. On the basis of your results in parts A and B, describe the relationship between the polarity of the paper clip and the direction of the magnetic field at the point where the paper clip was placed.

D. Which of the paper clips in part B became magnetized in the shortest time interval? Where were these located relative to the magnet?

Which of the paper clips in part B took the longest to magnetize? Where were these located relative to the magnet?

Is there any relationship between the strength of the magnetic field at a particular location and how long it takes to magnetize a paper clip at that location? Explain how you can tell.

Are there any paper clips that did not become magnetized at all during the observation time? If so, give their locations and orientations relative to the magnet. Why do you think these did not become magnetized?

✔ Discuss your results with a staff member.

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Experiment 8.3

A. Obtain a large iron rod from your instructor. Find any north and south poles of the rod and note the strength of the poles.

Take the rod, a hammer, a compass, and a dip needle outside the building. Use the dip needle to find the direction of the earth's magnetic field and align the long axis of the rod along the magnetic field of the earth. (Refer to Experiment 7.5.) Remember to align the axis both vertically and horizontally.

Sketch the rod in your notebook. Show the direction of the magnetic field of the earth in your sketch. If your rod had any north and south poles indicate their location in your sketch.

(1) Hit the end of the rod with the hammer numerous times.

Check the ends of the rod for north and south magnetic poles using a compass needle. What do you observe?

If your rod had any north and south poles to start this experiment, are they still in the same location? Are they still the same strength?

(2) Flip the axis of the rod, keeping it along the line of the magnetic field of the earth. *Predict* what will happen to the polarity of the rod if you again hit the rod numerous times with the hammer.

Conduct the experiment and check the polarity of the rod again. What do you observe?

B. Use a compass to examine large ferromagnetic objects in your room, particularly those that have been in the same position for a long time (for example, filing cabinets, radiators, etc.). Check both the tops and bottoms of these objects with the compass.

Can you locate magnetic poles on these objects? If so, use sketches to describe where the poles are located. Draw a sketch that shows where in the room these objects are located.

C. Are the results of this experiment consistent with your observations from Experiment 8.2? Explain.

Experiment 8.4

A. Obtain an alcohol burner and a small ferromagnetic object that is not magnetized. Heat the ferromagnetic object until it is very hot, then quickly place it near a pole of a magnet. After the object is cool, use a compass needle to check its magnetic properties.

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What do you observe? If the object is now magnetized, use a sketch to indicate its polarity relative to how it was held near the magnet. Is the magnetic strength of the object greater than what you would expect if it had been held near the magnet, but not heated first?

B. Repeat part A with an identical ferromagnetic object. However, this time let it cool in a location far from any magnets.

How does the resulting magnetic behavior of this object differ from the one used in part A?

✔ Discuss this experiment with a staff member.

Exercise 8.5

Based on the results of Experiment 8.2, 8.3, and 8.4, summarize the conditions under which an object made of ferromagnetic material can be turned into a magnet and where the resulting poles would be located.

✔ Discuss your results with a staff member.

Experiment 8.6

A. Magnetize a small iron nail so that the head of the nail is the north pole. Use a compass needle to check its polarity.

Place the nail on a block of wood that rests on the floor. Hit the nail 15–20 times with a hammer or another large object.

Check the polarity and magnetic behavior of the nail with a compass. What do you observe? Is the nail as strongly magnetized as before?

B. Obtain a paper clip and magnetize it strongly. Use a compass needle to check its polarity.

Drop the paper clip on a cement floor. Is the paper clip still magnetized?

C. Summarize the procedure one can use to reduce or eliminate the magnetic behavior of an initially magnetized ferromagnet.

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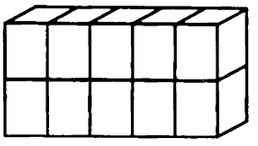
Section 9. A model for magnetic materials

Exercise 9.1



A. Ten small magnets are arranged as shown to make a large magnet stack.

Consider two such stacks: one in which all of the small magnets have the same orientation (e.g., the north poles all on the right and the south poles all on the left); the other in which alternating magnets have opposite orientations.

Which of the two stacks would be a stronger magnet? Explain.

B. In Section 5, you observed that when a magnet is broken into smaller pieces, each piece is a magnet with its own north and south poles.

Describe how the poles of the small magnet pieces are arranged within the magnet as a whole.

✔ Check your reasoning with a staff member.

As you have been working through this module, you may have begun to develop a mental picture of how magnets behave that helps you to make sense of your observations. In this section we draw on our observations to construct a scientific model for magnetic materials that we can use to account for our observations and that we can use to predict how magnets will behave in new situations.

Some important observations that we have made include the behavior of magnets when they are broken or stacked, and how a magnet can be made from a ferromagnetic object. These observations are consistent with a model in which a magnet is regarded as a collection of many smaller pieces, each of which acts like a tiny magnet. The magnetic interactions of a bar magnet can be thought of as the cumulative effect of many interactions arising from the small magnetic pieces composing the bar magnet.

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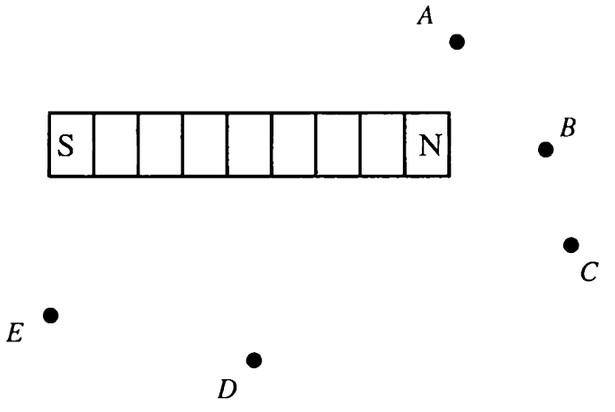
Experiment 9.2

In Experiment 7.1 you constructed a magnetic field line map for a bar magnet using a small compass as a probe. In this experiment you will consider what our model for magnets suggests about how a compass needle would behave if it could be placed inside a magnet.

A. Obtain some small magnets and assemble them into a stack that is about as long as the bar magnets that you have been using. All the magnets in your stack should have the same orientation. Place the stack on a large sheet of paper.

Use a small compass to find the direction of the magnetic field at the points *A–E* around the stack. Draw an arrow at each labeled point to show the direction of the field at that point.

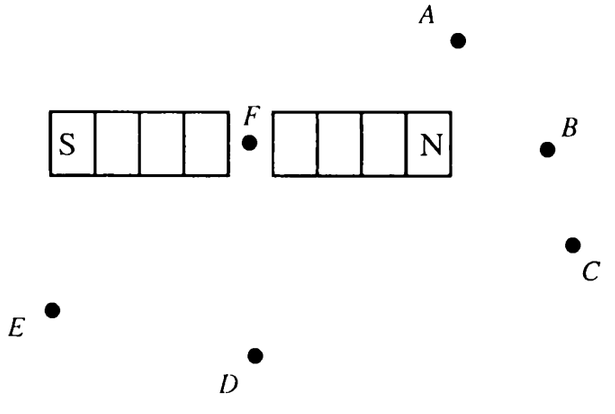
How does the direction of the magnetic field at each of these points compare to that of a bar magnet of the same length and strength?



B. Remove several of the small magnets from the middle of the stack. Take out enough magnets so that a compass can fit in the resulting gap.

Use a compass to find the field at the same points *A–E* as above.

Does the field of the stack at these locations change considerably when a few of the small magnets are removed?



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C. Use the compass to investigate the magnetic field at point *F*.

How does the direction of the field at point *F* compare to the *direction* of the field at point *D?*

How does the strength of the field at point *F* compare to the *strength* of the field at point *D?*

D. What do the results from this experiment suggest about the strength and direction of the magnetic field inside a bar magnet? Describe how your answer is consistent with the model for magnets developed so far.

Some of the observations you have made in this module concern the interaction of magnets with materials that we classified as ferromagnetic. Foremost among these observations were those in Section 8, in which we found that objects made of ferromagnetic materials can become magnetized when placed near a magnet. We have also observed that the center of a magnet, which is not a pole, is attracted to the poles of other magnets. These facts suggest that permanent magnets are made of ferromagnetic material. This close relationship between those objects we call "magnets" and those we call "ferromagnets" must be taken into account in any model that we develop.

Experiment 9.3

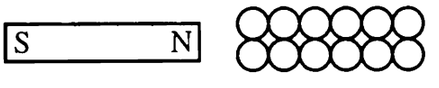
In this experiment we will explore how a set a compasses behave when placed near a bar magnet.

A. Obtain between 6–12 small compasses and place them close together.

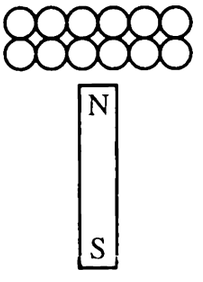
Do all your compasses initially point in the same direction?

B. Bring the north end of a bar magnet up near the set of compasses as shown in the two figures at right. On a separate piece of paper, sketch the alignment of each compass for each location of the magnet.

C. Repeat part B with the magnet reversed.



(1)



(2)

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Thus far, we have been thinking of magnets as consisting of many small magnetic pieces. We can think of ferromagnetic materials in this same way. We can account for interactions between magnets and objects made of ferromagnetic material by thinking of the small magnetic pieces in the ferromagnetic material as being free to rotate like a compass needle when other magnets are near.

Exercise 9.4

Use the model for ferromagnets to account for the following phenomena.

A. A ferromagnetic object that is not magnetized is attracted to a magnet.

B. Ferromagnetic objects that are not magnetized are not attracted to one another.

✔ Check your reasoning with a staff member.

Experiment 9.5

A. A magnet is placed near a similarly sized ferromagnetic bar as shown below.

Predict what the magnetic field of this arrangement would look like. Assume that the ferromagnetic bar is initially not magnetized. Explain your reasoning.

Set up the magnet and ferromagnet and check your answer by using either a compass or iron filings.

B. Predict how the situation in part A would change if now the ferromagnetic bar were reversed end-for-end. Explain.

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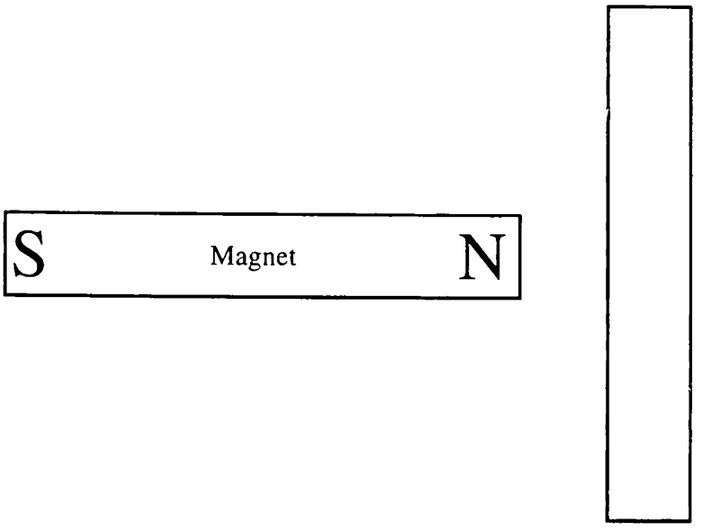
C. Predict the magnetic field that would result if the ferromagnet bar were initially placed as shown below. Explain your reasoning.



Set up the magnet and ferromagnet and check your answer by using either a compass or iron filings.

How does the presence of the ferromagnet affect the magneticf small magnets is agnet opposite the magnet?

Exercise 9.6

Below are some observations you may have made while conducting experiments in this module. Use the model that has been developed thus far to account for these observations. Explain your reasoning in each case.

A. A compass needle near a magnet may reverse polarity.

B. A pole of a strong magnet may attract both poles of a weaker magnet.

C. A ferromagnetic object becomes magnetized when it is placed very near a pole of a magnet.

D. An iron bar that is initially not magnetized may become a magnet when it is hit with a hammer.

How can you account for the resulting polarity of the bar?

Why do you think it helps to strike the bar?

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E. Large ferromagnetic objects that have been left in a room for a long time often have north and south poles.

F. A heated ferromagnet becomes a magnet when it is cooled in a magnetic field.

Exercise 9.7

Use your model for magnetic materials to answer the following questions. Explain your reasoning.

A. How might you account for one magnet being stronger than another?

B. What will happen to a strong magnet that is repeatedly dropped?

C. Suppose that you had a source of a very strong magnetic field.

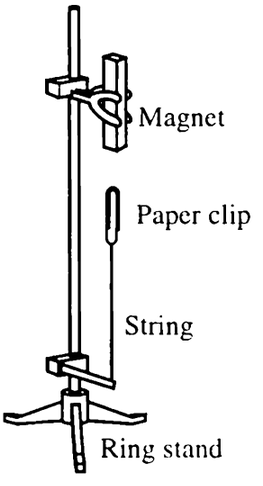
How could you use the magnetic field to make a magnet out of a ferromagnetic object?

Do you think there is a limit to how strong a magnet you could make out of a particular object?

✔ Check your reasoning with a staff member.

Experiment 9.8

Obtain a ring stand, two clamps, a piece of string, a strong magnet, and a ferromagnet (e.g., paper clip). Assemble the objects as shown at right. Make the string as short as you can without the paper clip falling.



Place thin objects made of various materials between the magnet and the paper clip.

Which types of materials cause the paper clip to fall and which do not?

How can you account for the behavior of the paper clip when ferromagnetic materials are placed between the magnet and the suspended ferromagnetic object? Explain on the basis of the model for magnetic materials and your observations from Experiment 9.5.

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Experiment 9.9

Discuss with your partner the model for magnetic materials that has been developed thus far.

Are there any other observations you have made thus far that you can account for using the model?

Are there any observations you have made that you cannot account for using the model?

✔ Check your reasoning with a staff member.

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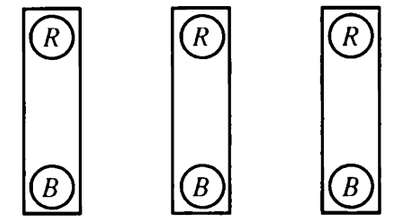
Supplementary problems for *Magnets*

Problem 1.1

A student is studying the behavior of three objects. She labels the objects 1, 2, and 3 and places red *(R)* and blue *(B)* stickers on either end of each object as shown.

She finds that:

- the end 1*R* attracts the end 2*R*
- the end 2*R* attracts the end 3*B*
- object 1 and object 3 do not interact



Object 1 Object 2 Object 3

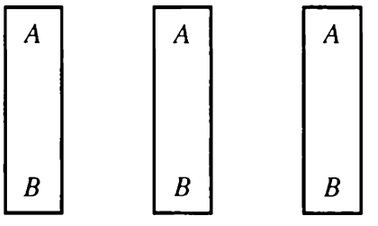
Classify the three objects using the same classes as you used in Experiment 1.1. Explain your reasoning. If you cannot classify some of the objects on the basis of the information given above, explain why not and describe experiments you could perform to help you classify the objects.

Problem 1.2

Three metal bars, labeled 1, 2, and 3, are marked *A* and *B* on either end as shown.

The following observations are made by a student:

- the end 1*A* attracts the end 2*B*
- the end 1*B* repels the end 3*A*
- the end 2*B* attracts the end 3*B*



Bar 1 Bar 2 Bar 3

A. To which of your three classes from Experiment 1.1 could bar 1 belong? Explain your reasoning and the characteristics that define each of your classes.

B. To which of your three classes from Section 1 could bar 2 belong? Explain your reasoning.

C. How would 2*A* interact if it were brought near 3*A*? If it is not possible to tell for certain, what are the possibilities? Explain your reasoning.

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Problem 2.1

A student in your class decides:

> *"A long time ago, someone decided to define 'like' ends of a magnet to be those that repel one another. They could have equally well have decided that 'like' ends are those that attract one another and 'unlike' ends repel."*

Do you agree with this student? Explain your reasoning.

What experiment(s) would you suggest this student perform in order to prove or disprove the claim? Describe the experiment and the results you believe the student should obtain. Explain how those results would support or refute the claim.

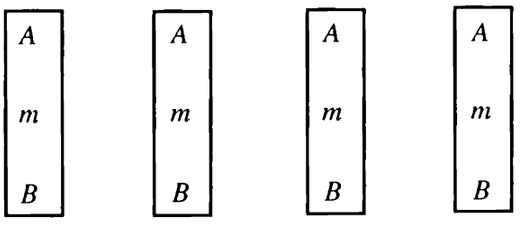
Problem 2.2

In Exercise 2.5 you inferred a general rule for the number and types of poles in a single magnet. Suppose you were to hear in the news tomorrow that someone found a magnet with three types of poles. Would you believe this claim? Are the experiments you have conducted thus far sufficient for you to prove or disprove this claim?

Problem 2.3

Consider four metal bars. The bars are numbered 1 through 4. The ends of each bar are labeled *A* and *B*, and the middle of each bar is labeled *m*. (See diagram at right.)

Assume that any objects that are magnets have their poles on the ends *A* and *B*.



A student makes the following observations:

- 1*A* attracts 2*A*
- 1*A* does not interact with 3*m*
- 1*A* repels 4*B*

A. Classify the four objects using the same classes as you used in Experiment 1.1. Explain your reasoning. If you cannot classify some of the objects on the basis of the information given above, explain why not and describe experiments you could perform to help you classify the objects.

B. How would 1*A* interact if it were brought near 4*m*? If it is not possible to tell, what are the possibilities? Explain your reasoning.

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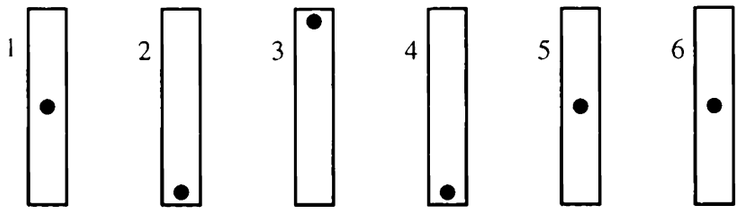
Problem 2.4

A student has taken 6 bar-shaped objects and tested their magnetic interactions with one another. However, the student has tested only one part of each object. The part that was tested is marked with a dot on the figure below.



The results of the experiment are given in the table below. ("A" indicates attraction, "R" indicates repulsion, and "–" indicates neither attraction nor repulsion.) Assume that the objects that are magnets have their poles on the ends and that all the magnets are equally strong.

	1	2	3	4	5	6
1		–	A	A	–	–
2	–		A	A	–	–
3	A	A		R	A	–
4	A	A	R		A	–
5	–	–	A	A		–
6	–	–	–	–	–	

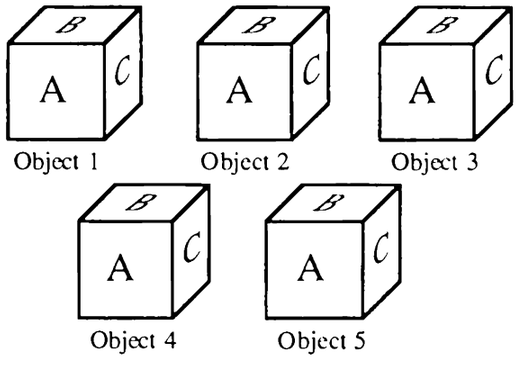
Classify the four objects using the same classes as you used in Experiment 1.1. Explain your reasoning. If you cannot classify some of the objects on the basis of the information given above, explain why not and describe experiments you could perform to help you classify the objects.

Problem 2.5

Consider the cubes shown below, numbered 1 through 5. Three of the faces of each cube are labeled *A*, *B*, and *C*.

A student observes that:

- surface 1*A* attracts surface 2*B*
- surface 1*A* attracts surface 3*A*
- surface 1*A* does not interact with surface 4*B*
- surface 3*A* does not interact with surface 4*B*
- none of the surfaces of cube 3 interact with any of the surfaces of cube 5



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Indicate whether each of the following statements is true or false. *For each part, explain your reasoning!*

A. Cube 1 is definitely a magnet.

B. Either cube 1 or cube 3 is definitely a magnet.

C. Cube 4 is definitely neither a magnet nor a ferromagnet.

D. Cube 5 might be a magnet.

Problem 3.1

Suppose you are given some string and two unmarked magnet (i.e., the "like" and "unlike" ends are not labeled). Design a new experiment to show that "like" ends repel and "unlike" ends attract. Base your experiment on your discoveries in Experiment 3.1. Explain your reasoning. Include in your answer a description of the criterion you would use to determine whether two poles are in fact "like" or "unlike."

Problem 3.2

A. Construct an operational definition for the north pole of a magnet.

B. Construct an operational definition for the south pole of a magnet.

Problem 3.3

Consider an unmarked *cubical* magnet. Using only this magnet, a paper clip, and some string, describe how you could find the direction of magnetic north. (Assume you know the approximate direction of true north.) Write clear, complete, and concise directions.

Would your method work with an unmarked, *irregularly shaped* magnet? Explain why or why not.

Problem 4.1

Fran and Shelby are attempting to determine which of them has the stronger magnet. After careful experiment, they reach the conclusion that the north pole of Fran's magnet is stronger than the north pole of Shelby's magnet. Fran is about to walk away satisfied when Shelby says,

> *"Wait! The south pole of my magnet might still be stronger than the south pole of your magnet."*

Does Shelby have a valid point? Justify your answer.

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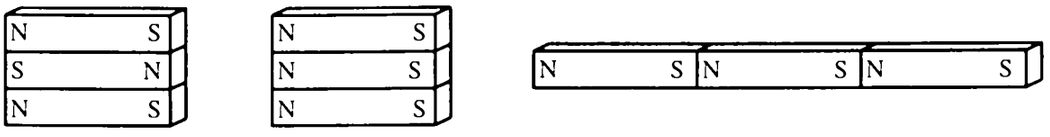
Problem 5.1

In the diagrams below, three identical magnets are arranged in three different ways. Which arrangement will result in the strongest magnet? Specify which of the three arrangements are stable and which are unstable. Explain your reasoning.

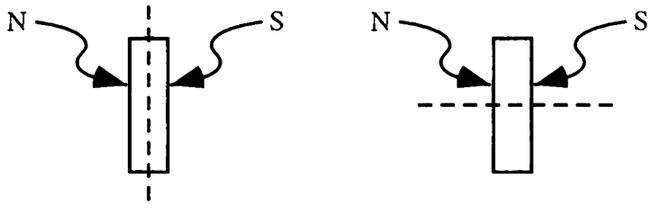


(1) (2) (3)

Problem 5.2

Consider a small, thin magnet. If the magnet were cut along the dashed line shown in case (1) would the two halves tend to move or stay attached? Explain your reasoning.

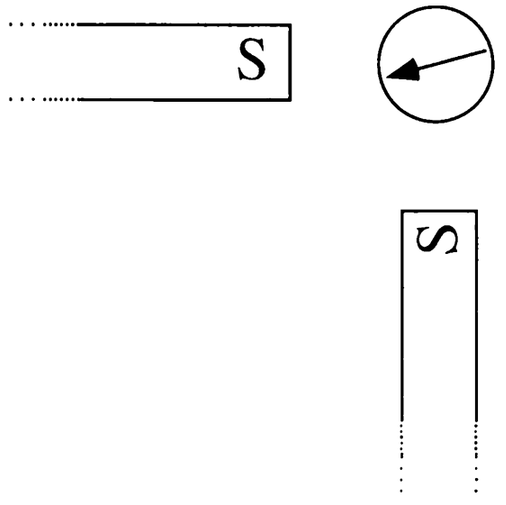
If the magnet were instead cut along the dashed line shown in case (2) would the two halves tend to move or stay attached? Explain.



(1) (2)

Problem 6.1

A student places two magnets as shown below, with their south poles equal distances from a compass. The student observes that the compass needle points in the direction shown. From this observation alone can you determine which magnet is stronger? Explain your reasoning.



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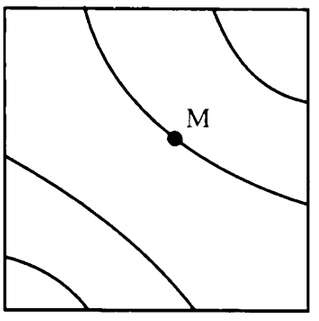
Problem 7.1

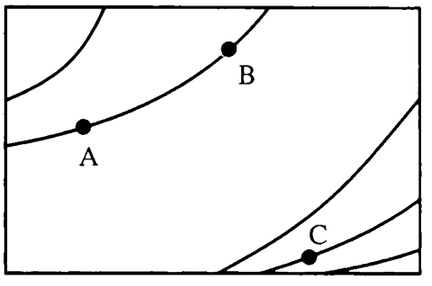
Some magnetic field lines in a region near several magnets are shown at right. The field lines in only a portion of the region are shown. Show the direction in which the needle of a compass would point when placed at point *M*. Explain your reasoning. If the direction cannot be determined, explain why not.



Problem 7.2

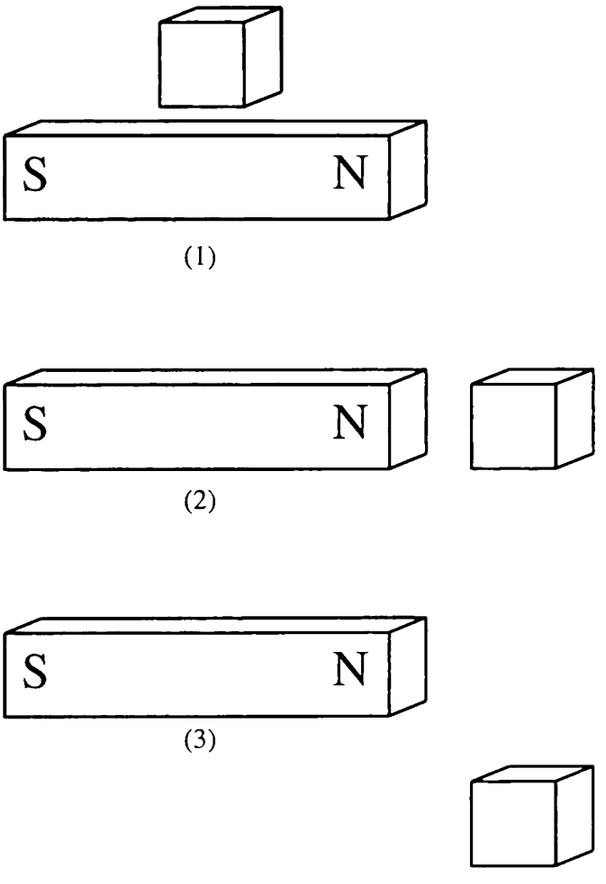
The diagram at right shows magnetic field lines near some magnets. Only a small portion of the region around the magnets is shown.

Compare the strength of the magnetic field at points *A*, *B*, and *C* (that is, rank them in order from largest to smallest). If you cannot compare the strength of the field at some of the points, explain why you cannot.



Problem 8.1

An initially unmagnetized iron cube has been placed near a bar magnet for a long time. Will the cube become magnetized in each of the three locations illustrated below? If so, where will the poles be located? Use the idea of a magnetic field in explaining your answers.



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Problem 8.2

A student has observed that paper clips become magnetized after they are in contact with a magnet. The student comes up with a model in which a magnet consists of smaller magnets that can be transferred from the magnet to a ferromagnet. What observations have you made that suggest this model is not appropriate? Explain your reasoning.

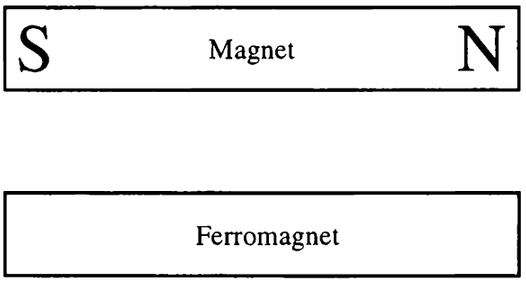
Problem 9.1

When the continents on the earth were first forming, the land was very hot and molten and contained many ferromagnetic materials. Describe where you would find the poles of a piece of iron in the ground after the continents had cooled.

It is proposed that the continents are not in the same locations around the earth today as they were when they first cooled. How might one check this hypothesis?

Problem 9.2

A magnet is placed near a similarly sized ferromagnetic bar as shown below. The ferromagnetic bar is initially not magnetized.



A. Sketch lines to represent the magnetic field of this arrangement. Explain how your sketch is consistent with the model for magnets and ferromagnets.

B. Does the presence of the ferromagnet make the field stronger or weaker:

(1) on the side of the ferromagnet opposite the magnet? Explain your reasoning.

(2) in the region to the left of the magnet and ferromagnet? Explain your reasoning.

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PHYSICS BY INQUIRY

Astronomy by Sight:

the sun, moon, and stars

In *Astronomy by Sight: the sun, moon, and stars,* we make observations of the motion of these objects in the sky. We identify patterns in the changes that occur during the course of a day and a month. We develop models that enable us to determine the present, past, and future appearance of the sky. The models also help us tell time and find geographic location and direction.

Note: *Astronomy by Sight: the sun, moon, and stars* is the first of two closely related modules. Students should finish working through this module before beginning *Astronomy by Sight: the earth and the solar system*, which appears in Volume II of *Physics by Inquiry.*

Section 1. Sun shadows

We begin *Astronomy by Sight* by making observations of the sun. However, we should not look directly at the sun.

> *Warning:* You can easily incur partial or complete blindness by looking directly at the sun. Always make your observations indirectly, without looking straight at the sun. *NEVER* look at the sun through an optical instrument such as binoculars or a telescope.

In this section, we develop an indirect method for locating the sun. We examine the shadow cast by a vertical stick during the day.

Experiment 1.1

Begin this experiment as soon after sunrise as possible.

Obtain a flat board with a nail driven straight into it that you can use as a shadow plotting board. Place a sheet of paper on the board so that the nail sticks through the center of the paper. Tape the paper onto the board so that it cannot move. Write the height of the nail and the date on the paper.

Place the board where it will be in direct sunlight for most of the day (e.g., in an unobstructed window). If necessary, use different locations for the morning and the afternoon. Secure the board so it is level and does not move.

On the paper, carefully indicate the locations of several nearby landmarks. Later, after you have removed the paper from the board, you will need to hold the paper in the same orientation that it had while on the board.

Mark the tip of the nail's shadow with a small dot. Write the time of the observation in pencil next to the dot. Repeat this procedure approximately every half hour during the entire day. Start early in the morning. Your shadow plot should span six to eight hours and consist of at least twelve observations. Organize with your classmates so that someone is responsible for marking a point at least every hour for most of the day.

> *Note:* If you must move the plot to keep it in sunlight, mark the tip of the shadow just before the plot is moved. Then, place the plot in a new location with the same orientation as before; the tip of the shadow should coincide with the last mark that you made.

Continue through Experiment 1.5 while you are waiting to add additional points to your shadow plot.

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Exercise 1.2

A. On a sheet of paper, sketch your prediction for how the completed shadow plot in Experiment 1.1 will look. Indicate the same landmarks on your sketch as you drew on your plot. Explain your reasoning.

What does the tip of the shadow tell you about the location of the sun? Draw a sketch to illustrate your answer.

How do you think the length of the shadow will change during the day? When will the shadow be the longest? When will it be the shortest? Explain your reasoning.

Place a dot on your diagram to show where you predict the shadow of the tip of the nail would be at noon. Where in the sky will the sun be located at this time? Explain.

What will the shadow look like at sunrise and at sunset? In each case, indicate the direction in which the shadow will point and describe the location of the sun. Explain.

How will the shape traced out by the shadow during the morning compare to the shape traced out during the afternoon? Explain.

What clock time will correspond to the "center" of the shadow plot?

How would your plot differ if the nail were twice as long?

B. Suppose you were to use your shadow plotting board to make a shadow plot on each of the following days:

- tomorrow
- a day six months from now
- the longest day of the year
- the shortest day of the year

Do you think the shadow plot for any of the days listed above will differ in shape from the plot you make today? Sketch any plots that you think will differ in shape and explain your reasoning.

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An object designed to record the shadow of the sun is called a *gnomon*. On modern sundials the gnomon is usually a triangular wedge of metal. The simplest gnomon, however, is a vertical stick. In Experiment 1.1, the nail served as a gnomon.

Experiment 1.3: Homework project

Make a shadow plot every sunny day during the next few weeks. Each plot should include at least the period from about 9:30 A.M. to 3:30 P.M. On each plot, record the height of the gnomon, the date, and some nearby landmarks (so that you have information about the orientation of the paper while the plot is being made).

After the first few weeks, you can stop making daily shadow plots; one plot every week or two will be sufficient.

As you make the plots, compare them to your predictions in Experiment 1.2.

Exercise 1.4

A. Discuss with your partner some of the facts that you know about the sun. Make a list of the items you discuss.

B. Separate your list into two categories: those items for which you have direct evidence and those that you know because you were told them or you read about them.

   In the case of those items for which you believe you have direct evidence, describe your evidence and explain how it is relevant.

C. Which of the items on your list do you think you could have come to know by watching the sun yourself (without materials or equipment to which you do not have ready access)? In each case, describe the observations you would need to make.

✔ Discuss this exercise with a staff member.

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Experiment 1.5

For this experiment, each member of your group will need a copy of the shadow plot that you made in Experiment 1.1.

A. Describe in words how the shadow of the gnomon changes during the course of one day.

How does the shape of the plot in the morning compare to the shape of the plot in the afternoon? What does this observation tell you about the motion of the sun?

B. At approximately what time did the shortest shadow occur? (You may not have plotted a point exactly at the moment of the shortest shadow. Lightly, and in pencil, draw a smooth curve on the plot to show the path of the tip of the gnomon. Try to judge where the shortest shadow occurred and estimate the time accordingly.)

Did the shortest shadow occur when the clock read 12:00 noon?

Estimate the time when the sun was highest in the sky. Explain your reasoning.

Was the sun ever directly overhead? Explain how you can tell.

C. Mark on the plot the approximate direction in which the sun rose and the direction in which the sun set. Explain your reasoning.

D. In Exercise 1.2 you made a prediction for the shape of the shadow plot. Describe any similarities and differences between your prediction and the actual plot. If there are any differences, explain what your analysis of the shadow plot has taught you about the motion of the sun.

E. To help you visualize the motion of the sun, move your outstretched hand to show the entire motion of the sun during the day.

✔ Discuss this experiment with a staff member.

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Experiment 1.6

Tape a completed shadow plot onto a piece of cardboard. Rotate the paper so that it has the same orientation as when the plot was made. Use a small piece of clay to hold a toothpick upright at the location of the original gnomon. Cut the toothpick so that it is the same height as the gnomon.

Perform parts A and B in a darkened room.

A. Obtain a flashlight and shine it at the toothpick. Describe how you must hold the flashlight so that:

- the shadow of the toothpick is very long
- the shadow of the toothpick is very short
- the shadow is the same length as the toothpick

Shadow plots cannot be started exactly at sunrise nor can they be ended exactly at sunset. Explain why this is true.

B. Hold the flashlight so that the tip of the shadow of the toothpick is at one of the marked points on your shadow plot.

How can you move the flashlight so that:

- the length of the shadow stays the same but its direction changes?
- the direction of the shadow stays the same but its length changes?
- the length and direction of the shadow remain the same?

C. We often say that the sun is "high" or "low" in the sky. Our usual meaning of "high up" is "a great distance above the ground." For example, at the top of a 50-story building you are high up.

What does "high" mean for the sun?

Can you tell from a shadow plot how far the sun is from the earth?

Can you tell from a shadow plot whether the distance from the earth to the sun changes during the course of a day?

Explain your reasoning in each case.

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D. For one of the marked points on your shadow plot, draw a side-view diagram of the gnomon and shadow. Make the diagram large; it should fill about half your paper. Keep the diagram to scale. (For example, if you draw the gnomon four times larger than it is in real life, then the shadow should also be four times larger.)

Repeat the procedure above for a second point on your shadow plot. Choose a point for which the length of the shadow is very different from the length of the first shadow you chose.

How does the "height" of the sun compare in the two cases?

Which aspect of your diagrams can you use to compare the "height" of the sun?

Experiment 1.7

For this experiment you will need three or four of the shadow plots that you have been making as part of Experiment 1.3.

A. For each plot, determine the time and length of the shortest shadow.

Does the shortest shadow seem to occur at the same time each day?

Does the shortest shadow seem to have the same length each day?

Does the shortest shadow seem to point in the same direction each day?

B. Compare the general shapes of the shadow plots. Does the motion of the sun seem to change appreciably from one day to the next? Explain how you can tell.

✔ Discuss your findings with a staff member.

One of the most striking features of a shadow plot is its symmetry. The trace of the tip of the shadow can be divided into two halves that are mirror images. The symmetry makes the midpoint of the motion stand out as a special event during the day. The time at which the sun is at the midpoint of its journey is called *local noon.*

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Exercise 1.8

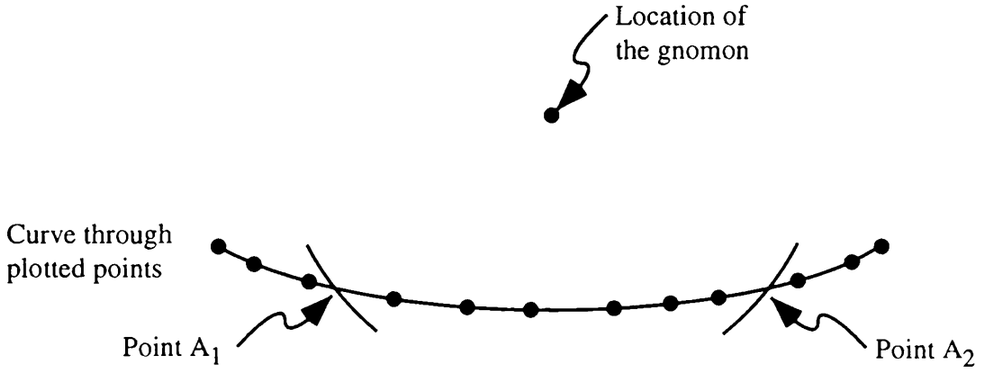
Write an *operational definition* for local noon. An operational definition is a concise description of the steps that one must take to measure a given quantity. In this case, the steps would describe how to find the instant at which local noon occurs. (See Section 5 of *Properties of Matter* for a detailed description of what is meant by an operational definition.)

✔ Check your operational definition with a staff member.

Experiment 1.9

The method below is sometimes useful to help estimate the time of the shortest shadow. For this experiment, use a copy of the shadow plot that you used in Experiment 1.5.

A. Sketch a smooth curve on the plot to show the path of the shadow of the gnomon. Place the point of a drawing compass at the location of the gnomon. Set the compass so that it intersects the path of the shadow about halfway between the shortest and longest shadows. Lightly draw two arcs that cut the path of the shadow. Label one of the intersections as A_1; the other as A_2. (See the diagram below).



B. Draw a line from each of the points A_1 and A_2 to the gnomon. Call the angle between the lines θ.

The shortest shadow lies along a line that bisects the angle θ. Explain why this is true.

A method for bisecting the angle θ is described in part C.

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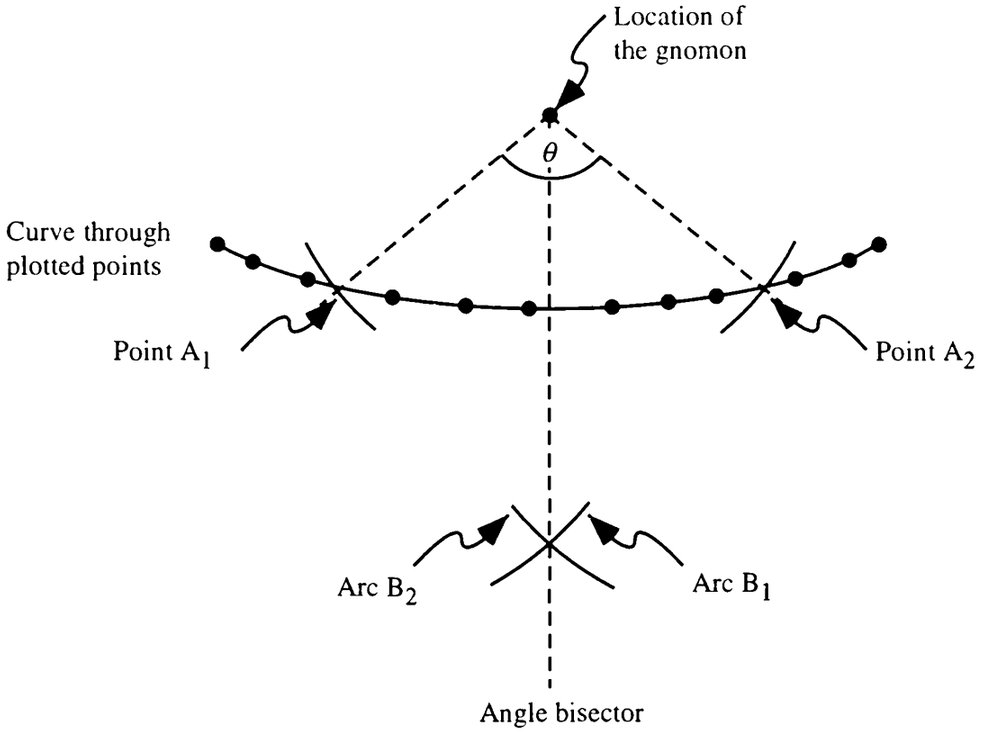
C. Widen the legs of the compass. Place the point of the compass on the point A_1. Draw a short arc through the region in which you expect the angle bisector to lie. (See arc B_1 in the diagram below.) Repeat for point A_2 and label the new arc B_2.



Draw a line from the base of the nail to the place where arcs B_1 and B_2 cross.

The line that you just drew bisects the angle θ. Discuss with your partner the above method for finding the angle bisector. Try to come up with a geometrical argument to convince yourself that the line does in fact bisect the angle.

Estimate the time of the shortest shadow.

D. In Experiment 1.5 you also found the time and direction of the shortest shadow for this same shadow plot. Which method do you think yields more accurate results?

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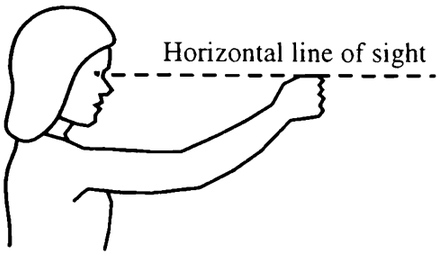
Section 2. Observing changes in the sky

In the preceding section we used the shadow of a nail to determine the location of the sun. We must use different methods for other objects. In this section, we develop a more general technique to determine the location of astronomical objects.

Experiment 2.1



A. Hold one arm straight out in front of you and make a fist with that hand.

Sight along the top of your fist and move your arm until your line of sight is horizontal.

Have your partner check that your line of sight is horizontal.

Is your arm horizontal when your line of sight is horizontal?

Write an operational definition for horizontal.

Suppose you were standing on the side of a hill. Would your definition of horizontal be different in that case?

B. Move your arm upward until your line of sight is pointing straight up.

Have your partner check that your line of sight is vertical.

Write an operational definition for vertical.

C. Make a fist and hold your arm out straight so that your line of sight to the top of the fist is horizontal. Place your other fist on top of the first one. Keep the top fist firmly in place and move the lower fist so it is on top. Continue this fist-over-fist process, counting each fist, until your line of sight is vertical. Your partner should make sure your fists remain steady.

Through what total angle have your fists moved?

How many fists correspond to the total angle? (Consider whether or not to include the original fist you held horizontal.)

How many degrees correspond to one of your fists held at arm's length? Explain your reasoning.

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D. Each of your partners should determine the angle corresponding to one fist held at arm's length.

   Do each of you measure approximately the same value?

E. Find the angle that corresponds to one of your fingers held at arm's length. Describe how you determined your answer.

✔ Check your technique and your results with a staff member.

Experiment 2.2

This experiment should be performed outdoors.

A. Make a fist and hold your arm straight out so that your line of sight to the top of the fist is horizontal. Have your partner check that your line of sight is horizontal.

   While your fist is held horizontal, look at a distant horizon.

   Is the horizon above your line of sight, level with your line of sight, or below your line of sight?

B. Your instructor will point out an object. Measure the angle between the horizon and the object indicated by your instructor. Use the "fists" technique from Experiment 2.1 to answer to within one finger's width.

   Measure the angle at least three times. Record your results. Wait until everyone has made their own measurements before comparing answers.

   Did you obtain the same angle each time?

C. Measure the angle between two points indicated by your instructor. Make at least three measurements. Record your results. Wait until everyone has made their measurements before comparing answers.

   Did you obtain the same angle each time?

D. If the sun is visible, measure the angular height of the sun above the horizon. Remember to do this without staring directly at the sun. Record your result.

E. Compare the angles that you measured above in parts B–D with the measurements made by other students in your class.

   Did everyone obtain the same answers? If not, should you conclude that someone made a mistake, or is there another explanation?

   Does the size of a person's fist or the length of a person's arm affect the angle that they measure? Explain.

✔ Discuss your results and your reasoning with a staff member.

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The angle between an object and the horizon is called the *altitude* of the object. In this module, when you use the "fists" technique to measure altitude, measure to within one finger's width.

Experiment 2.3

A. Obtain a recent shadow plot. Describe how you can use the shadow plot to determine the altitude of the sun at a particular time. Sketch a diagram that illustrates your answer.

   Use the plot to find the altitude of the sun at local noon.

   What is the altitude of the sun at sunrise or sunset? Can you use your plot to find the altitude of the sun at those times? Explain.

B. Three students have drawn diagrams of a gnomon and its shadow.

   - Student 1 drew the gnomon and shadow life size.
   - Student 2 drew the gnomon and shadow twice life size.
   - Student 3 drew the gnomon three times life size and the shadow twice life size.

   From which diagram could you directly measure the altitude of the sun? Explain.

C. You have two techniques for measuring the altitude of the sun. How are the methods related? Explain your answer.

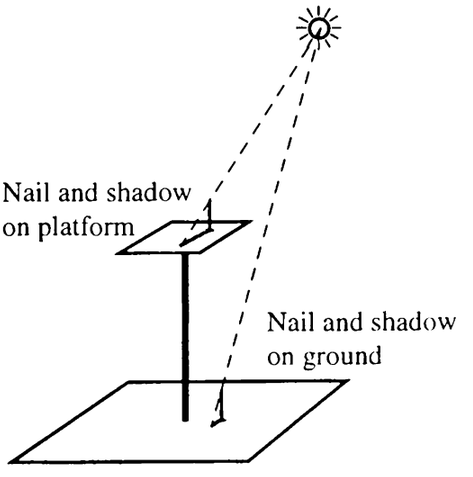
✔ Discuss your answers with a staff member.

Exercise 2.4

A. A student draws the diagram at right makes the following statement:

   "I think that the length of a shadow is different depending on where I measure it. For example, it should be longer at the top of a platform than it is at the bottom. My sketch shows this."

   Do you agree or disagree with the student? Explain your reasoning.



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B. Two shadow plots are made at the same time: one at the top of a tall cliff and the other at the base of the cliff. Both give the same value for the altitude of the sun.

Sketch a diagram similar to the one in part A that illustrates this situation.

What does the observation that the altitude of the sun is the same at both points imply about the distance of the sun from the earth? Explain your reasoning.

C. Reconsider your answer to part A in light of the information in part B. Discuss how the diagram might be misleading as it is drawn.

Exercise 2.5

A. Make a list of some of the astronomical objects that you might see if you were to look at the sky. Separate your list into three categories: objects you could see only during daylight hours, objects you could see only at night, and objects you could see both day and night.

B. Suppose you were to look at the sky over the following periods: a day, a week, a month, or longer. In each case, would you see changes in any of the objects that you listed in part A? If so, would the changes follow a regular pattern? Describe any changes and patterns that you would observe.

Exercise 2.6

In Exercise 1.4, you discussed your knowledge about the sun. In this exercise you will focus on the moon and stars.

A. Discuss with your partner some of the facts that you know about the moon. Make a list of all the items you mention.

(1) Separate your list into two categories: those items for which you have direct evidence and those that you know because you were told them or you read about them.

In the case of those items for which you believe you have direct evidence, describe your evidence and explain how it is relevant.

(2) Which of the facts on your list do you think you could have come to know by watching the moon yourself (without materials or equipment to which you do not have ready access)? In each case, describe the observations you would need to make.

B. Repeat part A, this time considering the stars instead of the moon.

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C. With your partner, discuss your understanding of the seasons. How do you account for the differences between winter, spring, summer, and autumn?

✔ Discuss this exercise with a staff member.

During the next few months, you will be making some of the observations that you and your fellow students described in Experiment 1.4 and Experiment 2.6. Make your observations carefully and record them accurately and systematically. Always exercise caution when making observations of the sun.

Experiment 2.7: Homework project

As part of your homework for the next few weeks, make the observations of the sun, moon, and stars described in parts A–C below. Write your observations in a notebook.

In each case, record:

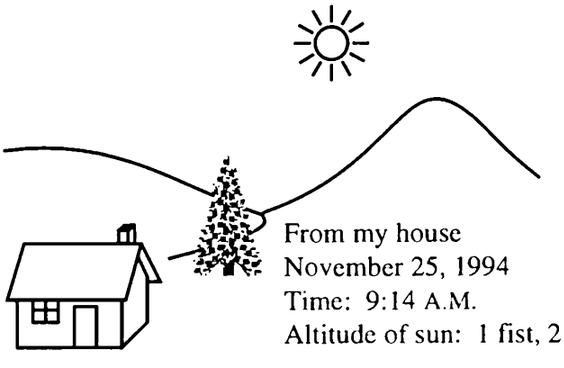
- the altitude of the object (to within one finger's width),
- the date and time of the observation,
- your physical location, and
- a sketch of the object. Show easily recognizable landmarks on your sketch. If there is a significant landmark in your area (for example, a mountain) mark its location on your sketch.

If, at any given time, you do not see the object for which you are looking, record that observation too! State whether you think you were unable to locate the object because of the weather.

If possible, make most of your observations from just one or two locations so that you can compare your observations from day to day.

A. *Sun observations*

At least one day every week, observe the location of the sun throughout the day. In particular, try to record the location of the sun at sunrise and sunset. Coordinate with your partner so that your combined observations span most of the day.



The figure above illustrates the information you should include with each observation.

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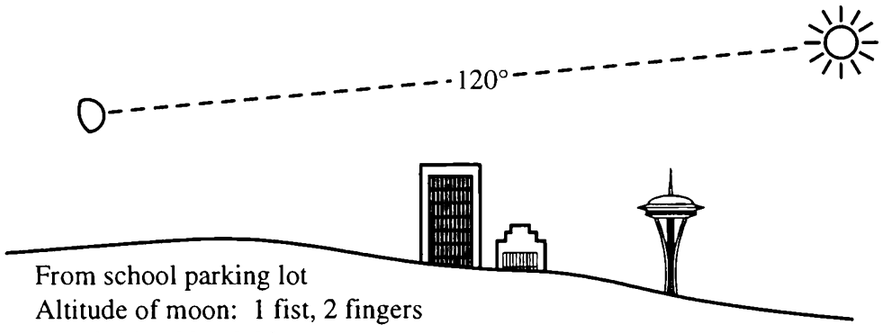
B. *Moon observations*

Repeat part A for the moon. In addition, whenever possible, use the fists technique to record the approximate angle between the lines of sight to the sun and moon. (See diagram below.) Be sure to include a sketch that shows the shape of the moon and how the moon is oriented with respect to the ground. If there are any stars near the moon, record the locations of the stars.



From school parking lot
Altitude of moon: 1 fist, 2 fingers
March 12, 1994, 2:20 P.M.

Try to make several moon observations every day.

C. *Star observations*

Select several stars in the early evening, each in a different part of the sky. To help you locate these stars on subsequent evenings, sketch the pattern of stars around each star that you choose. Include landmarks on your sketch as well.

Observe the stars that you have chosen once or twice during the night and again in the early morning. If, at any time, you cannot find your stars, record that information too.

Try to find your stars about once each week. It is important that you make some of your observations at about the same time every night.

D. As you make the observations listed in parts A–C above, you may notice other aspects of the motions of the sun, moon, and stars. You may also find other astronomical objects. Record any additional observations that you think are interesting.

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Section 3. The size and shape of the earth

Exercise 3.1

A surveyor notices that the sun is at the top of a distant mountain. He carefully measures the altitude and finds it is 2.1°.

Later, his partner, a mountain climber, tells him that he must be in error. She reasons:

> *"My map shows that the distance to the mountain is 60 miles and the top of the mountain is 2.7 miles above sea level. Since we are at sea level, I can draw the following diagram. I find an angle different from what you measured."*

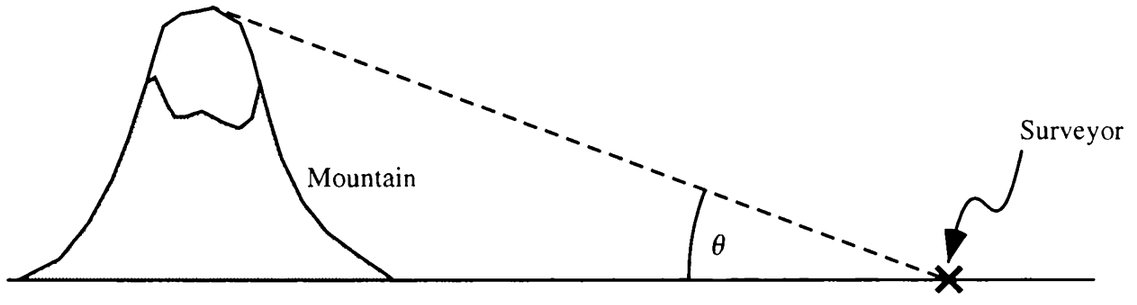

Figure drawn by mountain climber (not to scale).

A. Determine the altitude θ of the top of the mountain as predicted by the mountain climber. (*Hint:* Draw a scale diagram to help you answer this question. See Experiment 2.3.) Talk to a staff member if you need help in answering this question.

B. What possible explanations can you give for the discrepancy between the surveyor's measurement for the angle and the prediction by the second student?

Exercise 3.2

How might you account for the following observations?

A. On any given night, stars are visible from southern cities that are not visible from northern cities.

B. At local noon in one city a vertical stick casts a shadow, while at local noon in another city a vertical stick does not cast a shadow. The sky is clear in both cities.

✔ Discuss your answers to Exercises 3.1 and 3.2 with a staff member.

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Each of the observations in the preceding exercises suggests that the surface of the earth is curved, rather than flat. In the following exercises you will determine the size of the earth in two different ways.

Exercise 3.3

Eratosthenes (276–194 B.C.) used the following observations to calculate the size of the earth:

> At noon on a certain day of the year, the sun shines directly on the bottom of a well. At the same time, at a distance of about 500 miles north, the sun is 7.2° from being directly overhead.

On the basis of this information, calculate the approximate size of the earth. Explain your reasoning. The steps below can serve as a guide to help you.

A. Draw a large sketch of the earth. Mark points to indicate the location of each city.

> Sketch rays to represent light from the sun incident on the earth at each city.
>
> How should the directions of the two rays from the sun to the two cities compare if the sun is very far from the earth? Explain. (*Hint:* Refer to Exercise 2.4.)

B. Draw a line from the center of the earth to each of the two cities. Find the value of the angle θ between the two lines that you draw. Explain your reasoning.

> What fraction of a circle is represented by the angle θ? (*Hint:* There are 360° in a circle.) Explain.
>
> Calculate the circumference of the earth. Explain your reasoning.

✔ Check your reasoning with a staff member.

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Experiment 3.4

Perform this experiment if there is a large mountain visible in the distance on your horizon. The measurements that you will make in this experiment that are similar to those given in Exercise 3.1. If there is no such mountain nearby, continue to Exercise 3.5.

Suppose that you look across the top of an object of known height h and position yourself until you are just able see the mountain over the object. Let d represent the distance from you to the object. (See the figure below.)

A. Describe how you could use the known height h and the distance d to determine the altitude of the top of the mountain.

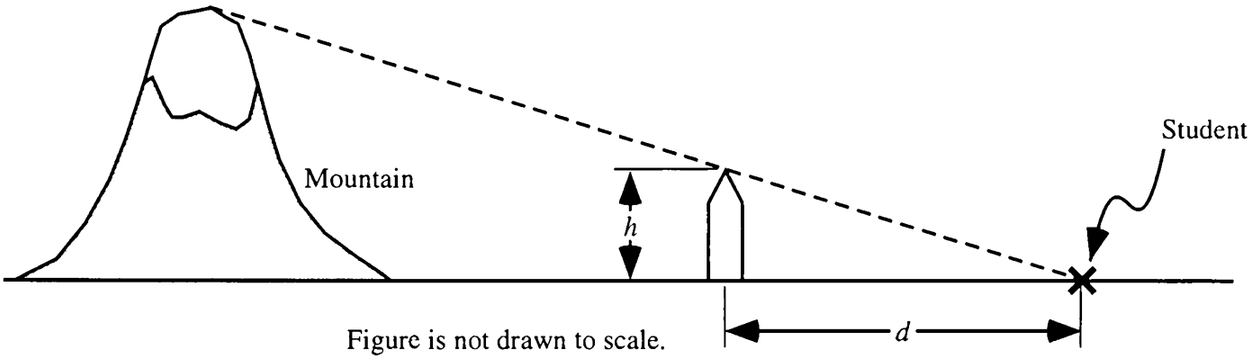

Figure is not drawn to scale.

B. Perform the experiment described above. Use a pair of metersticks for the object of height h. One person should hold the metersticks upright, while a second person sights along the top of the upper meterstick to the mountain. (The second person should lie on the ground in order to sight from the ground to the top of the mountain.) Adjust the height of the upper meterstick until the top of the mountain and the upper meterstick are aligned.

   Measure and record the lengths h and d. Both you and your partner should take turns making the measurements.

   Note: A small error in your measurements will make a big difference in your final answer. Make your measurements carefully.

   Is the area on which you are working horizontal? Why is this important?

C. Find the distance to the mountain and the accepted height of the mountain by looking it up in the library or by looking on a map. Use this information to find the angle that you would measure for the altitude of the mountain if the earth were flat.

D. Are the results that you obtained in parts A and B above consistent with the idea that the earth is not flat? Explain.

✔ Check your experiment with a staff member.

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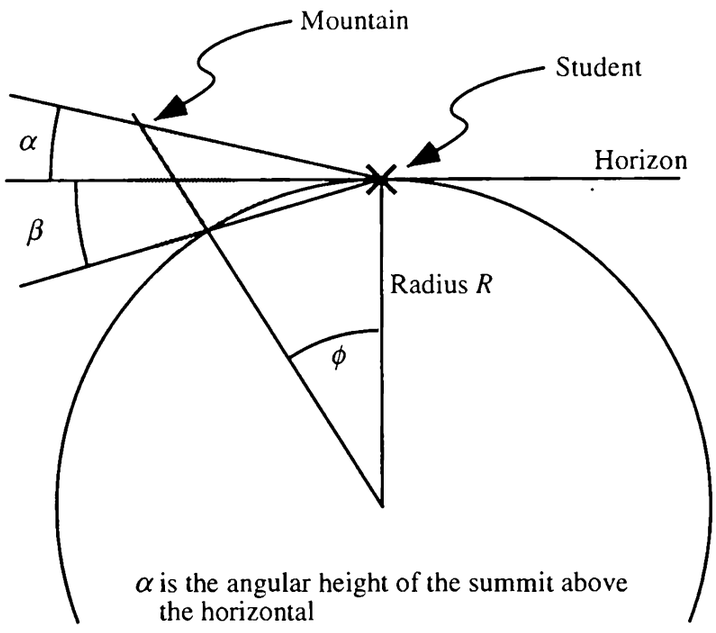
Exercise 3.5

The diagram below illustrates the situation described in Exercise 3.1 and Experiment 3.4. (The diagram is not drawn to scale.) If you performed Experiment 3.4, use your measured values.

A. Which one of the angles α, β, or ϕ corresponds to the altitude of the mountain as measured by the surveyor in Exercise 3.1?

The angles α and β taken together represent the approximate angle calculated by the mountain climber in Exercise 3.1.

Use the values from Experiment 3.1 to calculate β (the "missing angle").



α is the angular height of the summit above the horizontal

β is the "missing" angular height

ϕ is the angle between the student and the mountain measured from the center of the earth

B. Use geometry to find the angle ϕ in terms of the angle β.

Hints: (1) What kind of triangle is formed by the two radii and the line connecting the base of the mountain to the student? What is the mathematical relationship between the internal angles of the triangle? (2) How many degrees are in the angle formed by the line representing the student's horizon and the radius of the earth?

C. What fraction of a whole circle does the angle ϕ represent?

The distance from the student to the mountain is 60 miles. Use this information to calculate the circumference of the earth. Explain your reasoning.

Is your result consistent with the result you obtained in Exercise 3.3?

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Section 4. Daily motion of the sun

Experiment 4.1

Do not start this section until you have made shadow plots that span at least three weeks. Use copies of the plots, not the originals.

A. For each plot, use the method developed in Experiment 1.9 to mark the direction of the shadow of the gnomon at local noon.

In Experiment 1.7 you examined the noontime shadow of a gnomon over a period of several days and answered the following questions:

Does the shortest shadow occur at the same time every day?

Does the shortest shadow have the same length every day?

Does the shortest shadow point in the same direction every day?

Now that you have shadow plots that span a period of several weeks, would you change your answer to any of the questions above? If so, how would you change your answers? Explain.

B. In Section 1, we defined local noon as the instant when the sun is halfway through its journey in the sky.

Do you think the clock time of 12:00 noon is based on local noon? Explain your reasoning.

Which aspect or aspects of the sun's location at local noon remain the same from day to day? Explain how you can tell.

✔ Discuss this experiment with a staff member.

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As you have seen in the preceding experiment, the trace of the shadow of an object can be used to define a unique direction. The shortest shadow of an object always lies along a unique line that we call the north-south line. If you face south, your right shoulder points *west*, your left shoulder points *east,* and *north* is directly behind you. These directions are sometimes called *true* or *geographic* north, south, east, and west. Together they are called the *cardinal points.*

Exercise 4.2

Students often make the following statements:

"The sun is directly overhead at noon."

"The sun rises in the east and sets in the west."

Do you agree or disagree with either of these two statements? Explain your reasoning.

In this module, we will use the directions north, south, east, and west to refer to the locations of objects. We can also use the angle measured clockwise from north (as viewed from above). This angle is called the *azimuth.* Directions in a horizontal plane can be specified either in terms of the cardinal points or in terms of the azimuth.

Exercise 4.3

Identify the azimuth for each of the following directions:

(1) north
(2) east
(3) south
(4) west
(5) northeast
(6) southeast
(7) southwest
(8) northwest

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Experiment 4.4: Homework project

As part of Experiment 2.7 you have been making observations of the sun, moon, and stars. For your observations thus far, you have been using landmarks on the ground to record the direction in which an object lies. In the future, include information about the azimuth or the direction relative to the cardinal points.

In this section we continue to investigate the motion of the sun over the course of a single day. We do this by analyzing the shadow plots you made in Section 1.

Experiment 4.5

For this experiment, use a recent shadow plot that you recorded.

A. Write down a description of how the sun appears to move across the sky for an observer facing due south on the day the shadow plot was recorded.

B. Use your shadow plot to find the altitude and azimuth of the sun for at least eight different times during the day. Make a table that includes the following information: the time of day, the altitude of the sun, and the azimuth of the sun.

   Make a graph of *altitude* versus *time of day*. Next to each point you plot on the graph, write the corresponding direction that an observer would have to face to see the sun.

   At what time of day was the altitude the largest?

   What was the altitude of the sun at sunrise? What was the altitude of the sun at sunset?

C. At what time(s) of day is the altitude of the sun changing the most quickly? Explain how you can tell from the graph.

   At what time(s) of day is the altitude of the sun changing the most slowly? Explain.

✔ Discuss this experiment with a staff member.

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A physical model is often helpful in visualizing the motion of objects across the sky. Throughout this module, we will use such a model to help us visualize the motion of the sun, the moon, and the stars.

Experiment 4.6

Set up a light bulb at approximately eye level in the center of the room. In this experiment, the lamp represents the sun and your head represents the earth.

A. Stand facing the "sun" (light bulb) and rotate slowly in a circle. Keep your eyes facing straight ahead of you as you turn in place.

Note that the "sun" moves across your field of view while you rotate. Imagine a small observer standing on the "earth" (your head).

Should you rotate clockwise or counterclockwise in order for the observer to see the "sun" move from east to west?

B. Describe how you must stand so that the time is local noon for the observer represented by your eyes.

Describe how you must stand so that it is approximately 6:00 A.M. for the observer (i.e., sunrise).

Describe how you must stand so that it is approximately 6:00 P.M. for the observer (i.e., sunset).

C. Make a top view diagram of your head and the bulb to show your orientation at each of the times listed below. (Make a separate diagram for each time.)

(1) 6 A.M.
(2) local noon
(3) 6 P.M.
(4) midnight
(5) 9 P.M.

D. Compare the actual motion of the sun across the sky with the apparent motion of the bulb as you rotate.

How are the motions similar and how are they different?

How might you modify your model so that the motion of the bulb more accurately reflects the motion of the sun across the sky?

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E. In parts A–D of this experiment you rotated in place while the bulb representing the sun stood still. Suppose, instead, that you were to stand still while the bulb moved around your head.

In which direction should the bulb rotate around you so that the observer would see it rise in the east and set in the west?

Compare the two models for the relative motion of the earth and the sun. Do you have a basis for deciding which is a better model on the basis of the observations of the sun that you have made thus far? Explain your reasoning.

✔ Discuss this experiment with a staff member.

We will return to this physical model for the sun and earth several times in this module. As we make additional observations of the sky, we may need to modify the model so that it more accurately incorporates our findings.

Experiment 4.7

A. Use masking tape to mark a line on the floor of your room that runs north–south according to your shadow plots.

B. You and your partners should each obtain a magnetic compass. Hold your compass in your hand so that the needle swings freely.

Place the compass at various locations in the room: at different heights above the floor, at different places on table surfaces, near walls, windows, and so on.

What do you observe about the behavior of the compass?

How does the behavior of your compass compare to the behavior of your partner's compass?

What can you say about the behavior of the compass when it is far from other objects in the room?

C. Compare the direction that the compass needle points with true north. Does the needle point exactly in the direction of true north, to the right of true north, or to the left of true north?

If the compass does not point toward true north, measure the angle between the compass needle and true north.

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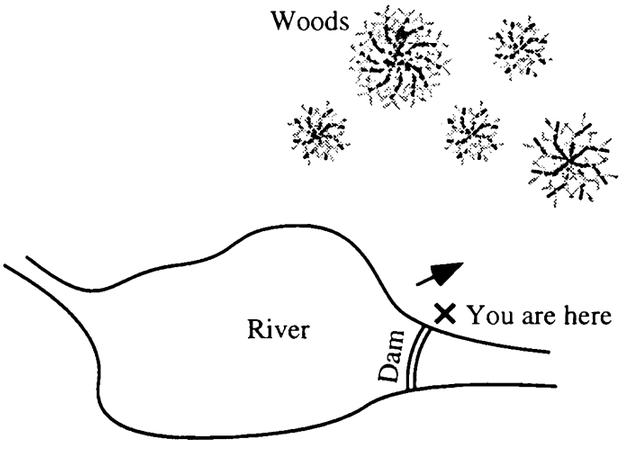
The compass points in a direction called *magnetic north.* The angle between magnetic north and true north is called the *magnetic declination.* Compass needles rarely point directly north–south. In Washington state, compass needles point about 22° to the right of true north. In Maine, they point about 15° to the left of true north. The value you obtained is good for your immediate region, say within a few hundred miles.

Exercise 4.8

Imagine that you are standing by a dam in a river as shown on the map at right. Your compass points in the direction shown by the arrow. You are looking for a cabin in the woods directly north of the dam.



A. Find the direction of true north on the map by making use of the compass heading and the correct magnetic declination for your region. Draw a line on the map to indicate true north.

B. Suppose that you had no compass. How could you determine the direction from the dam to the cabin?

✔ Check your reasoning with a staff member.

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Section 5. Phases of the moon

In this section, we begin to develop a model that we can use to account for the behavior of the moon. A scientific model is a set of rules for a particular system that makes it possible to explain and account for the behavior of the system. To be useful, your model should not only account for your observations but also enable you to predict the location and phase of the moon. The first step in building our model will be to incorporate the features that can account for the behavior we have already observed. As we learn more about the sun, moon, and stars we will add to our model or change it if we need to do so. At any time, we should be ready to discard any part of our model that is in conflict with our observations and that cannot be modified to be consistent with them.

Do not start this section until you have collected data for at least six to eight weeks.

Exercise 5.1

During the past few weeks, you and your classmates have been observing the moon. By combining all these observations, we can piece together a more complete description of the motion and appearance of the moon.

A. Together with your classmates create a "Moon Observation Summary Chart." On the chart, record the observations you and your classmates have made of the moon. All the observations for each day should be grouped together in your chart.

B. Examine the data in your chart and try to find patterns in the behavior of the moon. Discuss with your partner any features of the data that you find interesting or noteworthy.

Base your answers to the following questions on your moon observations.

C. During the course of a single day:

Does the appearance of the moon vary? If so, how? Support your answers by giving specific examples from your data.

Does the angle between the sun and the moon vary? If so, how does the angle change? Support your answers by giving specific examples from your data.

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D. During the entire period of your observations:

Does the appearance of the moon vary? If so, describe in your own words and with diagrams how the shape changes.

Does the angle between the sun and the moon vary? If so, how does the angle change? Support your answers by giving specific examples from your data.

E. Is there any repeating pattern to the way the shape of the moon changes? If so, describe the pattern.

Does the moon change abruptly from one shape to the next or is the change gradual?

Do you observe any pattern to the change in shape of the moon or do the shapes occur in random order? If there is a pattern, draw a sketch that illustrates the sequence of shapes.

F. Were there any days when you or your classmates did not see the moon at any time during the day or night, even though the sky was clear?

Discuss with your partner whether you should include a step in your sequence to represent the times when the moon was not visible.

Exercise 5.2

Pick a day at the beginning of your observations and call it "day zero." Number the subsequent days sequentially, including days for which you have no data.

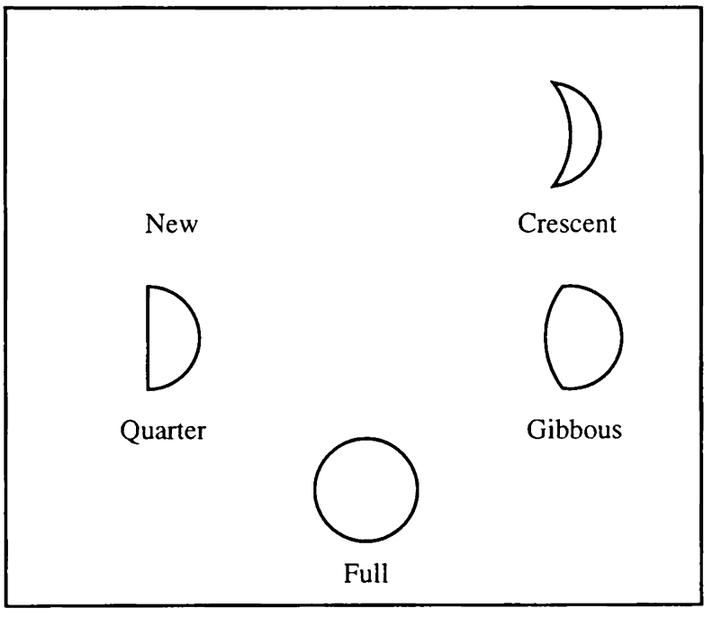
A. Choose a day on which the moon had a distinctive shape. As carefully as possible, estimate the days on which this same shape recurred. Record the shape and day numbers in the first row of a table like the one below. Include more columns in the table if necessary.

Shape of the moon	Days on which the shape was observed		
	Day	Day	Day

B. Repeat part A for several different shapes of the moon.

C. Are there always approximately the same number of days between successive observations of a particular shape of the moon? If so, roughly how much time passes before a given shape reappears? Is this interval the same for all the shapes?

The diagram below shows the names given to the various shapes or *phases* of the moon. During the *new moon,* we do not see the moon at all. As you have discovered, the moon goes through a cycle from the new moon, through the full moon and back to the new moon in about 30 days. The duration of this cycle is called the *synodic period* of the moon. When the bright part of the moon is growing, we say that the moon is *waxing.* When the bright part is shrinking, we say the moon is *waning.*



There is no special time at which the cycle of phases begins. By tradition, however, the new moon is always considered to be the beginning of the cycle of phases. The waxing quarter moon is usually called the *first quarter* moon, the waning quarter moon is usually called the *third quarter* moon.

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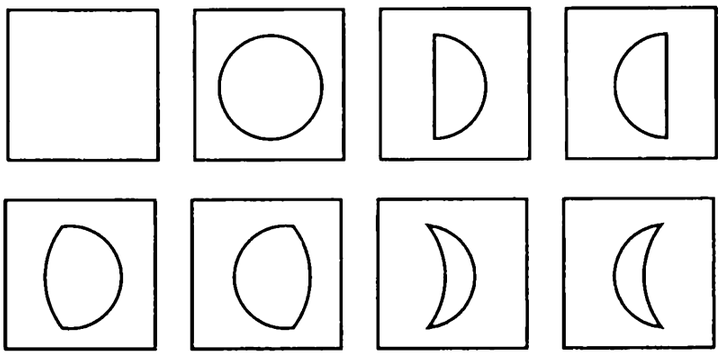
Exercise 5.3

The figure below shows several phases of the moon. Draw a sketch that shows these phases in the same order as you observed them during the course of a month. Label each phase according to the convention described in the preceding text (e.g., waxing gibbous moon, waning crescent moon, and so on).



Exercise 5.4

Base your answers to the following questions on your moon observations.

A. Are the sun and the moon ever in the sky at the same time?

Where is the sun relative to the moon:

- when the moon is full?
- when the moon is in the first quarter phase?
- when the moon is in the third quarter phase?

Support your answers by giving examples from your observations.

B. Consider your observations of the sky at noon during each of the following phases of the moon:

- the first quarter phase
- the full moon
- the third quarter phase

During each of these three phases, was the moon in the sky at noon? If so, in what part of the sky was the moon? Support your answers by giving specific examples from your data.

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C. When the moon has a particular shape, is the angle between the sun and the moon always the same? Illustrate your answer with specific examples from your data.

What is the phase of the moon right now? On the basis of your data, predict when you could look into the sky to see the moon and where in the sky it would be located at that time.

Based on your data, can you tell where the new moon is located in the sky relative to the sun? Explain your reasoning.

Experiment 5.5

Darken the room and place a bright light bulb at about eye level. Hold a ball in front of you. The bulb represents the sun, the ball represents the moon, and your head is the earth.

A. Move the ball around your head and note that the appearance of the lit portion of the ball changes in a way that is reminiscent of the phases of the moon.

How much of the ball is actually lit at any time? How is your answer to this question consistent with your observation that the "phase" of the ball changes as you move it around your head?

Can you reproduce all the phases of the moon in their correct order using the ball and light bulb? If so, should you move the ball clockwise or counterclockwise around your head?

B. In light of your observations in part A, come up with a plausible explanation for the phases of the moon. Discuss your answer with your partner.

Is the answer you give the only possible way to account for the phases of the moon? Try to suggest another explanation that can also account for the observed phases of the moon.

C. In light of the results of this experiment, discuss the following statement made by a student:

"The phases of the moon are caused by the shadow of the earth."

✔ Check your reasoning with a staff member.

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As you have just seen, one possible explanation for the observed phases of the moon is that they are the result of sunlight being reflected off the surface of the moon. We do not claim to have "proved" that this is really what happens, just that this model can account for some of our observations of the phases of the moon. If this model is to be useful, it should account for other observations we have made and allow us to predict future events.

Experiment 5.6

For this experiment, you will need your Moon Observation Summary Chart.

A. Look in your summary chart for each of the recorded values of the angle between the sun and the moon. In each case, note:

- the approximate phase of the moon,
- the approximate angle between the sun and the moon, and
- the number of days since the last new moon.

Organize the data in a table like the one below:

Phase of the moon	Angle between sun and moon	Number of days since new moon

B. Answer the following questions based on the data in the table above:

How does the angle between the sun and the moon change with time? Describe the way in which the angle varies during one synodic period of the moon.

Note: It is conventional to regard the angle between the sun and moon as varying from 0° to 360° during one synodic period of the moon. Change the angles in the table above so they are consistent with this way of recording the angle.

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C. Make a graph of the angle between the sun and the moon versus the number of days since the previous new moon. On the horizontal axis, indicate the day of each of the following phases: the new moon, the first quarter moon, the full moon and the third quarter moon. Sketch the phases underneath the appropriate days on the horizontal axis.

What does your graph suggest about the angle between the sun and the moon when the moon is full?

Experiment 5.7

For this experiment, you will need the light bulb and ball from Experiment 5.5.

A. Move the ball around you so that all the phases of the moon are reproduced.

For each of the phases listed below, estimate the angle between your line of sight to the ball and your line of sight to the bulb.

Record your measurements in a table like the one below.

Phase	Angle
New	_____
Waxing crescent	_____
First quarter	_____
Waxing gibbous	_____
Full	_____
Waning gibbous	_____
Last quarter	_____
Waning crescent	_____

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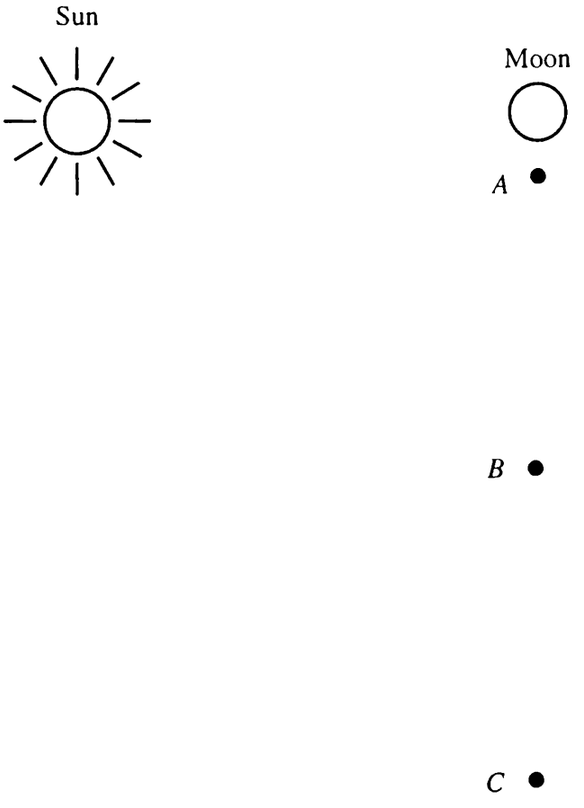
B. The diagram below shows the sun, the moon, and three locations: *A*, *B*, and *C*. The sizes of the sun and the moon are not drawn to scale.



Point A is much closer to the moon than the sun. For points A and B the relative distances to the sun and moon are more nearly equal.

Describe the phase of the moon that an observer at each point would see. Explain.

What approximate value would each observer measure for the angle between the lines of sight to the sun and moon?

C. Which of the three locations *A*, *B*, and *C* best corresponds to the actual relative distances between the earth, sun, and moon? Base your answer on your observations of the angle between the lines of sight to the sun and moon. Explain your reasoning.

What can you conclude about the relative *sizes* of the moon and sun? (*Hint:* Consider the observation that the sun and moon appear the same size in the sky.) Explain.

D. Are the angles that you measured in part A of this experiment consistent with the angles that you measured between the real moon and sun (see part A of Experiment 5.6)? If not, account for any discrepancies. If necessary, repeat part A of this experiment.

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E. Plot the data from the table in part A of this experiment as points on your graph from Experiment 5.6. Use a different color to distinguish these points from the original points. Compare the two plots. Is the model we have developed to account for the phases of the moon consistent with your observations?

✔ Check your reasoning with a staff member.

Experiment 5.8

A. Use the light bulb and ball to predict the approximate time at which different phases of the moon will rise and set.

Record your results in a table like the one shown below.

Phase	Rise time	Set time

B. Compare the data in your table with the observed rising and setting times of the moon.

During each of its phases, did the moon rise and set at the times listed above? If not, how can you account for any discrepancies?

✔ Check your reasoning with a staff member.

Exercise 5.9

Use the light bulb and ball to illustrate your answers to these questions.

A. Does the physical model we are using suggest how an eclipse of the sun might occur? If so, under what circumstances does an eclipse of the sun occur?

B. Does the physical model we are using suggest how an eclipse of the moon might occur? If so, under what circumstances does an eclipse of the moon occur?

C. Does the physical model we are using suggest whether the moon and the sun follow the same general path across the sky in the course of a day?

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Exercise 5.10

A student makes the following statement:

> *"I have observed the moon frequently and noticed that the same side of the moon always faces us on the earth. Thus, there is a 'dark side of the moon' that is always in shadow and never visible from earth."*

Do you agree with this student? Explain why or why not.

Experiment 5.11

Perform this experiment with at least two other people. One person should be designated the earth, the others should play the roles of the sun and the moon.

A. Suppose that the earth is stationary and does not rotate.

The people representing the sun and moon should stand to show the apparent locations of the sun and moon at local noon when the moon is new. They should then move to show the motion of the sun and moon during a single day (ending with local noon).

Where should the moon be located at noon the following day?

How do the motions of the sun and moon compare?

Describe how you would move to illustrate the motion of the sun and moon over the course of one synodic period of the moon.

What does this model for the motion of the sun and moon suggest about whether the angle between the sun and moon changes during the course of a single day? Explain.

B. Suppose that the sun is stationary and the earth and moon are free to move.

How should the people representing the earth and moon stand at noon when the moon is new?

The people representing the earth and moon should now move to show the passage of time through local noon the following day.

Describe how you would move to illustrate the motion of the earth and moon over the course of one synodic period of the moon.

C. The motions in parts A and B above represent two different physical models for the motion of the sun, earth, and moon. On the basis of your observations thus far, do you have any evidence for preferring one model over the other? Explain.

✔ Check your reasoning with a staff member.

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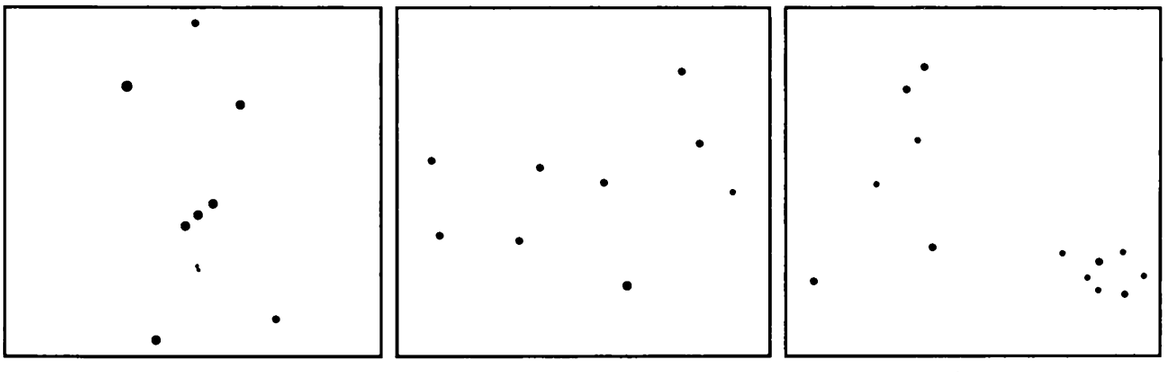
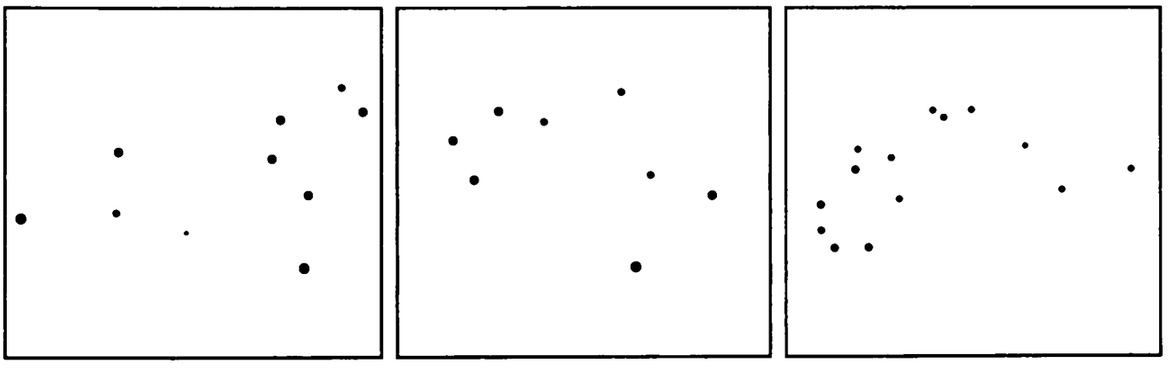
Section 6. Daily motion of the stars

Each individual star has its own brightness and color. Most stars seem faint, but some are quite bright. It is possible to become familiar with the brightest stars. Bright stars often have a slight color; some are noticeably blue or red. A star's color is part of its identity and is helpful in recognizing it.

Brightness and color, however, are not enough to let us identify all stars. The traditional way of recognizing an individual star is to remember its location relative to a pattern of other stars. Historically, the patterns were identified with people, animals, or other objects. Such patterns are called *constellations.* Below are several examples.



Orion Virgo Pisces



Leo Sagittarius Aquarius

In this section, you and your classmates will begin to study the motion of the stars in detail. Your instructor will organize a night session in which you and your classmates will make the observations described in Experiments 6.1 and 6.2. Be sure to take this module, a notebook, a flashlight, and a pencil or pen.

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Experiment 6.1

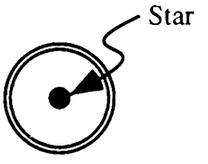
During the night session, several stars and constellations will be pointed out to you.

A. Record the names and locations (azimuth and altitude) of several of the constellations that you see in various parts of the sky. Also record the date and time of your observations. Include at least one star or constellation in the northern and in the southern parts of the sky. You will need these observations for a subsequent experiment.

B. Record the names and locations of three or more bright stars. Indicate the color of each. If you know the name of the constellations in which the stars lie, record that information also. In each case, record the date and time of your observation.

Experiment 6.2

During the night session, hollow tubes will be placed on stationary stands and aimed toward stars in different parts of the sky. Help your instructor arrange the tubes so that a star appears to lie directly in the center of each tube. (See diagram at right.)



Star centered in tube

A. Look through the tubes at various intervals during the evening.

   Describe your observations.

B. For each of the different sighting tubes illustrated on the following page, look through the tube about 15 minutes to one-half hour after the tube has been aimed at a star. For each tube:

   - Record the position of the star relative to the sighting tube in the second observation.
   - Describe whether the star is generally moving from east to west, west to east or neither.
   - State whether the star is rising, setting, or neither.

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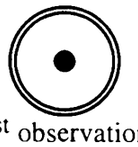
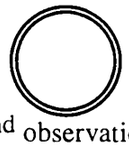
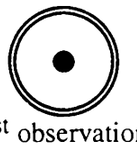
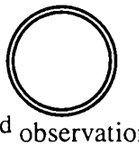
(1) Star low in the southeast


1st observation

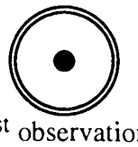

2nd observation

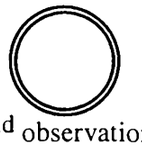
(2) Star high in the south


1st observation

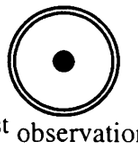

2nd observation

(3) Star low in the southwest


1st observation


2nd observation

(4) Star high in the northeast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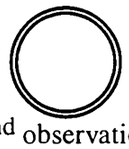
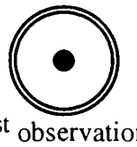

2nd observation

(5) Star low in the north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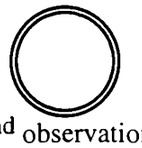
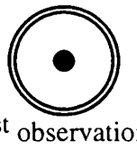
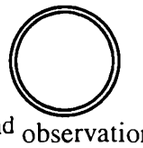
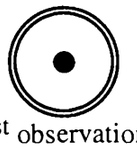

2nd observation

(6) Star high in the northwest


1st observation


2nd observation

(7) Polaris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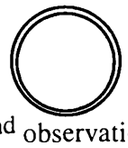
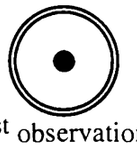
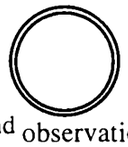

2nd observation

(8) Star high in the north


1st observation


2nd observation

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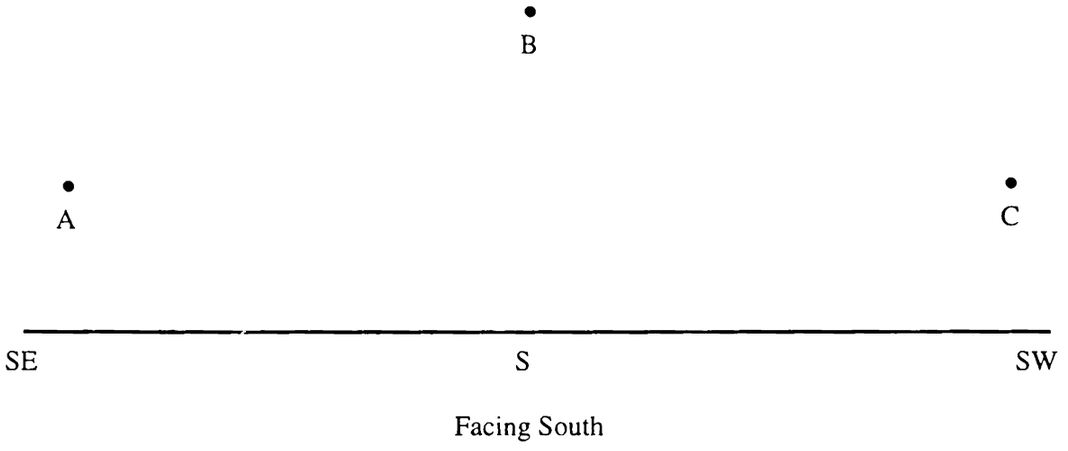
Exercise 6.3

A. The diagram below shows several stars in the southern sky. Draw an arrow on each star to show the direction in which it appears to move.

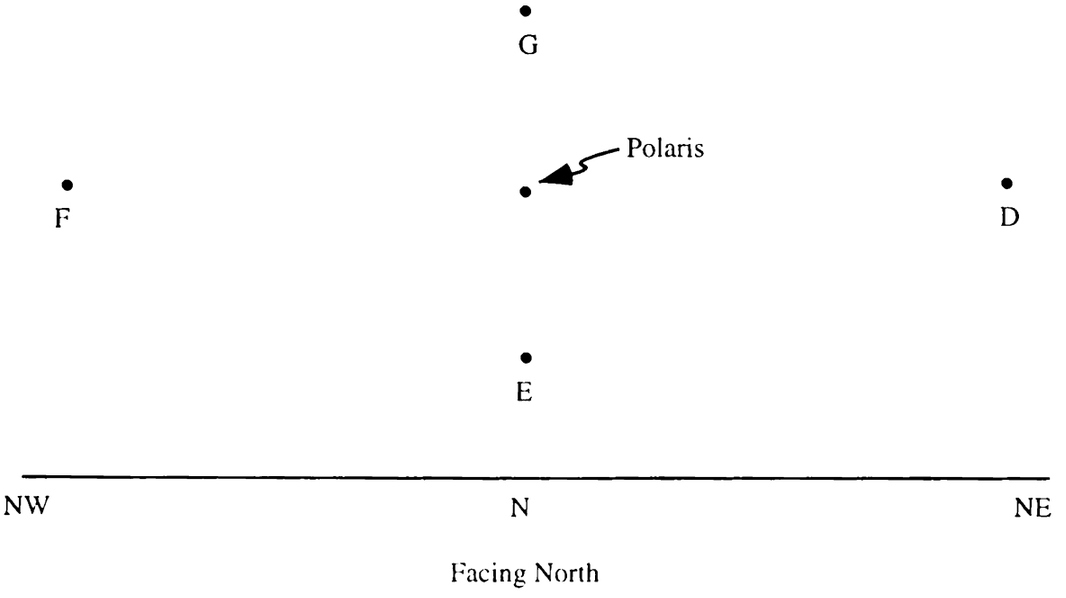
Describe the motion of stars in the southern part of the sky. How does their motion compare to the motion of the sun and the moon?



Facing South

B. The diagram below shows several stars in the northern sky. Draw an arrow on each star to show the direction in which it moves.

Describe the motion of the stars in the northern part of the sky. How does their motion compare with the motion of sun and the moon?



Facing North

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C. Discuss with your partner the apparent motion of each of the stars or constellations.

Are there any stars:

- That appear to move from east to west (in the same way as the sun)? If so, where in the sky are these star(s) located?
- That appear to move from west to east (in the opposite way as the sun)? If so, where in the sky are these star(s) located?
- That do not appear to move? If so, where in the sky are these star(s) located?

In Sections 4 and 5, you used a light bulb to represent the sun and your head to represent the earth. In this experiment, we will extend this model to include the stars.

Experiment 6.4

Place a light bulb in the center of the room to represent the sun. Obtain a ball to represent the earth, and a small figure to represent an observer on the earth at your location.

A. Hold the earth ball in such a way that it is approximately the same time for the observer that it was for you during your night observation session.

(1) Imagine yourself as the observer on the earth ball looking at Polaris.

Point in the direction, relative to the earth ball, that the observer should look to see Polaris. Note the point on the wall, ceiling, or floor at which you are pointing and place a piece of paper at that point. Write *Polaris* on the paper.

(2) Your instructor will choose a constellation that you observed at the night session. Repeat part 1 above for this constellation.

(3) Imagine that the observer on the earth ball is looking at a star near the northern horizon. Repeat part 1 above for this star.

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B. Suppose that the sun and stars are free to move but that the earth remains fixed.

   Describe how the light bulb sun must move so that the observer would see it move in the same way that you see the sun move.

   Describe how the "stars" that you marked in the room must move so the observer would see them move the same way you see the corresponding stars move.

C. In part B, your viewpoint was that the sun and stars moved while the earth remained fixed. Now suppose that the sun and the stars remain stationary, but the earth rotates.

   As you did in part A of Experiment 4.6, spin the "earth" slowly in a circle so that the observer would see the "sun" move from east to west.

   For the observer, do the "stars" you labeled in part A appear to move in the same way that the corresponding stars moved during your night observation session? If not, resolve any inconsistency.

D. In the model in part B, the sun and stars move while the earth remains fixed. In the model in part C, the sun and stars are stationary while the earth rotates.

   On the basis of the observations that you have made thus far, do you have any reason for preferring one model over the other?

✔ Discuss your reasoning with a staff member.

For an object that spins, the line around which it rotates is called the *axis of rotation.* Along the axis of rotation of the earth there are two points in the sky that do not appear to move. These are called the *north* and *south celestial poles.* (*Celestial* refers to the sky, *pole* means the axis of rotation of a sphere.) In the northern hemisphere, there is a star, Polaris, very nearly on the north celestial pole.

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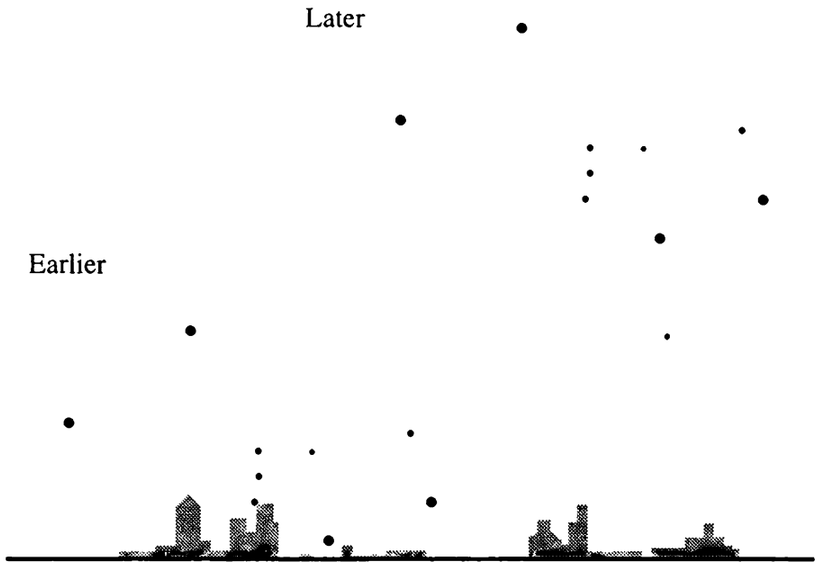
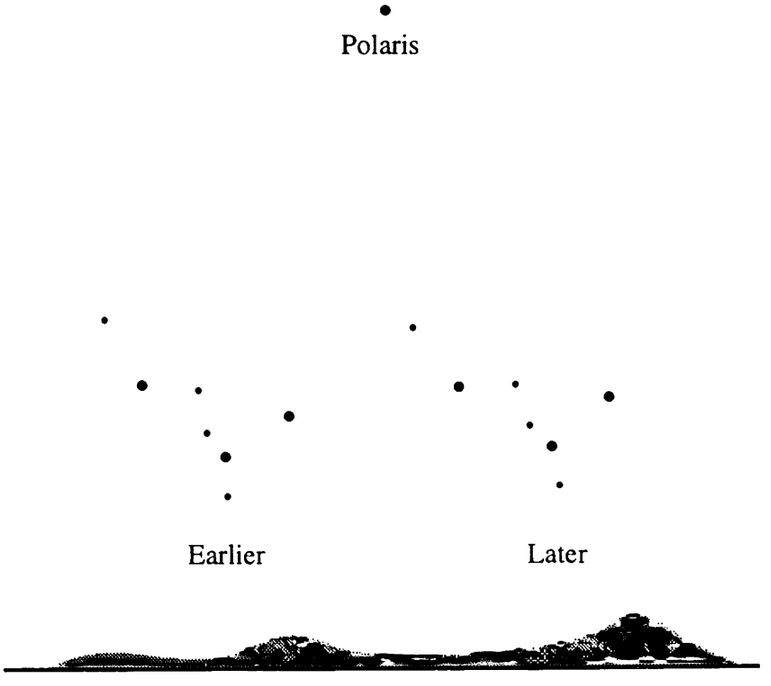
Exercise 6.5

In which direction(s) could a camera be pointed to take pictures like those below? Explain your reasoning.

A.



B.



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Exercise 6.6

A camera mounted on a tripod is aimed at the night sky in the northern hemisphere. The diagram below illustrates the picture obtained if the shutter is held open for a long period.

A. How can you account for the fact that the stars are streaks rather than dots in the photograph?

   What feature of the stars do you think is responsible for some streaks being thicker than others?

B. How would the photograph differ if the shutter were open longer? Explain.

   Estimate the time interval for which the shutter was open. Explain.

C. Which star do you think is located almost at the center of the photograph? Explain.

D. Is Polaris precisely at the north celestial pole? Explain your reasoning.

✔ Discuss your answers with a staff member.

Exercise 6.7

A. Does the altitude of a given star change during the night? Is your answer true for every star? Explain.

B. What is the azimuthal direction of a given star when its altitude is the greatest? Is your answer true for every star? Explain your reasoning.

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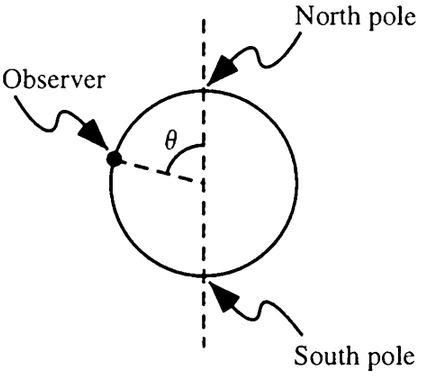
Section 7. Describing locations on the earth

In Section 6 we observed that there is a point in the sky around which the stars appear to move. This rotation allowed us to define a line through the earth: the axis of rotation. We call the two points of intersection of this line with the earth the *north* and *south poles* of the earth. In this section, we use these two points as part of a coordinate system that we can use to specify the locations of different points on the earth.

Exercise 7.1

The diagram at right shows an observer at an unknown location on the earth. The dashed line indicates the axis of rotation of the earth. The north and south poles are labeled.



A. Copy the diagram onto a large sheet of paper. Draw a line on your diagram to show the approximate direction in which the observer would look to see the north star (Polaris).

   Hint: Pick a point on the axis of rotation and imagine that Polaris is located at that point. Draw the line of sight from the observer to that point. However, Polaris is very far from the earth. How does the line of sight change as the point representing Polaris is moved away from the earth? As the distance increases, the line of sight approaches a particular direction. Indicate that direction on your diagram.

B. On your diagram, mark the angle α that represents the altitude that the observer would measure for Polaris. *Hint:* First draw the horizontal line of sight for the observer.

C. Find the angle θ that represents how far the observer is from the north pole. Express your answer in terms of the altitude of Polaris (α).

D. Use your measurements for the altitude of Polaris to find the angle θ for your particular location.

   Use your measurement for the circumference of the earth (Section 3) to determine how far you are from the north pole.

✔ Check your answers to this exercise with a staff member.

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The part of the earth's surface that lies at an angle of 90° from the north pole is called the *equator.* Rather than measuring angles from the north pole, as you did in the experiment above, it is conventional to specify a location by the angle from the equator. The angle from the equator is called the *latitude.*

Exercise 7.2

A. Determine the latitude of each of the following:

- the north pole
- the south pole
- the equator
- your current location. (Base your answer on your measurement of the altitude of the north star.)

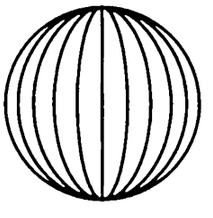
B. Obtain an earth globe that has latitude marked on it and check your answers to part A.

C. Find each of the following locations on the globe. What altitude would you measure for the north star from each place?

- the north pole
- the south pole
- the equator
- Cairo, Egypt

D. Is it possible for two different cities to have the same latitude? If not, explain why not. If so, give an example and tell what features the cities must share in common.

The latitude of a city is not enough to specify a particular spot on the earth. The latitude tells how far north or south a city is from the equator, but we still need to specify the relative east–west location.



We can imagine lines from the north pole to the south pole like those shown at right. These lines are called *meridians.* The meridian that passes through your location is called your *local meridian* or *local terrestrial meridian.* On globes, one often sees several meridians drawn in, but many others could be added.

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Experiment 7.3

A. Obtain a globe, a bright light bulb, and a small nail or tack. Hold the nail on the globe so that it makes a small gnomon at your location on the earth. Place the bulb far from the globe so that it acts like the sun. Watch the shadow of the nail as you rotate the globe.

When is the shadow the smallest? Does the shadow at this time point toward the north pole?

In Section 4, we defined north as the direction of the shortest shadow. In Section 7, the north pole was defined by the axis of rotation of the earth. Convince yourself that the shortest shadow does, in fact, point directly toward the north pole.

B. Hold the globe in such a way that it corresponds to the time at which the shortest shadow occurs.

What clock time at your location is represented by this arrangement?

Would this arrangement represent the time of the shortest shadow for observers at any other locations? If so, what do these locations have in common?

C. Arrange the globe so that it is local noon in New York City. Rotate the globe until it is local noon in Los Angeles. Estimate how much time elapses between these two events.

The earth is divided into different time zones. Explain why this is done.

Within a time zone, everyone uses the same clock time. Do all observers within the time zone agree upon the clock time that corresponds to local noon? Explain.

The abbreviation A.M., which we use to denote morning hours, is short for the Latin *ante meridiem.* (*Ante* means "before.") Similarly, P.M. stands for *post meridiem.* (*Post* means "after.") Meridiem is a form of the Latin word for *the middle of the day,* or *noon.* The Romans also used this same word to mean *south.* It is clear from the etymology of this word that the three ideas *noon, south,* and *meridian* are all very closely related.

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Longitude is a way of labeling meridians. There is no special place to start labeling; by tradition everyone starts with the meridian that passes through the old Greenwich Observatory in Greenwich (London), England. Other meridians are labeled by how many degrees they are east or west of Greenwich. For example, New York and Bogota are nearly on the same meridian. Their meridian is 74° west of Greenwich, so their longitude is 74°W.

Exercise 7.4

Sort the following list of cities into groups with approximately the same longitude. What is the longitude of each group?

Algiers, Algeria	Karachi, Pakistan	Nairobi, Kenya
Damascus, Syria	Lagos, Nigeria	Paris, France
Kabul, Afghanistan	Manila, Philippines	Shanghai, China

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Section 8. Celestial clocks and calendars

Throughout history, observations of the sky have played an important role. This is partly due to the importance of these observations in describing the passage of time. On a daily basis, human activity is greatly affected by the presence of daylight. On an annual basis, knowledge of the seasons is crucial in the growing of food. In this section we explore ways in which we can use our observations of the sky to measure time.

Experiment 8.1

A. For this experiment, use a photocopy of a shadow plot you have completed *within the last few days*. The time corresponding to each recorded shadow should be included on the plot.

   Tape the completed shadow plot onto a piece of cardboard. Place a toothpick at the location of the original gnomon and cut the toothpick so it is the same length as the gnomon.

      Describe how you could use this apparatus to tell the approximate time of day.

   Take your apparatus outside and predict the current time. Use a clock to check your prediction. Repeat this procedure at several different times during the day.

B. For this part of the experiment, you will need photocopies of shadow plots that span the past few months. The time corresponding to each recorded shadow should be included on each plot.

   (1) Tape each of the shadow plots onto a different piece of cardboard. Make a gnomon for each plot. Try to predict the current time of day from each shadow plot.

      Use a clock to check your predictions.

   (2) Repeat part 1 at several different times during the day.

C. Can you use the shadow plot from one day during the year to tell the approximate time of day on any other day during the year? Explain.

✔ Check your answers to this exercise with a staff member.

As you may have observed in the preceding experiment, the direction of a shadow at a particular time of day varies somewhat during the year. However, it is possible to ignore this effect to determine the approximate time of day. An instrument that uses the sun to

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tell the time of day is called a *sundial.* Sundials have been used for thousands of years. Most are based on the same basic principles as your sundial in the preceding experiment.

In the rest of this section we examine other ways to use the sky to tell time.

Experiment 8.2

For this exercise, you will need shadow plots that span several months.

A. Determine the altitude of the sun at local noon for each shadow plot.

   Is the altitude always the same? If not, does it change gradually from day to day or does it change haphazardly?

B. Predict how, if at all, the length of the shortest shadow will vary over a long period of time.

C. Ask a staff member for a set of shadow plots spanning several years. On graph paper, plot the length of the shortest shadow versus the date. Plot a long enough time period that you can see a pattern to the graph.

   Give an operational definition for a year.

   How could you use your graph to determine the number of days in a year?

D. Imagine that you were on a deserted island and had lost track of time. Describe how you could make a calendar that would allow you to determine the current date.

✔ Discuss this experiment with a staff member.

Exercise 8.3

Examine your observations of the sky over the past few months. Describe a method that you could use to determine the date from (1) observations of sunrise and sunset and (2) from observations of the stars.

In Volume I, we have developed models that we can use to account for the daily motion of the sun, moon, and stars, as well as the monthly changes of the moon. In Volume II, we extend our model so that we can account for changes that take place over a longer period of time.

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Supplementary problems for *Astronomy by Sight: the sun, moon, and stars*

Problem 1.1

For shadow plots from at least 3 different days, use the method from Experiment 1.9 to

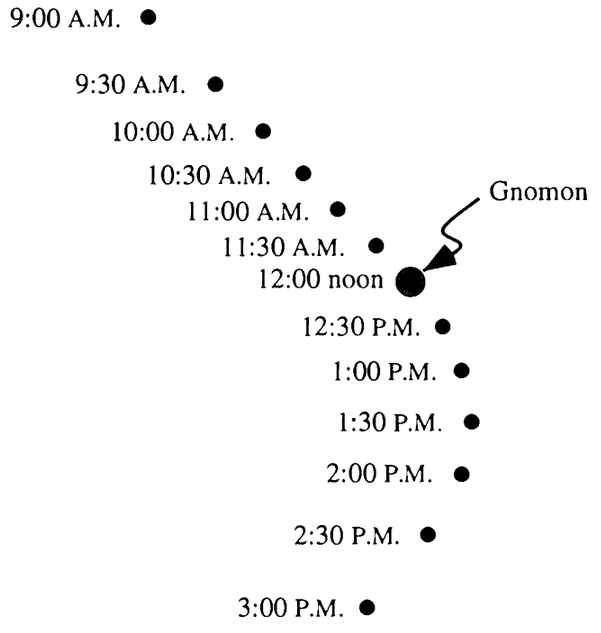
(1) determine the direction of the shortest shadow,

(2) determine the length of the shortest shadow, and

(3) estimate the time of the shortest shadow.

Problem 1.2

Consider the hypothetical shadow plot shown below. In what ways, if any, is this motion *inconsistent* with what you have observed so far? Explain.



Problem 2.1

Describe how one could use a board, a nail, a hammer, a sheet of paper, a ruler, a protractor, and a pencil to determine the altitude of the sun. In your description, you may use the word "perpendicular," but not "horizontal."

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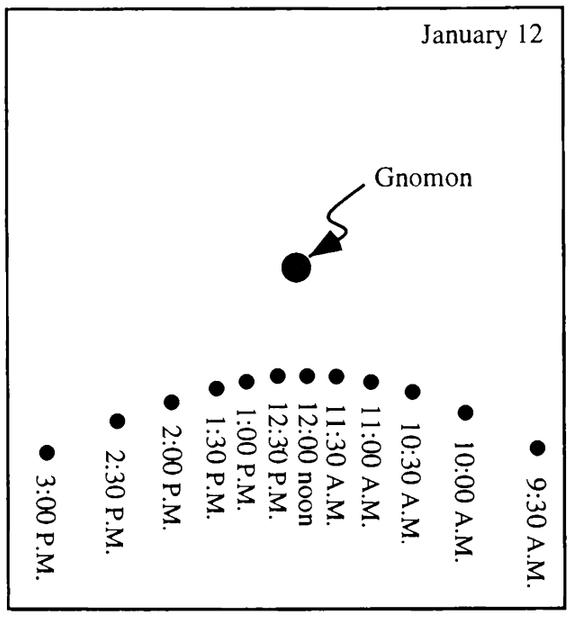
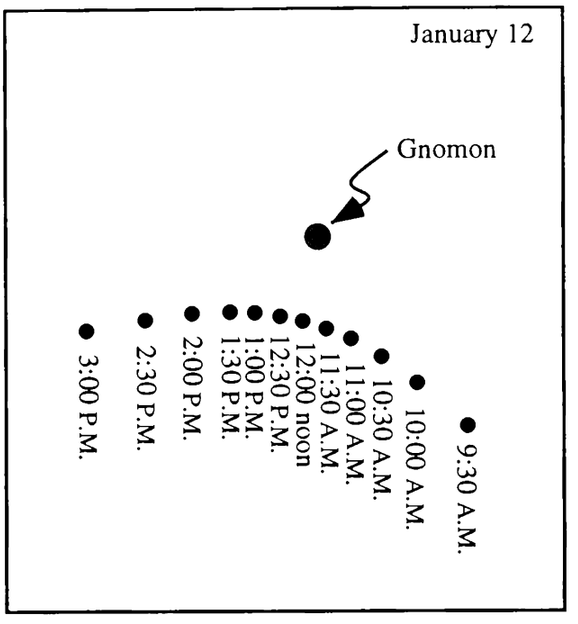
Problem 2.2

As a part of their astronomy homework assignment, two students, who happen to be neighbors, made shadow plots on the same day. Before turning in their assignments, the students compared their shadow plots, shown below.



First student's shadow plot



Second student's shadow plot

Upon seeing the differences in their shadow plots, the first student exclaimed,

> *"Your shadow plots looks wrong for two reasons: your shadows are too short, and your shortest shadow points in the wrong direction. How could that be?"*

Could these shadow plots have been made on the same day, only a few meters apart? If so, how can you account for both of these differences in the student's shadow plots? Explain your reasoning.

Problem 3.1

Two students who live on the equator want to estimate the circumference of the earth. The students synchronize their watches before one student leaves on vacation 600 miles due west of their home.

Describe an experiment that these students can do to estimate the circumference of the earth. In your description, you may use any of the terms that have been operationally defined in class up to this point. In your answer, include realistic results for each of the measurements you describe.

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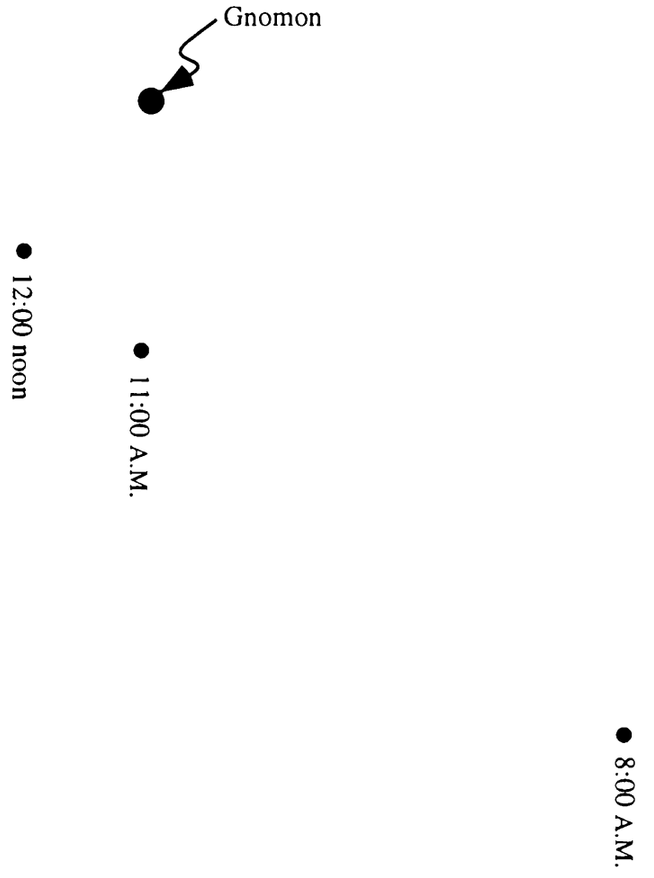
Problem 4.1

One day, the sun rose at 6:00 A.M., due east and set at 6:00 P.M., due west. That day, a student made a shadow plot using a gnomon that was 5.0 cm tall. The student, however, did not record many observations, as shown below.



A. How can you use the sparse data that the student recorded to infer the locations of the shadow of the tip of the gnomon at two times in the evening? Explain your reasoning.

B. Graph the *altitude* of the sun versus time of day *from sunrise until sunset.* Use graph paper, and clearly label your axes. Show how you determined the altitudes.

C. Graph the *azimuth* of the sun versus time of day *from sunrise until sunset* on a separate sheet of graph paper. Clearly label your axes. Explain how you determined the azimuths.

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Problem 4.2

A. Consider the model you developed in Experiment 4.6, where your head represents the earth and a light bulb represents the sun.

   (1) Make a top view diagram of your head and the bulb to show 3 P.M. for an observer on the earth.

   (2) Make a separate top view diagram of your head and the bulb to show 9 A.M. for the same observer.

B. Draw a diagram showing the sun and the earth. On the diagram, indicate the location of four observers: one for whom it is 3 A.M., one for whom it is 3 P.M., one for whom it is 9 A.M., and one for whom it is 9 P.M. Clearly label each observer with the corresponding time. Explain how you determined the locations of the observers.

C. Using your diagram from part B, estimate the distance (measured along the surface of the earth) between the observer for whom it is 3 A.M. and the observer for whom it is 9 A.M.

Problem 5.1

A physics student made an observation of the moon on a particular day. When she came to class to report the observation, she realized that she had not recorded the time of day for the observation. However, she had noted that the moon looked like the letter *C* and that she was facing due south and looking up and straight ahead to see the moon.

A. Approximately what time of day did she make her observation? Explain how you can tell.

B. Approximately what time did the moon rise that day? Approximately what time did it set? Explain how you determined your answers.

C. Suppose one week later the student looked for the moon again *at the same time of day.* Would she see the moon again? If so, roughly what direction would she have to look? If not, explain why not.

Problem 5.2

A. Approximately what time does a waxing crescent moon rise? Approximately what time does it set? Explain your reasoning.

B. Repeat part A for a full moon.

C. Repeat part A for a third quarter moon.

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Problem 5.3

You look outside and see a first quarter moon. Suppose that an astronaut were on the moon looking at the earth.

In answering the following questions, use clear diagrams to support your answers. Assume that the astronaut is able to see both the sun and the earth at all times.

A. Make a sketch of the earth as seen by the astronaut, clearly indicating which portion of the earth would be illuminated.

B. What angle would the astronaut measure between the sun and the earth? Explain your reasoning.

C. Suppose the astronaut were to look at the earth 6 hours later. How, if at all, would the illuminated portion of the earth appear different (as seen by the astronaut)?

D. Suppose the astronaut were, instead, to look at the earth 3 days later. How, if at all, would the illuminated portion of the earth appear different (as seen by the astronaut)?

Problem 5.4

A. You are up late one night doing your physics homework. The time is 3:00 A.M. You look out of your window and see the moon above the southern horizon.

   Answer the following questions, and in each case explain your reasoning.

   (1) What is the phase of the moon? Make a sketch to accompany your answer.

   (2) What is the approximate angle between the sun and the moon?

   (3) Approximately what time did the moon rise?

   (4) Approximately what time will the moon set?

   (5) Draw a diagram that shows the relative positions of the sun, the moon, and the earth. Clearly label your location on the earth as well as the illuminated portions of the earth and the moon.

B. You are working on your next homework assignment one week later. At 3:00 A.M., you remember seeing the moon the preceding week and decide to take another look.

   (1) Will you be able to see the moon in the same place in the sky again? Explain.

   (2) Answer the questions in part A for this new date.

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Problem 5.5

In the middle of the night, a student notices a quarter moon rising due east.

A. Did the student see a first quarter moon or a third quarter moon? Explain how you can tell.

B. Is length of time that the moon is above the horizon greater than, less than, or equal to the length of time that the sun is above the horizon? Explain your reasoning. (*Hint:* Is the phase of the moon *exactly* the same when the moon rises and when it sets?)

Problem 5.6

A. Suppose the sun and the moon were both approximately the same distance from the earth. Make a sketch that shows the relative locations of the sun, moon, and earth when a first quarter moon would be observed on earth. Approximately what angle would an observer on earth measure between the sun and moon in this case? You may assume that the sun, moon, and earth are all approximately the same size.

B. Suppose the sun and the earth were both the same distance from the moon. Make a sketch that shows the relative locations of the sun, moon, and earth when a first quarter moon would be observed on earth. Approximately what angle would an observer on earth measure between the sun and moon in this case?

Problem 6.1

Suppose that you are looking due north and see a star on the horizon. What is the maximum altitude the star will have? How many hours later will the star have that altitude? Explain your reasoning. (*Hint:* Describe the motion of the star and sketch a diagram that shows the motion.)

Problem 7.1

A sailor sets out to cross the Atlantic from Baltimore to Lisbon. The sailor has been attempting to travel due east because his point of departure and intended point of arrival are at the same latitude, 38° north. After some time at sea, he measures the altitude of Polaris and finds that it is 46°.

A. Explain how the sailor knew that he was not on course?

B. Did his boat drift northward or southward? Explain.

C. How far north or south is the sailor from 38° north latitude? Use your calculated value for the circumference of the earth in answering this question. Explain how you determined this distance.

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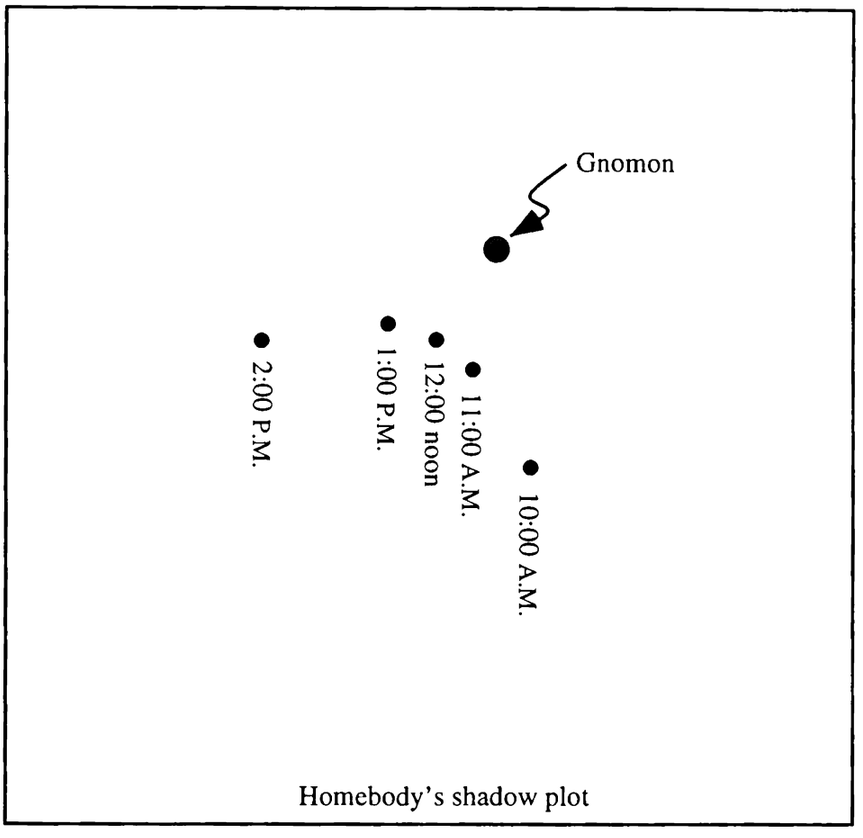
Problem 7.2

Two students synchronize their watches, then one student travels due east a distance corresponding to one time zone. The next day, each student records a shadow plot. The two shadow plots are taken with the paper in the same orientation relative to true north and the gnomon in the same part of each page.



Homebody's shadow plot

A. Describe how the general shape of the traveler's shadow plot will compare to the homebody's, which is shown above. Explain your reasoning.

B. Explain how the times recorded by the two students at the time of the shortest shadow would differ. (Remember, the students synchronized their watches before the traveler left.)

C. The homebody's gnomon is 1.5 cm tall. What would the homebody measure for the altitude of the sun at local noon? Would you expect that the traveler's measurement of the altitude of the sun at local noon would be greater than, less than, or equal to the homebody's measurement of the altitude of the sun at local noon? Explain.

D. Suppose the traveler used a gnomon that was twice as long as the homebody's. Using a color other than black, draw the traveler's shadow plot directly on top of the homebody's plot, including the times of the shadows. Explain the reasoning you used to determine the points that you drew on the second shadow plot.

E. Determine the altitude of the sun for the homebody and the traveler at local noon for the traveler. Explain how your results are consistent with your physical model.

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Appendix: Tables

Table of densities

Substance	Density (g/cm^3)	Substance	Density (g/cm^3)
Solids		*Liquids*	
Platinum	21.4	Mercury	13.60
Gold	19.3	Carbon Tetrachloride	1.59
Uranium	19.0	Saturated salt water	1.20
Tungsten	19	Ethanandiol (antifreeze)	1.11
Lead	11.3	Water	1.00
Silver	10.5	Liquid air (–193 °C)	0.87
Copper	8.9	Methanol	0.79
Nickel	8.7	Oils:	
Brass, yellow	8.5	crude	0.86 – 0.95
Iron	7.9	furnace oil	0.90
Tin, gray	5.8	kerosene	0.82
Titanium	4.5	gasoline	0.74
Aluminum	2.7		
Magnesium	1.7	*Gases*	
Magnetite	5	Chlorine	0.0030
Granite	2.7	Sulfur dioxide	0.0027
Glass	2.4 – 5.9	Carbon dioxide	0.0018
Solid Salt	2.2	Oxygen	0.0013
Plastics	0.8 – 1.2	Air	0.0012
Ice	0.92	Nitrogen	0.0012
Paraffin wax	0.9	Carbon monoxide	0.0012
Oak	0.6 – 0.9	Methane	0.00067
Bamboo	0.3 – 0.4	Helium	0.00017
Balsa	0.1	Hydrogen	0.000084

Note: All of the densities given in the table above are for 20° C and normal sea level pressure, except for the density of liquid air. All figures have been rounded off and are not exact. The units are g/cm^3.

Table of specific heats

Substance	Specific heat (cal/g/C°)	Substance	Specific heat (cal/g/C°)
Aluminum	0.21	Air	0.24
Copper	0.092	Water	1.00
Carbon	0.17	Methanol	0.60
Gold	0.031	Ammonia	1.13
Hydrogen	3.41	Salt	0.21
Iron	0.11	Brass	0.092
Lead	0.030	Granite	0.19
Magnesium	0.25	Marble	0.21
Mercury	0.033	Wood	0.42
Nickel	0.11	Glass	
Nitrogen	0.25	Flint	0.12
Oxygen	0.22	Crown	0.16
Silicon	0.17		
Silver	0.056		
Tin	0.052		
Zinc	0.092		

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Latent heats and boiling and melting points

Substance	Heat of fusion (cal/g)	Melting point (°C)	Heat of vaporization (cal/g)	Boiling point (°C)
Aluminum	94	660	2524	2057
Copper	49	1083	1146	2336
Carbon		3550		4200
Gold	16	1063	377	2600
Hydrogen			108	–253
Iron	64	1535	1516	3000
Lead	5.9	327		1621
Magnesium		651		1110
Mercury	3.0	–39	71	357
Nickel	56	1455		2900
Nitrogen	6.1	–210	48	–196
Oxygen	3.3	–218	51	–183
Silicon	337	1420		2355
Silver	26	961		1950
Tin	14	232		2270
Zinc	23	419		907
Water	80	0	540	100
Methanol	24	–98	239	65
Ammonia	108	–75	327	–33
Salt	124	804		1413
Brass		900		

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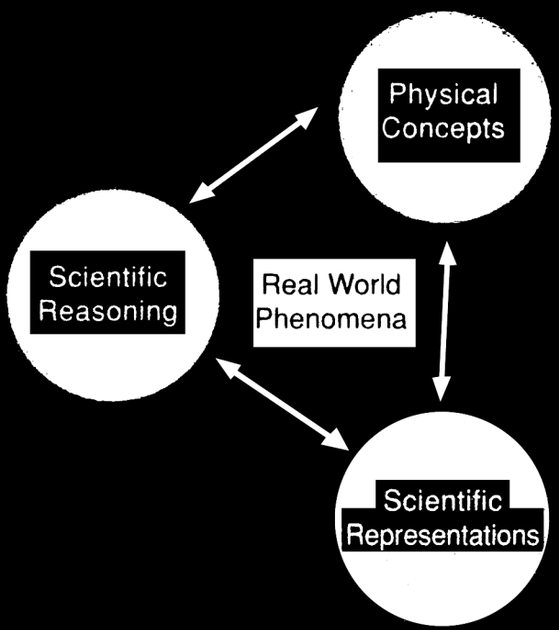

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